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A GENERAL THEORY OF TAX-SMOOTHING

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A general theory of tax-smoothing*

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Abstract

This paper extends the dynamic theory of optimal fiscal policy with a representative agent in several environments by using a generalized version of recursive preferences. I allow markets to be complete or incomplete and study optimal policy under commitment or discretion. The resulting theories are interpreted through the *excess burden of taxation*, a multiplier, whose evolution gives rise to different notions of "tax-smoothing." Variants of a law of motion in terms of the *inverse* excess burden emerge when we allow for richer asset pricing implications through recursive preferences. I highlight a common unifying principle of taxation and debt issuance in all environments that revolves around interest rate manipulation: issue new debt and tax more in the future if this can lead to lower interest rates today.

Keywords: Excess burden, tax smoothing, recursive utility, commitment, discretion, state-contingent debt, incomplete markets, martingale, fiscal hedging.

JEL classification: D80; E62; H21; H63.

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1 Introduction

The theory of normative fiscal policy revolves around the use of debt returns and distortionary taxes in order to maximize the utility of the representative household subject to financing some exogenous government expenditures. Environments can differ in terms of the type of debt that is available to the government (state-contingent or not), or in terms of the pricing of debt. Furthermore, optimal policy can be designed under different timing protocols that capture, for example, commitment or discretion. In all these setups, the government is always facing the question whether to tax *today* or issue debt and *postpone* tax distortions to the future. This basic question is in the heart of every dynamic fiscal policy problem.

This study emphasizes that the underlying principle behind several tax-smoothing environments is always the *same* and is captured by the optimal choice of debt and its associated *revenue*. The dynamic tradeoffs of issuing debt are always governed by an optimality condition that takes schematically the following form:

$$\text{(Average) Future Taxes} = \Phi \times \text{Marginal Revenue from Debt} \tag{1}$$

The left-hand side of (1) denotes the marginal *cost* of issuing new debt. New debt is costly because it has to be repaid with distortionary taxes, so the left-hand side is always a measure of the tax burden at a future date or state. The right-hand side of (1) denotes the marginal *benefit* of issuing debt. The marginal benefit depends naturally on how much additional revenue the government is raising. Selling debt for a particular date or state generates revenue that relaxes the government budget, leading to less taxes today. The shadow value of relaxing the government budget is captured by the multiplier $\Phi > 0$. This multiplier is called throughout the paper the *excess burden of taxation*, and serves as an indicator of tax distortions at the second-best.

Despite its apparent simplicity, equation (1) captures some basic economics. It gives the following prescription to the policy-maker: issue more debt and tax more tomorrow, if you can achieve higher marginal revenue from debt issuance today. Therefore, independent of the particular environment, we always know that if the planner can make debt effectively *cheaper*, he should issue more debt and tax more in the future.

The crucial element in (1) is the marginal revenue part. This depends on three factors: a) preferences, since they determine the pricing of debt through the stochastic discount factor; b) market structure, i.e. the degree of state-contingency of government debt; c) timing protocol: for example, commitment, that is, a setup where the policymaker commits to a plan designed at the initial period, or discretion, that is, a setup where the current policymaker optimizes every period, taking into account the fact that future policymakers will re-optimize without upholding old promises.

In the current paper I analyze *all* three aspects of the marginal revenue channel in (1). To illustrate the mechanisms I build a simple economy without capital, where a representative household works, pays distortionary labor taxes and invests in government securities. The government has to finance a stochastic stream of exogenous government expenditures.

I build a theory of taxes and debt that is based on realistic properties of debt returns and use a generalized version of recursive preferences to achieve that. Recursive preferences bring additional curvature in the pricing of future risks, by introducing aversion to future utility volatility, a fact which generates a higher market price of risk. This leads to a better match of asset-pricing facts, making recursive preferences the preferred choice in asset-pricing and macro-finance.[1] I allow an arbitrary concave certainty equivalent of continuation utility and perform the analysis of optimal policy under full generality. Time-additive utility is nested, since it corresponds to risk *neutrality* with respect to future utility risks. Whenever necessary, I consider as parametric examples a certainty equivalent that exhibits constant absolute risk aversion with respect to continuation utilities, constant relative risk aversion or the logarithmic case. This allows me to nest in my analysis, besides the standard time-additive case, cases like the preferences of Epstein and Zin (1989) and Weil (1990) (EZW henceforth), the preferences of Swanson (2018), and the risk-sensitive preferences of Hansen et al. (1999) and Tallarini (2000).

In terms of the other two determinants of the marginal revenue channel, I am *expansive*: I consider complete markets for government debt as in Lucas and Stokey (1983) or the case of non-contingent debt, as in Aiyagari et al. (2002). Furthermore, I consider *both* the case of commitment *and* the case of discretion, that is, the case of a Markov-perfect policymaker that keeps track only of the natural state variables and takes into account that future policymakers do the same. This is the notion of Markov-perfect policy in Krusell et al. (2004) and Klein et al. (2008).

Optimality condition (1) can be re-expressed in each of the respective four environments in terms of the excess burden of taxation. Consider first the case of commitment, and, to put the results into context, recall the time-additive and complete markers environment of Lucas and Stokey (1983). With time-additive utility, there is *no* curvature with respect to future utilities. As a result, the marginal revenue part in (1) is *constant*, making the planner to *equalize* the excess burden of taxation over states and dates. This result makes the labor tax effectively constant, leading optimally to *tax-smoothing*. There are no drifts in terms of tax rates and debt, and there is no endogenous persistence generated by optimal policy.

In contrast, when we turn to general recursive preferences, the tax-smoothing result *breaks down*, even if the *same* principle in (1) holds. The planner is manipulating the added sensitivity

[1] Indicative contributions from a voluminous literature are Tallarini (2000), Bansal and Yaron (2004), Piazzesi and Schneider (2007), Hansen et al. (2008), Gourio (2012), Rudebusch and Swanson (2012), Petrosky-Nadeau et al. (2013), Ai and Bansal (2018) and Ai et al. (2024).

of the stochastic discount factor with respect to the future to make debt cheaper, having typically an incentive to issue more state-contingent debt against good times of low fiscal shocks, and less state-contingent debt against bad times of high fiscal shocks. A law of motion for the *inverse* excess burden of taxation emerges, that holds for any certainty equivalent used. Tax rates are volatile, typically higher for good shocks and lower for bad shocks and there is high endogenous persistence. Moreover, there is a *positive* drift in the excess burden of taxation for all three parametric cases considered, imparting a positive drift in taxes and debt. So a general result about the optimal *back-loading* of tax distortions emerges, leading to the accumulation of high government debt in the long-run.

Turning to the case of commitment and non-contingent debt of Aiyagari et al. (2002), who endogenize the analysis of Barro (1979), the marginal revenue part in (1) with time-additive utility is again *constant*. The absence of sufficient markets in government debt, makes the planner "average" distortions over time. This leads to an excess burden (and therefore tax rate) that behaves similarly to a random walk. In contrast, with recursive preferences, the "averaging" of tax distortions over time *breaks down*. Optimality condition (1) translates into a law of motion in terms of the inverse *average* excess burden of taxation. Instead of "averaging" tax distortions, the planner has an incentive to issue more non-contingent debt for the future and to put more taxes on bad states of the world, to take advantage of the household's aversion to utility volatility. This action increases average marginal utility, *reducing* therefore interest rates.

When I drop the commitment assumption and turn to Markov-perfect policy, new forces emerge. Lack of commitment to policies designed in the past, generates an incentive for the current policymaker to *postpone* taxes and bequeath more state-contingent debt to the future policymaker. This policy increases future period marginal utility, since future consumption falls, and reduces rates. This force, due to period marginal utility, is the only one operating in the time-additive case. With recursive utility, the period marginal utility force is still present, but it is intertwined with the incentives to manipulate the additional curvature in the pricing kernel through continuation utilities. In particular, the current policymaker has an incentive to issue more (less) state-contingent debt and increase (decrease) taxes against good (bad) times. Thus, the incentives to manipulate period marginal utility and continuation utilities align with each other in good times of low government expenditure shocks, and oppose each other in bad times of high government expenditure shocks. In contrast, when debt is non-contingent, both the period utility and continuation utility channel lead to an incentive to postpone taxes to the future.

Related literature. The literature analyzing optimal fiscal policy with time-additive utility is vast. For the case of commitment and complete markets, the seminal contributions are Lucas and Stokey (1983), in an economy without capital, or Chari et al. (1994) and Zhu (1992), in economies with capital. The case of market incompleteness has been considered by Aiyagari et al. (2002) and Bhandari et al. (2017) in economies without capital, and Farhi (2010), in economies with capital.

Dropping the commitment assumption and following the Markov-perfect protocol of Klein et al. (2008), the main contribution in a deterministic economy is Krusell et al. (2004). Occhino (2012) considers stochastic setups with complete markets and government expenditures that provide utility.[2] When markets are incomplete, relevant studies are Martin (2009), who considers optimal time-consistent fiscal and monetary policy, and Karantounias and Valaitis (2024), who consider optimal time-consistent taxation and debt issuance with default.

Optimal fiscal policy with recursive preferences in setups with commitment and complete markets has been considered in Karantounias (2018), who used EZW utility. Relative to EZW utility, the use of a general certainty equivalent in this paper allows to demonstrate the generality of the law of motion of the inverse excess burden of taxation, the importance of the coefficient of absolute risk aversion with respect to continuation utilities for the manipulation of the pricing kernel, and the ubiquity of the optimal back-loading of tax distortions. Moreover, the current paper, besides emphasizing the unifying principle in (1), considers three additional important environments, for which *little* is known: commitment and incomplete markets and discretion with either complete or incomplete markets.

Organization. Section 2 lays out the two market structures I consider, that is, the complete markets economy of Lucas and Stokey (1983), and the incomplete markets economy of Aiyagari et al. (2002), and delves into the recursive preferences used. Section 3 analyzes the optimal policy problem with complete markets and commitment. Section 4 analyzes the respective problem with non-contingent debt. Section 5 drops the commitment assumption and treats the optimal policy design problem with either state-contingent or non-contingent debt. Section 6 concludes and an Appendix follows.

2 Economy

Time is discrete and the horizon is infinite. To make my points about optimal debt issuance and the determination of tax distortions over states and dates, I use an economy without capital and a representative household as in Lucas and Stokey (1983) and Aiyagari et al. (2002). Government expenditures are exogenous, stochastic, and live in a finite set.[3] Let g_t denote the spending shock at time t and let $g^t \equiv (g_0, g_1, ..., g_t)$ denote the partial history of shocks up to period t with probability $\pi_t(g^t)$. There is no uncertainty at $t = 0$, so $\pi_0(g_0) \equiv 1$. The operator E denotes

[2]See also Debortoli and Nunes (2013) for a deterministic setup with utility-providing government consumption.

[3]Introducing technology shocks is straightforward in this setup.

expectation with respect to π. Whenever I use a recursive formulation of the policy problem throughout the paper, I make the additional assumption that shocks are Markov with transition density $\pi(g'|g)$.

The resource constraint of the economy reads

$$c_t(g^t) + g_t = h_t(g^t), \tag{2}$$

where $c_t(g^t)$ consumption and $h_t(g^t)$ labor. The notation indicates the measurability of these functions with respect to the partial history g^t. Total endowment of time is normalized to unity, so leisure is $l_t(g^t) = 1 - h_t(g^t)$.

2.1 Preferences

The household is valuing stochastic streams of consumption and leisure using a recursive utility criterion of Kreps and Porteus (1978),

$$V_t = u(c_t, 1 - h_t) + \beta H^{-1}\big(E_t H(V_{t+1})\big), \tag{3}$$

where u an increasing and concave function of consumption and leisure, H an increasing and *concave* function of continuation utility (or continuation *value*- the two terms are used interchangeably), and $A(x) \equiv -H''(x)/H'(x)$ the respective coefficient of absolute risk aversion. For future reference, let $\epsilon_{cc} \equiv -u_{cc}c/u_c, \epsilon_{ch} \equiv u_{cl}h/u_c$ denote the own and cross elasticity of the period marginal utility of consumption, and let $\epsilon_{hh} \equiv -u_{ll}h/u_l, \epsilon_{hc} \equiv u_{cl}c/u_l$, denote the own and cross elasticity of the marginal disutility of labor.

Preferences (3) imply that the household exhibits *aversion* to volatility in continuation utilities. Current utility is the sum of period utility u and the *certainty equivalent* (CE) of continuation utility, $\mu_t \equiv H^{-1}\big(E_t H(V_{t+1})\big)$. To preserve concavity of the utility recursion, I further assume that the certainty equivalent μ_t is a *concave* function of future utilities, V_{t+1}.[4] Time-additive utility is nested in specification (3) by assuming a linear H, $H(x) = x$. In that case, the household exhibits risk *neutrality* with respect to continuation utility risks, and the certainty equivalent reduces to expected future utility, $\mu_t = E_t V_{t+1}$.[5]

Consider now the stochastic discount factor (SDF henceforth) that corresponds to the preferences in (3). We have[6]

[4] A sufficient condition for that is that absolute risk tolerance, $-H'(x)/H''(x)$, is a weekly concave function. See Gollier (2004, p. 322).

[5] See Backus et al. (2004) for an expansive survey on dynamic preferences over uncertainty.

[6] We have $\partial V_t/\partial c_{t+1} = \beta\pi_{t+1}(g_{t+1}|g^t)H^{-1\prime}(E_t H(V_{t+1}))H'(V_{t+1})\frac{\partial V_{t+1}}{\partial c_{t+1}}$ and $\frac{\partial V_t}{\partial c_t} = u_{ct}$. Recall that

$$S_{t+1} = \beta m_{t+1} \frac{u_{c,t+1}}{u_{ct}}, \quad \text{where} \quad m_{t+1} \equiv \frac{H'(V_{t+1})}{H'(\mu_t)} > 0. \tag{4}$$

Due to the fact that the household dislikes variation not only in consumption, but also in continuation utilities, the *total* marginal utility of an increase in future consumption c_{t+1}, $\partial V_t/\partial c_{t+1}$, and therefore, the SDF, has *two* components: a) the *period* marginal utility of consumption, $u_{c,t+1}$, and b) the *scaled* marginal utility from *continuation* value, m_{t+1}, that is, the marginal utility of V_{t+1}, $H'(V_{t+1})$, divided by the marginal utility of the certainty equivalent of continuation values, $H'(\mu_t)$. Clearly, in the time-additive, or in the deterministic, case we have $m_{t+1} \equiv 1$. Moreover, for some parametric cases of H, the scaled marginal utility of continuation values m_{t+1} is associated with a *change of measure*, as we will shortly see.

The two components in the SDF capture respectively aversion to future consumption risk, and aversion to future continuation value risk. I conduct the entire analysis throughout the paper with the preferences in (3), without assuming a particular specification of H. Whenever I need concrete illustrations, I use three parametric examples.[7]

Constant absolute risk aversion (CARA). Assume that H is an exponential function,

$$H(x) = -A^{-1} \exp(-Ax), A > 0. \tag{5}$$

This function delivers constant absolute risk aversion with respect to continuation utility risks, $A(x) = A$. The certainty equivalent μ_t and the scaled marginal utility of continuation value m_{t+1} are respectively

$$\mu_t = -A^{-1} \ln E_t \exp(-AV_{t+1}) \quad \text{and} \quad m_{t+1} = \frac{\exp(-AV_{t+1})}{E_t \exp(-AV_{t+1})}. \tag{6}$$

The constant absolute risk aversion case is of particular interest because the random variable m_{t+1} can be interpreted as a conditional likelihood ratio, or a change of measure, since $m_{t+1} \geq 0$ and $E_t m_{t+1} = 1$. I refer to the induced probability measure as the *continuation-value* adjusted measure, $\pi_t \cdot M_t$, where $M_t \equiv \prod_{i=0}^{t} m_i, m_0 \equiv 1$.

Specification (5) is associated with various familiar cases in the literature. First, it corre-

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$H^{-1\prime}(x) = 1/H'(H^{-1}(x))$. Thus, $H^{-1\prime}(E_t H(V_{t+1})) = 1/H'(\mu_t)$ and $\partial V_t/\partial c_{t+1}$ becomes $\partial V_t/\partial c_{t+1} = \beta \pi_{t+1}(g_{t+1}|g^t) m_{t+1} u_{c,t+1}$. Consequently, the marginal rate of intertemporal substitution is $\frac{\partial V_t/\partial c_{t+1}}{\partial V_t/\partial c_t} = \pi_{t+1}(g_{t+1}|g^t) S_{t+1}$, where S_{t+1} given by (4).

[7] All parametric examples furnish a concave certainty equivalent, since absolute risk tolerance is either constant or linear (see footnote 4).

sponds to the risk-sensitive preferences of Hansen et al. (1999) and Tallarini (2000).[8] Furthermore, if we make the assumption that the period utility is logarithmic in the composite good of consumption and leisure, then the case of an exponential H can be reinterpreted as the case of Epstein and Zin (1989) and Weil (1990) preferences with unitary intertemporal elasticity of substitution.

Constant relative risk aversion (CRRA). Assume that $u > 0$ and let H be a power function,

$$H(x) = \frac{x^{1-\alpha} - 1}{1-\alpha}, x > 0, \tag{7}$$

where $\alpha > 0$ and $\alpha \neq 1$. We have $A(x) = \alpha/x$ and the respective CE and m_{t+1} take the form

$$\mu_t = \left(E_t V_{t+1}^{1-\alpha}\right)^{\frac{1}{1-\alpha}} \quad \text{and} \quad m_{t+1} = \left(\frac{V_{t+1}}{\mu_t}\right)^{-\alpha} = \kappa_{t+1}^{-\frac{\alpha}{1-\alpha}}, \tag{8}$$

where $\kappa_{t+1} \equiv \frac{V_{t+1}^{1-\alpha}}{E_t V_{t+1}^{1-\alpha}} > 0$. Note that $E_t \kappa_{t+1} = 1$, so κ_{t+1} defines again a change of measure that applies a continuation-value adjustment, with an induced measure $\pi_t \cdot K_t$, $K_t \equiv \prod_{i=0}^{t} \kappa_i$, $\kappa_0 \equiv 1$. So when aversion towards future risk is expressed with a power function, m_{t+1} corresponds to a conditional likelihood ratio raised in the power $-\alpha/(1-\alpha)$. The power case corresponds to the preferences used by Swanson (2018). No additional assumptions on the shape of u are made, besides the requirement that $u > 0$.[9] If we restrict the period utility u to also take a power form, then we have the case of EZW utility.[10]

Logarithmic case. Assume that $u > 0$. Consider the logarithmic function

$$H(x) = \ln(x), \tag{9}$$

which corresponds to the case of $\alpha = 1$ in (7). The certainty equivalent becomes $\mu_t = \exp(E_t \ln V_{t+1})$. Define $v_t \equiv \ln V_t$ and express the utility recursion as

[8] There is an alternative interpretation of risk-sensitivity in terms of the multiplier preferences of Hansen and Sargent (2001), which are designed to capture fear of model misspecification, and the household's desire for robust decision rules.

[9] The case of $u < 0$ can be treated in a similar way by using the increasing and concave function $H(x) = -\frac{(-x)^{1+\gamma}}{1+\gamma}, x < 0, \gamma > 0$. We have $A(x) = \gamma/(-x)$, $\mu_t = -\left(E_t(-V_{t+1})^{1+\gamma}\right)^{\frac{1}{1+\gamma}}$ and $m_{t+1} = \frac{(-V_{t+1})^{\gamma}}{(-\mu_t)^{\gamma}} = \kappa_{t+1}^{\frac{\gamma}{1+\gamma}}$, where $\kappa_{t+1} \equiv \frac{(-V_{t+1})^{1+\gamma}}{E_t(-V_{t+1})^{1+\gamma}} > 0$, a change of measure again since $E_t \kappa_{t+1} = 1$.

[10] See for example Swanson (2018).

$$v_t = \ln\big[u_t + \beta \exp(E_t v_{t+1})\big]. \tag{10}$$

Similarly, the scaled marginal utility of continuation value becomes

$$m_{t+1} = \Big(\frac{V_{t+1}}{\exp(E_t \ln V_{t+1})}\Big)^{-1} = \exp\Big(-(v_{t+1} - E_t v_{t+1})\Big), \tag{11}$$

so $E_t \ln m_{t+1} = 0$.[11]

Continuation utilities and the SDF. The concavity of the certainty equivalent μ_t in continuation utilities implies that an increase in V_{t+1} reduces m_{t+1}, and therefore, it decreases the SDF, all else equal.[12] Thus, increases in continuation values act as increases in future consumption; they reduce the price of state-contingent claims.

2.2 Market structure and government policy

Complete markets. Consider an environment with complete markets as in Lucas and Stokey (1983). There are no lump-sum taxes and the government resorts to a linear labor tax $\tau_t(g^t)$ in order to finance government expenditures. The representative household consumes, works at the pre-tax wage rate $w_t(g^t)$, and trades in a full set of Arrow securities with the government. These securities have price $p_t(g_{t+1}, g^t)$ and promise one unit of consumption if the state next period is g_{t+1} and zero otherwise. The household maximizes utility (3) subject to the budget constraint,

$$c_t(g^t) + \sum_{g_{t+1}} p_t(g_{t+1}, g^t) b_{t+1}(g^{t+1}) = (1 - \tau_t(g^t)) w_t(g^t) h_t(g^t) + b_t(g^t), \tag{12}$$

and the feasibility conditions on consumption and labor, $c_t \geq 0, h_t \in [0,1]$. The household is also subject to a typical no-Ponzi game condition and starts with an initial debt b_0. The government's dynamic budget constraint reads

[11]Note that m_{t+1} cannot be interpreted in the logarithmic case as a change of measure, since $E_t m_{t+1} = E_t \exp(\ln m_{t+1}) > \exp(E_t \ln m_{t+1}) = 1$, due to the convexity of the exponential function.

[12]To see that, note that $\frac{\partial \mu_t}{\partial V_{t+1}} = H^{-1\prime}(E_t(H(V_{t+1})))\pi_{t+1}(g_{t+1}|g^t)H'(V_{t+1}) = \pi_{t+1}(g_{t+1}|g^t)\frac{H'(V_{t+1})}{H'(\mu_t)} = \pi_{t+1}(g_{t+1}|g^t)m_{t+1}$. Consequently, $\frac{\partial^2 \mu_t}{\partial V_{t+1}^2} = \pi_{t+1}(g_{t+1}|g^t)\frac{\partial m_{t+1}}{\partial V_{t+1}} \leq 0$, due to the concavity of μ_t. Therefore, $\frac{\partial m_{t+1}}{\partial V_{t+1}} \leq 0$.

$$b_t(g^t) = \tau_t(g^t)w_t(g^t)h_t(g^t) - g_t + \sum_{g_{t+1}} p_t(g_{t+1}, g^t)b_{t+1}(g^{t+1}). \tag{13}$$

Incomplete markets. Consider an environment with incomplete markets as in Aiyagari et al. (2002), where the government can issue only non-contingent debt. In particular, the government issues a risk-free discount bond, which promises one unit of consumption for each realization of the shock next period, and trades at price $q_t(g^t)$. The household's budget constraint reads

$$c_t(g^t) + q_t(g^t)b_t(g^t) = (1 - \tau_t(g^t))w_t(g^t)h_t(g^t) + b_{t-1}(g^{t-1}). \tag{14}$$

Note that $b_t(g^t)$ indicates now the holdings of government debt at the beginning of period $t+1$, for each realization of the shock at $t+1$. The level of initial debt is b_{-1} and, as before, $c_t \geq 0, h_t \in [0,1]$. The household is also subject to individual borrowing constraints that will be assumed to be non-binding, so that we focus on an interior solution of the household's problem. Similarly, the government's budget constraint is

$$b_{t-1}(g^{t-1}) = \tau_t(g^t)w_t(g^t)h_t(g^t) - g_t + q_t(g^t)b_t(g^t). \tag{15}$$

2.3 Equilibrium and optimality conditions

Government policy is summarized by a stochastic process for taxes and debt $\{\tau, b\}$, where debt is either state-contingent or non-contingent.

Definition 1. *("Complete markets") A competitive equilibrium with taxes is a policy of state-contingent taxes and state-contingent debt $\{\tau, b\}$, prices of Arrow securities $\{p\}$, wages $\{w\}$, and an allocation $\{c, h, b\}$ such that a) Given $\{\tau\}$ and $\{p, w\}$, $\{c, h, b\}$ solves the household's maximization problem; b) given $\{w\}$, firms maximize profits; c) markets clear, that is, the resource constraint (2) holds.*

Definition 2. *("Incomplete markets") A competitive equilibrium with taxes is a policy of state-contingent taxes and non-contingent debt $\{\tau, b\}$, prices of non-contingent debt $\{q\}$, wages $\{w\}$, and an allocation $\{c, h, b\}$ such that a) Given $\{\tau\}$ and $\{q, w\}$, $\{c, h, b\}$ solves the household's maximization problem; b) given $\{w\}$, firms maximize profits; c) markets clear, that is, the resource constraint (2) holds.*

Note that given the household's budget and the resource constraint (2), the government's budget constraint is satisfied, which is why we did not include it in the definition of the equilibrium.

Optimality conditions. Given the linear production function and the competitive labor markets, profit maximization implies $w_t(g^t) = 1, \forall t, g^t$. Furthermore, for both market structures, the household's labor supply condition equalizes the marginal rate of substitution between consumption and leisure to the after-tax wage,

$$\frac{u_l(g^t)}{u_c(g^t)} = 1 - \tau_t(g^t). \tag{16}$$

In the complete market case, the portfolio of Arrow securities is determined by the condition

$$p_t(g_{t+1}, g^t) = \pi_{t+1}(g_{t+1}|g^t) S_{t+1}(g^{t+1}), \tag{17}$$

where S_{t+1} the SDF in (4). Condition (17) equalizes the price of a state-contingent claim to the marginal rate of intertemporal substitution. Instead, with non-contingent debt we have

$$q_t(g^t) = \sum_{g_{t+1}} \pi_{t+1}(g_{t+1}|g^t) S_{t+1}(g^{t+1}), \tag{18}$$

which equalizes the price of risk-free debt to the average SDF.

These optimality conditions, together with the resource constraint (2), the household's budget and the respective transversality conditions characterize fully the competitive equilibrium.

Revenue from debt issuance. As a prelude to the policy problem, consider the problem of a benevolent planner that chooses distortionary taxes and debt so that the utility of the representative household is maximized and government expenditures are financed. What matters in this decision is the current tax rate versus the revenue that the planner can raise by issuing new debt. In the complete market setup, debt revenue is captured by the market value of the government portfolio, $\sum_{g_{t+1}} p_t(g_{t+1}, g^t) b_{t+1}(g^{t+1})$. With non-contingent debt, the proper object is $q_t(g^t) b_t(g^t)$. The planner is a large player who takes into account how his decisions affect debt revenue through both the direct effects of larger debt positions, and the indirect *pricing* effects. For both market structures, the revenue raised will depend a) on the SDF S_{t+1}, which entails continuation utilities, and b) on the timing protocol, that is, on assumptions on the commitment ability of the policymaker. Different timing protocols lead to different interest rate manipulation incentives through the SDF. I start first with the case of commitment.

3 Optimal policy with complete markets under commitment

Consider a policymaker that chooses tax rates to maximize that utility of the representative household at $t = 0$ and commits to this policy. So the planner chooses a stochastic process for τ and b subject to the resource constraint, the budget constraints and the optimality conditions coming from the competitive equilibrium. I follow the primal approach of Lucas and Stokey (1983) and eliminate the tax rate and Arrow securities prices from (12) by using the optimality conditions (16) and (17). This allows me to express the budget constraint of the household in terms of allocations $\{c, h, b\}$ and continuation utilities $\{V_{t+1}\}$,

$$u_{ct}b_t = \underbrace{u_{ct}c_t - u_{lt}h_t}_{\text{primary surplus}} + \underbrace{\beta E_t m_{t+1} u_{c,t+1} b_{t+1}}_{\text{market value of gov. portfolio}}, \tag{19}$$

where continuation utilities V_{t+1} follow recursion (3) and affect (19) through $m_{t+1} = H'(V_{t+1})/H'(\mu_t)$. Equation (19) denotes the dynamic *implementability* constraint, that is, the constraint that allocations have to satisfy so that they are implemented as a competitive equilibrium with taxes. In all environments I consider, the term $u_c c - u_l h$ denotes consumption net of after-tax income in marginal utility units. This term is equal in equilibrium to the *primary surplus* in marginal utility units. Thus, we may think of (19) as the government budget (13).

3.1 Recursive formulation

I follow the method of Kydland and Prescott (1980) and use debt in period marginal utility units, $z_t \equiv u_{ct}b_t$, as a *pseudo-state* variable, to capture the commitment of the planner to his past promises. The initial value of z is chosen optimally.[13] Let the value of the commitment problem from period one onward be denoted as $V(z, g)$, when the state at $t = 1$ is (z, g). Let $Z(g')$ denote the space where $z'_{g'}$, that is, the state-contingent debt position in marginal utility units, lives. The Bellman equation takes the form

$$V(z, g) = \max_{c \geq 0, h \in [0,1], z'_{g'} \in Z(g')} u(c, 1-h) + \beta H^{-1}\Big(\sum_{g'} \pi(g'|g) H\big(V(z_{g'}, g')\big)\Big)$$

subject to

[13] See the Appendix for the statement of the initial period problem.

$$z = u_c(c, 1-h)c - u_l(c, 1-h)h + \beta \sum_{g'} \pi(g'|g) m'_{g'} z'_{g'} \tag{20}$$

$$c + g = h, \tag{21}$$

where

$$m'_{g'} = \frac{H'(V(z'_{g'}, g'))}{H'(\mu)}, \quad \text{and} \quad \mu = H^{-1}\Big(\sum_{g'} \pi(g'|g) H\big(V(z'_{g'}, g')\big)\Big). \tag{22}$$

Thus, the planner is effectively choosing taxes and state-contingent debt subject to the government budget constraint (20) and the resource constraint (21). The value function $V(z'_{g'}, g')$ shows up in the implementability constraint through $m'_{g'}$, since it determines the market value of the government portfolio, due to the aversion of the household to utility volatility.

3.2 Excess burden with complete markets

Let $\Phi \geq 0$ denote the multiplier on the implementability constraint (20). I call this multiplier the *excess burden of taxation*. The excess burden of taxation serves as an indicator of tax distortions throughout the paper. Note that $\Phi = 0$ when lump-sum taxes are available. Let also $R(\{z'_{g'}\}_{g'}) \equiv \sum_{g'} \pi(g'|g) m'_{g'} z'_{g'}$ denote the revenue – in period marginal utility units– from debt issuance.[14] The optimality condition with respect to $z'_{g'}$ for an interior solution takes the form

$$\underbrace{-V_z(z_{g'}, g')}_{\text{MC: } \Phi'_{g'}} = \Phi \cdot \big[\overbrace{1 \;\; \underbrace{-V_z(z'_{g'}, g') \cdot \eta'_g}_{\text{continuation value effect}}}^{\frac{\partial R}{\partial z'_{g'}}}\big], \tag{23}$$

where

$$\eta'_{g'} \equiv A\big(V(z'_{g'}, g')\big) z'_{g'} - A(\mu) \sum_{g'} \pi(g'|g) m'_{g'} z'_{g'}, \tag{24}$$

that is, the debt position in period marginal utility units, adjusted by absolute risk aversion at $V'_{g'}$, $A(V'_{g'})$, relative to the value of the government portfolio, $\sum_{g'} \pi(g'|g) m'_{g'} z'_{g'}$, adjusted by absolute risk aversion at the certainty equivalent μ, $A(\mu)$. For brevity, and bearing always in

[14] I leave the dependence of the revenue on the current g implicit throughout the paper.

mind the proper adjustment with period marginal utility and absolute risk aversion, I call $\eta'_{g'}$ the *relative debt position*.

Optimality condition (23) takes the marginal cost/marginal benefit form (1) that I stressed in the introduction. The left-hand denotes the welfare cost of new debt, since it has to be repaid with distortionary taxes. The right-hand side denotes the marginal revenue that the planner is raising, times the shadow value of relaxing the government budget, Φ. By issuing more debt, the planner is raising more revenue, but he has also to see how additional debt affects equilibrium prices through continuation values. To understand this mechanism, note that an increase in debt *reduces* continuation values, $V_z(z'_{g'}, g') < 0$. With time-additive utility, there are *no* pricing implications, due to risk-neutrality with respect to V. With recursive utility though, a decrease in continuation values increases the price of an Arrow security at g' due to the aversion to utility volatility, captured by the curvature $A(V'_{g'})$, and generates additional revenue $-V_z(g')A(V'_{g'})z'_{g'}$. However, prices of state-contingent claims are interconnected through the certainty equivalent of continuation utilities in (4). A decrease in $V'_{g'}$ reduces μ, increasing its marginal utility, $H'(\mu)$, putting therefore *downward* pressure on all prices of the state-contingent claims at $\hat{g} \neq g'$. This *reduces* revenue by the amount $-V_z(g')A(\mu)\sum_{g'} \pi(g'|g)m'_{g'}z'_{g'}$, that is, by the curvature at the CE, $A(\mu)$, times the value of the entire government portfolio, $\sum_{g'} \pi(g'|g)m'_{g'}z'_{g'}$.

Consequently, the *relative* debt position $\eta'_{g'}$ captures the *net* revenue effect of an increase in $z'_{g'}$. By using the envelope condition $V_z(z, g) = -\Phi$, that associates the marginal cost of debt to the excess burden of taxation, we can connect (23) to the allocation of the excess burden of taxation over states and dates.

Proposition 1. *("Excess burden with complete markets under commitment")*

Turn into sequence notation and assume that $\Phi_t > 0$. The inverse of the excess burden follows the law of motion

$$\frac{1}{\Phi_{t+1}} = \frac{1}{\Phi_t} - \eta_{t+1}, t \geq 0 \tag{25}$$

where $\eta_{t+1} \equiv A(V_{t+1})z_{t+1} - A(\mu_t)E_t m_{t+1} z_{t+1}$, the relative debt position in marginal utility units.

Proof. See the Appendix. □

For the time-additive (or the deterministic) case, we have $\eta_{t+1} \equiv 0$, so the planner is making optimally the excess burden of taxation *constant* over states and dates, $\Phi_{t+1} = \Phi_t = \bar{\Phi}$, the usual source of tax-smoothing in time-additive models. In contrast, the excess burden is not constant anymore with recursive utility, although the *same* principle (1) holds, as in the time-additive case. The law of motion in terms of the *inverse* excess burden of taxation is valid for *any* H and any period utility u used, and not only for risk-sensitive preferences or EZW utility.[15]

The law of motion (25) implies that if there are shocks g' and $\hat{g}$ next period for which the relative debt position is respectively positive, $\eta_{t+1}(g') > 0$, and negative, $\eta_{t+1}(\hat{g}) < 0$, then $\Phi_{t+1}(g') > \Phi_t > \Phi_{t+1}(\hat{g})$. The reason is intuitive: the rise in prices that accompanies an increase in debt is *beneficial* at a state of the world where the planner is relatively issuing more debt, $(\eta_{t+1}(g') > 0)$, since it increases its revenue, making effectively state-contingent debt cheaper. As a result, the planner postpones distortions and taxes more at g' versus today, as measured by the excess burden. Instead, the rise in prices is *harmful* for states of the world for which the planner is relatively issuing less debt (or even buys assets), $(\eta_{t+1}(\hat{g}) < 0)$, as it decreases its revenue. As a result, the planner will tax more in the current period and less at $\hat{g}$ next period.

To gain more insight on (25) and the relative debt position, consider our three parametric examples.

Proposition 2. *("Parametric examples and drifts")*

- ***Constant absolute risk aversion:*** *Let H be as in (5), with m_{t+1} given in (6). Then, $\eta_{t+1} = A \cdot [z_{t+1} - E_t m_{t+1} z_{t+1}]$, so $E_t m_{t+1} \eta_{t+1} = 0$. $1/\Phi_t$ is a martingale with respect to $\pi \cdot M$, so Φ_t is a submartingale with respect to $\pi \cdot M$, $E_t m_{t+1} \Phi_{t+1} \geq \Phi_t$.*

- ***Constant relative risk aversion:*** *Let H be as in (7), with m_{t+1} given in (8) and $\alpha \neq 1$. Then, $\eta_{t+1} = \alpha \cdot [V_{t+1}^{-1} z_{t+1} - E_t \kappa_{t+1} V_{t+1}^{-1} z_{t+1}]$, so $E_t \kappa_{t+1} \eta_{t+1} = 0$. $1/\Phi_t$ is a martingale with respect to $\pi \cdot K$, so Φ_t is a submartingale with respect to $\pi \cdot K$, $E_t \kappa_{t+1} \Phi_{t+1} \geq \Phi_t$.*

- ***Logarithmic case:*** *Let H be as in (9), with m_{t+1} given in (11). Then, $\eta_{t+1} = V_{t+1}^{-1} z_{t+1} - E_t V_{t+1}^{-1} z_{t+1}$, so $E_t \eta_{t+1} = 0$. $1/\Phi_t$ is a martingale with respect to π, so Φ_t is a submartingale with respect to π, $E_t \Phi_{t+1} \geq \Phi_t$.*

Proof. See the Appendix. □

For all three parametric examples, the excess burden of taxation Φ_t exhibits a *positive* drift, with respect to the associated measure listed in proposition 2. Thus, there is a robust incentive for the planner to *back-load* tax distortions and accumulate debt with recursive preferences.

Moreover, the parametric cases allow us to think more about how the relative debt position η_{t+1} is associated with good (bad) shocks, that is, low (high) government expenditures. The relative debt position depends either on debt in period marginal utility units, z_{t+1} (exponential H), or debt in period marginal utility units, adjusted also by continuation utility, $V_{t+1}^{-1} z_{t+1}$ (power or logarithmic H). Without loss of generality, assume that the shock takes two values,

[15] This is a generalization of Karantounias (2018), who considered the case of EZW utility. See also the same paper for a thorough quantitative analysis.

with $g_H > g_L$. The government hedges adverse fiscal shocks by issuing high debt against good shocks next period ($g_{t+1} = g_L$), and low debt against bad shocks next period ($g_{t+1} = g_H$), so $b_{t+1}(g_L) > b_{t+1}(g_H)$. If this ranking of state-contingent positions in good and bad times holds also for z_{t+1} (for the exponential case), or $V_{t+1}^{-1} z_{t+1}$ (for the power or logarithmic case), we have $\eta_{t+1}(g_L) > 0 > \eta_{t+1}(g_H)$, so $\Phi_{t+1}(g_L) > \Phi_t > \Phi_{t+1}(g_H)$. Hence, the planner puts more tax distortions on good times and less tax distortions on bad times.

To conclude the analysis, the following proposition connects the tax rate to the excess burden of taxation Φ_t and the elasticities of the period utility function u, $\epsilon_{i,j}, i, j = c, h$.

Proposition 3. *("Optimal tax rate")*

- *The optimal tax rate for $t \geq 1$ takes the form*

$$\tau_t = \frac{\Phi_t(\epsilon_{cc,t} + \epsilon_{ch,t} + \epsilon_{hh,t} + \epsilon_{hc,t})}{1 + \Phi_t(1 + \epsilon_{hh,t} + \epsilon_{hc,t})}. \tag{26}$$

- *Assume a period utility function with constant elasticities in c and h,*

$$u(c, 1-h) = \frac{c^{1-\rho} - 1}{1-\rho} - a_h \frac{h^{1+\phi_h}}{1+\phi_h}, \tag{27}$$

  so $\epsilon_{cc} = \rho, \epsilon_{hh} = \phi_h$ and $\epsilon_{ch} = \epsilon_{hc} = 0$. Then, the optimal tax rate for $t \geq 1$ follows the law of motion

$$\frac{1}{\tau_{t+1}} = \frac{1}{\tau_t} - \frac{1}{\rho + \phi_h} \cdot \eta_{t+1}, \tag{28}$$

  where $\eta_{t+1} \equiv A(V_{t+1}) z_{t+1} - A(\mu_t) E_t m_{t+1} z_{t+1}$, with $z_{t+1} \equiv c_{t+1}^{-\rho} b_{t+1}$, that is, the respective relative debt position for (27).

- *For the three parametric examples for H, the optimal tax rate for (27) has the same (sub) martingale properties as Φ_t, listed in proposition 2.*

Proof. See the Appendix. □

Due to constant period elasticities, the utility function (27) furnishes perfect tax-smoothing in the time-additive case, since the excess burden is constant. Instead, with recursive utility any variation in the tax rate is due to Φ_t. As we noted with the inverse excess burden of taxation in (25), the law of motion for the *inverse* tax rate in (28) holds for *any* H used. The only difference from (25) is the constant $1/(\rho + \phi_h)$ that multiplies the relative debt position η_{t+1}, a fact which allows us to derive the same (sub)martingale results for the three parametric examples. Thus, the entire discussion about the drifts of Φ_t and the allocation of the excess burden of taxation across good or bad times, can be recast in terms of the actual tax rate τ_t.

4 Optimal policy with incomplete markets under commitment

Consider the second market structure which features non-contingent debt as in Aiyagari et al. (2002). The price of non-contingent debt is determined by the average SDF, $q_t = \beta E_t m_{t+1} u_{c,t+1}/u_{c,t}$, as we saw in (18). Consequently, price manipulation through continuation values means that the planner is trying to affect "average" marginal utilities next period, $\beta E_t m_{t+1} u_{c,t+1}$, through m_{t+1}. The non-contingency of debt implies that the marginal revenue from debt issuance is contrasted to the *average* tax distortions across states next period.

4.1 Preliminaries

To set up the policy problem with incomplete markets and commitment, we need to capture the promises of the planner *across* states. For that reason, we express the utility recursion (3) in terms of the certainty equivalent, *before* the realization of the shock at time t:

$$\mu_{t-1} = H^{-1}\Big(E_{t-1} H\big(u(c_t, 1-h_t) + \beta \mu_t\big)\Big). \tag{29}$$

Following the primal approach, use the labor supply condition (16) and the Euler equation (18) to eliminate tax rates and interest rates from the household's budget constraint (14). This leads to the following implementability constraint with incomplete markets,

$$u_{ct} b_{t-1} = u_{ct} c_t - u_{lt} h_t + \beta E_t m_{t+1} u_{c,t+1} b_t. \tag{30}$$

Define $B_t \equiv E_t m_{t+1} u_{c,t+1} b_t$, which is now a measure of debt in *average marginal utility* units. This variable captures the planner's past promises of period marginal utility and continuation values across states and serves as a state variable in a recursive formulation of the problem. The implementability constraint (30) becomes then

$$\frac{u_{ct}}{E_{t-1} m_t u_{ct}} B_{t-1} = u_{ct} c_t - u_{lt} h_t + \beta B_t. \tag{31}$$

I also assume that the planner is subject to some upper (lower) debt (asset) limits, which may be stricter than the natural borrowing limits. I follow Farhi (2010) and express the debt limits directly in terms of the variable B_t, so $\underline{B}_t \leq B_t \leq \bar{B}_t$.

For future reference, define the random variable $x_t \equiv \frac{u_{ct}}{E_{t-1} m_t u_{ct}} \geq 0$. In a world with time-additive utility, this variable reduces to $x_t = \frac{u_{ct}}{E_{t-1} u_{ct}}$, and integrates to unity, $E_{t-1} x_t = 1$. Thus, it captures the *risk-adjusted* change of measure, since it adjusts for the household's aversion to consumption volatility. In a world with recursive utility, we can define one more random variable that has an interpretation of a change of measure with respect to π, $n_t \equiv m_t \cdot x_t \geq 0$, with $E_{t-1} n_t = 1$. I call n_t the *consumption and continuation-value adjusted* change of measure, since it entails an adjustment for both consumption risk (through x_t), and for continuation-value risk (through m_t).[16] If we followed the terminology of Bansal and Yaron (2004) and Hansen et al. (2008), these adjustments would correspond to short- (x_t) and long-run risk (m_t). The induced measure $\pi_t \cdot N_t$, $N_t \equiv \prod_i^t n_i$, is instrumental in our analysis of tax smoothing with incomplete markets.[17]

4.2 Recursive formulation

Let $W(B_-, g_-)$ denote the optimal value of the certainty equivalent, when the state is (B_-, g_-), where the underscore "_" denotes previous period.[18] The Bellman equation takes the following form:

$$W(B_-, g_-) = \max_{c_g \geq 0, h_g \in [0,1], B_g \in [\underline{B}_g, \bar{B}_g]} H^{-1}\Big(\sum_g \pi(g|g_-) H\big(u(c_g, 1-h_g) + \beta W(B_g, g)\big)\Big)$$

subject to

$$\frac{u_c(c_g, 1-h_g)}{\sum_g \pi(g|g_-) m_g u_c(c_g, 1-h_g)} B_- = u_c(c_g, 1-h_g) c_g - u_l(c_g, 1-h_g) h_g + \beta B_g, \forall g \tag{32}$$

$$c_g + g = h_g, \forall g \tag{33}$$

[16] Note that $n_t = S_t / E_{t-1} S_t$, where S_t the stochastic discount factor in (4).

[17] When m_t can be interpreted as a change of measure with respect to π, (as in the exponential case (5)), then $E^m_{t-1} x_t = 1$, where E^m refers to the expectation with respect to m. In that case, x_t would be a change of measure with respect to the induced continuation-value adjusted measure $\pi_t \cdot M_t$, so n_t would be a product of two conditional likelihood ratios.

[18] In the case of independent shocks, we would not need to keep track of g_-.

where m_g is shorthand for scaled marginal utility of continuation values,

$$m_g = \frac{H'\big(u(c_g, 1-h_g) + \beta W(B_g, g)\big)}{H'\Big(H^{-1}\big(\sum_g \pi(g|g_-)H(u(c_g, 1-h_g) + \beta W(B_g, g))\big)\Big)}, \forall g. \tag{34}$$

The planner chooses state-contingent consumption, labor and "debt", (c_g, h_g, B_g), and faces an implementability and resource constraint for each realization of g. As in the complete markets case, value functions show up in the constraints through the determination of the price of risk-free debt.

4.3 Excess burden with incomplete markets

Consider first the optimality condition with respect to B_g, which determines the optimal allocation of tax distortions over states and dates. In the Appendix I show that the first-order condition with respect to B_g for an interior solution takes the form

$$\underbrace{-W_B(B_g, g)}_{\text{MC: average future excess burden}} = \Phi_g + \underbrace{\Big(\sum_g \pi(g|g_-)n_g\Phi_g\Big)}_{\text{value of relaxing IC across } g} \cdot \underbrace{\big(-W_B(B_g, g)\big)\cdot \xi_g b_-}_{\text{change in revenue due to } \partial W}, \tag{35}$$

where, recalling the definition of the state variable B, b_- stands for non-contingent debt issued for period t, $b_- = B_-/\sum_g \pi(g|g_-)m_g u_c(c_g, 1-h_g)$, Φ_g the excess burden of taxation, that is the (scaled) multiplier on the implementability constraint (32), n_g the consumption and continuation-value adjusted change of measure at g, and

$$\xi_g \equiv A(V_g)u_c(c_g, 1-h_g) - A(\mu_-)\sum_g \pi(g|g_-)m_g u_c(c_g, 1-h_g), \tag{36}$$

the *relative marginal utility position* at g, adjusted properly by absolute risk aversion at V_g and μ_-.[19]

Equation (35) takes the same form as (1). The left-hand side denotes again the cost of issuing debt, since it has to be repaid with distortionary taxes. From the envelope condition we have $W_B(B_-, g_-) = -\sum_g \pi(g|g_-)n_g\Phi_g$, so the left-hand side denotes future *average* tax distortions, since in contrast to (23), debt is non-contingent. The right-hand side of (35) has two parts: the first part denotes the relaxation of the government budget constraint at g which bears shadow

[19] V_g is shorthand for $V_g = u(c_g, 1-h_g) + \beta W(B_g, g)$.

benefit Φ_g. This is coming from new debt issuance, without taking into account any price-manipulation through continuation values. But increasing debt reduces utility and increases therefore prices($-W_B > 0$), since it affects "average" marginal utility, $E_{g|g_-} m_g u_{c,g}$. Since the SDF across different states is interconnected through the certainty equivalent, the benefit or cost of this action in terms of revenue depends on the *relative* marginal utility position, ξ_g, times the amount of non-contingent debt b_-. Note that since debt is non-contingent, and since the planner operates under commitment, his marginal utility and continuation value promises are bound to the "average" value of the promises he is committing to, as captured by the state variable B_-. Thus, any price change through B_g affects the implementability contraints at $\tilde{g} \neq g$. This is why the price effect of continuation values in (35) is multiplied by the shadow valuation of relaxing the budget constraints *across* shocks g, $\sum_g \pi(g|g_-) n_g \Phi_g$.

After providing this interpretation, we can use sequence notation and finally derive an inverse law of motion for the average excess burden of taxation.

Proposition 4. *("Excess burden of taxation with incomplete markets and commitment")*

- *The law of motion for the excess burden of taxation for $t \geq 0$ is*

$$\frac{1}{E_t n_{t+1} \Phi_{t+1}} = \frac{1}{\Phi_t} - \frac{E_{t-1} n_t \Phi_t}{\Phi_t} \cdot \xi_t b_{t-1}, \tag{37}$$

  where $\xi_t \equiv A(V_t) u_{ct} - A(\mu_{t-1}) E_{t-1} m_t u_{ct}$. At the initial period we have $\xi_0 \equiv 0$, so $E_0 n_1 \Phi_1 = \Phi_0$.

- *Assume the planner issues non-contingent debt for period t, $b_{t-1} > 0$. If $\xi_t > 0$, the average excess burden increases, $E_t n_{t+1} \Phi_{t+1} > \Phi_t$. If $\xi_t < 0$, the average excess burden decreases, $E_t n_{t+1} \Phi_{t+1} < \Phi_t$.*

- *Parametric examples:*
  - ***Constant absolute risk aversion:*** *Assume H is as in (5). Then $\xi_t = A[u_{ct} - E_{t-1} m_t u_{ct}]$, so ξ_t takes positive and negative values with $E_{t-1} m_t \xi_t = 0$.*
  - ***Constant relative risk aversion:*** *Assume H is as in (7). Then $\xi_t = \alpha[V_t^{-1} u_{ct} - E_{t-1} \kappa_t V_t^{-1} u_{ct}]$, so ξ_t takes positive and negative values with $E_{t-1} \kappa_t \xi_t = 0$.*
  - ***Logarithmic case:*** *Assume H is logarithmic. Then $\xi_t = V_t^{-1} u_{ct} - E_{t-1} V_t^{-1} u_{ct}$, so ξ_t takes positive and negative values with $E_{t-1} \xi_t = 0$.*

Proposition 4 furnishes a law of motion that involves the inverse *average* excess burden of taxation. Note the similarity of the law of motion in (37) to the respective law of motion with complete markets in (25), taking into account the proper modifications for market incompleteness. Instead of the future excess burden in (25) we have the average excess burden in (37). With market completeness the term $E_{t-1}n_t\Phi_t/\Phi_t$ in (37) would drop since the planner is not bound to keep track of his promises across states. And the relative debt position in marginal utility units (adjusted by absolute risk aversion) η_t in (25), becomes naturally with incomplete markets the relative marginal utility position (adjusted by absolute risk aversion) ξ_t *times* non-contingent debt for t, b_{t-1}.

Excess burden with time-additive utility. The tax-smoothing results of Aiyagari et al. (2002), who provide the foundation of the Barro (1979) analysis, are nested in (37). To see that, note that with time-additive utility we have $m_t \equiv 1$, $\xi_t \equiv 0$ and $n_t = x_t = u_{ct}/E_{t-1}u_{ct}$. Then (37) becomes $E_t x_{t+1}\Phi_{t+1} = \Phi_t$, which is the martingale result of Aiyagari et al. (2002). Thus, with time-additive utility, "average" (with respect to the risk-adjusted measure) tax distortions are *constant*, when debt is non-contingent. The Aiyagari et al. formula for the excess burden is still in the general form of (1), since it equates "average" future tax distortions with the benefit of relaxing the current government budget. The marginal revenue part of the analysis is elementary, since with time-additive utility the price manipulation we have been highlighting is absent.

Excess burden with recursive utility. Proposition 4 shows that the "averaging" of tax distortions of Aiyagari et al. (2002) breaks *down* with recursive utility, due to the price manipulation through continuation values. To understand the mechanism, recall that with complete markets, a planner that issues state-contingent debt for t tries to increase the value of the government debt portfolio, $E_{t-1}m_t u_{ct} b_t$, by shifting away tax distortions from states of the world where the "debt" position (adjusted by period marginal utility and absolute risk aversion) is relatively small ($\eta_t < 0$), towards states of the world where the "debt" position is relative large ($\eta_{t+1} > 0$). These efforts make state-contingent debt effectively cheaper and increase the value of the government portfolio. Instead, when the planner issues non-contingent debt for t, $b_{t-1} > 0$, he wants instead to increase "average" marginal utility, $E_{t-1}m_t u_{ct}$, which is inversely related to the interest rate. The way to achieve that is by shifting tax distortions away from states of the world where "marginal utility" (adjusted by absolute risk aversion), is relatively low, $\xi_t < 0$, towards events where "marginal utility" is relatively high, $\xi_t > 0$. These actions increases "average" marginal utility, lowering therefore the interest rate of non-contingent debt.

Turning to the parametric examples, ξ_t is determined by period marginal utility u_{ct} for the constant absolute risk aversion case, or $V_t^{-1}u_{ct}$ for the constant relative risk aversion or the logarithmic case. Both period marginal utility and V_t^{-1} are high in bad times of high spending, since consumption and utility fall, and low in good times of low spending. If $g_H > g_L$, we expect then $\xi_t(g_H) > 0 > \xi_t(g_L)$, so the planner has an incentive to *amplify* tax distortions in bad times. Moreover, from proposition 4 we get that average tax distortions for next period increase in bad times, $E_t n_{t+1}\Phi_{t+1} > \Phi_t(g_H)$, whereas average tax distortions decrease in good times, $E_t n_{t+1}\Phi_{t+1} < \Phi_t(g_L)$.

Quasi-linear period utility. Finally, consider a period utility function that is quasi-linear in consumption, as for example utility function (27) for $\rho = 0$. The SDF in this case simplifies to $S_{t+1} = \beta m_{t+1}$, so all curvature is due to aversion to continuation value risk, and the change of measure n becomes $n_t = m_t / E_{t-1} m_t$. In the complete markets world of proposition 1, the excess burden of taxation is still time-varying, with a relative debt position equal to $\eta_{t+1} = A(V_{t+1}) b_{t+1} - A(\mu_t) E_t m_{t+1} b_{t+1}$, since the prices of state-contingent claims are *not* constant, allowing room to manipulate them through continuation values. In a world with non-contingent debt though, we need to differentiate across the different parametric cases. In the CARA case, the relative marginal utility position is identically equal to zero, $\xi_t = A[1 - E_{t-1} m_t] = 0, \forall t$, $n_t = m_t$, and (37) implies that the excess burden of taxation is on average *constant* with respect to the continuation-value adjusted change of measure m, $E_t m_{t+1} \Phi_{t+1} = \Phi_t$. Consequently, we get a similar "averaging" result as in the time-additive case of Aiyagari et al. (2002), who would feature averaging with respect to π, $E_t \Phi_{t+1} = \Phi_t$, in the quasi-linear case.

The reason for the *muting* of the effects of continuation utilities in the CARA case for a quasi-linear period utility function is simple: there is *no* room for price manipulation, since the price of non-contingent debt is *constant* and equal to the subjective discount factor, $q_t = \beta E_t m_{t+1} = \beta$. The "averaging" result of the excess burden of taxation still breaks down, despite quasi-linear period utility, if we considered instead the CRRA or logarithmic case, since the price of non-contingent debt would not be constant anymore.[20]

To conclude, we connect the excess burden to the optimal tax rate, as we did in proposition 3.

Proposition 5. *("Optimal tax rate with incomplete markets under commitment")*

- *The optimal tax rate for $t \geq 1$ takes the form*

$$\tau_t = \frac{\Phi_t(\epsilon_{cc,t} + \epsilon_{ch,t} + \epsilon_{hh,t} + \epsilon_{hc,t}) - (\epsilon_{cc,t} + \epsilon_{hc,t})\left[\Phi_t - E_{t-1} n_t \Phi_t\right] \frac{b_{t-1}}{c_t}}{1 - (E_{t-1} n_t \Phi_t)\xi_t b_{t-1} + \Phi_t(1 + \epsilon_{hh,t} + \epsilon_{hc,t}) - \epsilon_{hc,t}\left[\Phi_t - E_{t-1} n_t \Phi_t\right] \frac{b_{t-1}}{c_t}} \tag{38}$$

- *For the constant elasticity case (27), the tax rate for $t \geq 1$ is*

[20] As seen in proposition 4, the respective relative marginal utility positions for a quasi-linear u are $\xi_t = \alpha[V_t^{-1} - E_{t-1}\kappa_t V_t^{-1}]$ for the CRRA case, and $\xi_t = V_t^{-1} - E_{t-1} V_t^{-1}$ for the logarithmic case.

$$\tau_t = \frac{\Phi_t(\rho+\phi_h) - \rho\big[\Phi_t - E_{t-1}n_t\Phi_t\big]\frac{b_{t-1}}{c_t}}{1-(E_{t-1}n_t\Phi_t)\xi_t b_{t-1} + \Phi_t(1+\phi_h)}, \tag{39}$$

where Φ_t *follows* (37) *with* $\xi_t \equiv A(V_t)c_t^{-\rho} - A(\mu_{t-1})E_{t-1}m_t c_t^{-\rho}$.

Proof. See the Appendix. □

The optimal tax rate in (38) or (39) reflects the period elasticities, the excess burden of taxation, the benefits and costs of manipulating the price of non-contingent debt through period marginal utility under commitment, which is the term that entails the difference $\Phi_t - E_{t-1}n_t\Phi_t$, and the relative marginal utility position ξ_t, that captures the manipulation of the prices through continuation values. In the time additive case of Aiyagari et al. (2002), (38) simplifies to

$$\tau_t = \frac{\Phi_t(\epsilon_{cc,t}+\epsilon_{ch,t}+\epsilon_{hh,t}+\epsilon_{hc,t}) - (\epsilon_{cc,t}+\epsilon_{hc,t})\big[\Phi_t - \Phi_{t-1}\big]\frac{b_{t-1}}{c_t}}{1+\Phi_t(1+\epsilon_{hh,t}+\epsilon_{hc,t}) - \epsilon_{hc,t}\big[\Phi_t - \Phi_{t-1}\big]\frac{b_{t-1}}{c_t}}, \tag{40}$$

since $\xi_t \equiv 0$, $m_t \equiv 1$, $n_t = x_t = u_{ct}/E_{t-1}u_{ct}$, and $E_{t-1}x_t\Phi_t = \Phi_{t-1}$.

5 Optimal policy without commitment

Consider the case of no commitment to previous promises. I assume a Markov-perfect policymaker that keeps track only of the "natural" state variables, that is, debt and the exogenous shock, (b, g). This has two implications: a) the current policymaker tries to devalue current debt (or else "legacy" debt), an incentive that shows up only in the initial period of the commitment problem, since there are no initial promises to uphold b) the current policymaker understands how his actions affect the actions of the future policymaker. In particular, the current policymaker understands that issuing more debt for the future makes the future policymaker tax more, reducing future consumption. This increases future marginal utility, lowering current interest rates. Thus, the lack of commitment creates an incentive to postpone taxation and issue more debt, since government debt becomes cheaper.

5.1 A quick digression: time-additive utility

To make the mechanism clear, consider first an environment with time-additive utility and complete markets. This would be a version of the deterministic setup of Krusell et al. (2004)

with shocks and complete markets.[21]

Let $\mathcal{C}(b'_{g'}, g')$ and $\mathcal{H}(b'_{g'}, g') = \mathcal{C}(b'_{g'}, g') + g'$ denote the policy functions for consumption and labor of the *future* policymaker at $(b'_{g'}, g')$. The problem of the current policymaker is as follows.

$$V(b,g) = \max_{c \geq 0, h \in [0,1], b'_{g'}} u(c, 1-h) + \beta \sum_{g'} \pi(g'|g) V(b'_{g'}, g') \tag{41}$$

subject to

$$u_c(c,1-h)b = u_c(c,1-h)c - u_l(c,1-h)h + \beta \sum_{g'} \pi(g'|g) u_c(\mathcal{C}(b'_{g'}, g'), 1 - \mathcal{H}(b'_{g'}, g'))b'_{g'}$$

$$c + g = h.$$

Let $c(b,g)$ and $h(b,g)$ denote the policy functions of the current policymaker. The time-consistency requirement is $c(b,g) = \mathcal{C}(b,g), h(b,g) = \mathcal{H}(b,g), \forall (b,g)$.

Let $R(\{b'_{g'}\}_{g'}) \equiv \sum_{g'} \pi(g'|g) u_c(\mathcal{C}(b'_{g'}, g'), 1 - \mathcal{H}(b'_{g'}, g'))b'_{g'}$ denote the revenue from debt issuance in marginal utility units, when the future policymaker follows $(\mathcal{C}, \mathcal{H})$. Assign the multiplier Φ on the implementability constraint (42). For the sake of the discussion, assume that $u_{cl} \geq 0$. The first-order necessary condition for state-contingent debt at a *locally* smooth Markov-perfect equilibrium takes the form[22]

$$-V_b(b'_{g'}, g') = \Phi \cdot \Big[\overbrace{u'_c + \underbrace{(u'_{cc} - u'_{cl})\mathcal{C}_b(b'_{g'}, g')b'_{g'}}_{\text{period MU effect}}}^{\partial R / \partial b'_{g'}} \Big]. \tag{42}$$

As usual, optimality condition (42) has the same interpretation as (1) and it takes the form of a Generalized Euler Equation (GEE), in the spirit of Klein et al. (2008).[23] The left-hand side denotes the marginal cost of issuing more debt, whereas the right hand-side denotes the marginal revenue from debt issuance when there is no commitment, times the excess burden of taxation. The first-term in the debt marginal revenue (u'_c) is the mechanical increase in revenue stemming from issuing more state-contingent debt, *given* prices. This would be the only effect with time-

[21]See Occhino (2012) or Debortoli and Nunes (2013) respectively for a stochastic or deterministic setup with government expenditures that provide utility.

[22]Existence of a smooth Markov-perfect equilibrium is not guaranteed. See Krusell et al. (2004) and Karantounias and Valaitis (2024) for extensive discussions.

[23]In the derivation I use the fact $\mathcal{C}_b = \mathcal{H}_b$. Primes $'$ denote next period. For example, u'_c is shorthand for $u_c(\mathcal{C}(b'_{g'}, g'), 1 - \mathcal{H}(b'_{g'}, g'))$.

additive utility and commitment. In contrast, at a Markov-perfect equilibrium, the current policymaker takes into account that the future policymaker that inherits $b'_{g'}$ reduces consumption ($\mathcal{C}_b < 0$) and increases future tax rates in order to repay. These actions increase future marginal utility, leading to an increase of the *price* of the state-contingent debt ($(u'_{cc} - u'_{cl})\mathcal{C}_b > 0$). This increase in prices increases the revenue to the current policymaker, if he *sells* debt against g', $b'_{g'} > 0$.

We can rewrite (42) in terms of the current and future excess burden of taxation. In particular, use the envelope condition $V_b = -\Phi u_c$, update it one period and eliminate $V_b(b'_{g'}, g')$ to get

$$\Phi'_{g'} = \Phi\big[1 + \frac{u'_{cc} - u'_{cl}}{u'_c}\mathcal{C}_b(b_{g'}, g')b'_{g'}\big],$$

which in sequence notation becomes

$$\Phi_{t+1} = \Phi_t\big[1 + \frac{u_{cc,t+1} - u_{cl,t+1}}{u_{c,t+1}}\mathcal{C}_b(b_{t+1}, g_{t+1})b_{t+1}\big]. \tag{43}$$

Thus, if $b_{t+1} > 0$ and there is no commitment, the government postpones tax distortions $\Phi_{t+1} > \Phi_t$ and issues more debt, exactly because it is cheaper to do so.

5.2 Recursive utility and complete markets

Turn now to the problem of interest with recursive utility. In addition to the manipulation of the consumption of the future policymaker, the current policymaker will manipulate equilibrium prices though the continuation value channel in the stochastic discount factor.

Let $\mathcal{C}, \mathcal{H}$ denote the policy functions of the future policymaker. The Bellman equation takes the form

$$V(b, g) = \max_{c\geq 0, h\in[0,1], b'_{g'}} u(c, 1-h) + \beta H^{-1}\big(\sum_{g'} \pi(g'|g)H(V(b'_{g'}, g'))\big) \tag{44}$$

subject to

$$
\begin{aligned}
u_c(c,1-h)b &= u_c(c,1-h)c - u_l(c,1-h)h \\
&\quad \underbrace{+\beta \sum_{g'} \pi(g'|g) m'_{g'} u_c(\mathcal{C}(b'_{g'},g'), 1-\mathcal{H}(b'_{g'},g'))b'_{g'}}_{R(\{b'_{g'}\}_{g'})}
\end{aligned} \tag{45}
$$

$$
c + g = h, \tag{46}
$$

where $m'_{g'} \equiv \frac{H'(V(b'_{g'},g'))}{H'(\mu)}$, $\mu \equiv H^{-1}\left(\sum_{g'} \pi(g'|g) H(V(b'_{g'},g'))\right)$, and $R(\{b'_{g'}\}_{g'})$ a shorthand for the respective revenue in marginal utility units. As previously, the Markov-perfect requirement is $c(b,g) = \mathcal{C}(b,g)$ and $h(b,g) = \mathcal{H}(b,g), \forall (b,g)$.

Optimal debt issuance. The first-order condition with respect to $b'_{g'}$ at a locally smooth equilibrium reads

$$
-V_b(b'_{g'},g') = \Phi \cdot \overbrace{\left[u'_c + \underbrace{(u'_{cc} - u'_{cl})\mathcal{C}_b(b'_{g'},g')b'_{g'}}_{\text{period MU effect }(+)} \underbrace{-V_b(b'_{g'},g')\eta'_{g'}}_{\text{continuation value effect}}\right]}^{\partial R/\partial b'_{g'}}, \tag{47}
$$

where

$$
\eta'_{g'} \equiv \left[A(V(b'_{g'},g'))u'_c b'_{g'} - A(\mu) \sum_{g'} \pi(g'|g) m'_{g'} u'_c b'_{g'}\right], \tag{48}
$$

the relative debt position (adjusted by period marginal utility and absolute risk aversion).

Optimality condition (47) takes again the familiar form of (1). The marginal revenue at the right-hand side reflects now *two* channels: a) the *period* marginal utility channel, that is present in the time-additive case when there is no commitment and depends on the *gross* debt position $b'_{g'}$, and b) the *continuation value* channel, that emerges with recursive utility, which depends on the *relative* debt position $\eta'_{g'}$. We encountered the relative debt position $\eta'_{g'}$ in the analysis of optimal policy under commitment.[24]

Optimality condition (47), which is the respective Generalized Euler equation with recursive utility, can be expressed in terms of the excess burden of taxation Φ_t.

Proposition 6. *("Excess burden of taxation with complete markets and no commitment")*

[24] Recall that $z_t = u_{ct} b_t$ in the respective relative debt position of the commitment case (24).

- *The excess burden of taxation follows the law of motion*

$$\Phi'_{g'} = \frac{\Phi}{1-\eta'_{g'}\Phi}\big[1+\frac{u'_{cc}-u'_{cl}}{u'_c}\mathcal{C}_b(b'_{g'},g')b'_{g'}\big], \tag{49}$$

with $\eta'_{g'}$ *the relative debt position in marginal utility units, defined in* (48). *Taking inverses and turning into sequence notation, we have*

$$\frac{1}{\Phi_{t+1}} = [1+\frac{u_{cc,t+1}-u_{cl,t+1}}{u_{c,t+1}}\mathcal{C}_b(b_{t+1},g_{t+1})b_{t+1}]^{-1}\big[\frac{1}{\Phi_t}-\eta_{t+1}\big] \tag{50}$$

with $\eta_{t+1} \equiv [A(V_{t+1})u_{c,t+1}b_{t+1} - A(\mu_t)\cdot E_t m_{t+1}u_{c,t+1}b_{t+1}]$.

- *The optimal tax without commitment for* $t \geq 0$ *is*

$$\tau_t = \frac{\Phi_t\big[(\epsilon_{cc,t}+\epsilon_{hc,t})(1-\frac{b_t}{c_t})+\epsilon_{ch,t}+\epsilon_{hh,t}\big]}{1+\Phi_t\big(1+\epsilon_{hh,t}+\epsilon_{hc,t}(1-\frac{b_t}{c_t})\big)}. \tag{51}$$

Proof. See Appendix. □

As we expect, when the is no curvature with respect to continuation values and H is linear, we have $\eta_{t+1} \equiv 0$ and $m_{t+1} \equiv 1$, and the law of motion (50) simplifies to the law of motion of the time-additive case in (43).

Two potentially opposing incentives. As we saw in (47), the revenue from debt issuance depends both on the gross debt position $b'_{g'}$ and on the relative debt position $\eta'_{g'}$, reflecting the fact that the stochastic discount factor depends on period marginal utility and continuation values respectively. To see what this implies for the allocation of the excess burden across states and dates, assume that the policymaker is issuing state-contingent debt against g', $b'_{g'} > 0$, and assume that $u_{cl} \geq 0$. This generates an incentive to postpone tax distortions to g', $\Phi'_{g'} > \Phi$, as we can see from (49) (since $\frac{u'_{cc}-u'_{cl}}{u'_c}\mathcal{C}_b(b'_{g'},g')b'_{g'} > 0$). However, the continuation value channel is more nuanced. If $\eta'_{g'} > 0$, that is, if the policymaker is issuing relatively *more* debt in marginal utility units at g' than on average, then both incentives are *aligned*. The policymaker has an incentive to issue more debt against g' to reduce *both* future consumption *and* continuation value, with the ultimate purpose of increasing the price of the state-contingent claim sold. As a result, the government allocates more tax distortions at g', $\Phi'_{g'} > \Phi$. If instead $\eta'_{g'} < 0$, then the government has on the one hand an incentive to increase the price of the claim that he sells through period marginal utility. On the other hand, the government has an incentive to increase continuation values and reduce the price of the claim, since the claims sold at g' are relatively small relatively to the average value of claims. In that case the two incentives *oppose* each other. If the price manipulation incentive through period marginal utility is stronger, we have $\Phi'_{g'} > \Phi$. If the continuation value channel is stronger, we have $\Phi'_{g'} < \Phi$.[25]

Assume for example that the shock takes two values, $g_H > g_L$, and that the parameters are such so that it is optimal to issue debt for both states of the world, $b_{t+1} > 0$. Government fiscal hedging implies that $b_{t+1}(g_L) > b_{t+1}(g_H) > 0$. If this holds also for the relative debt position in marginal utility units, then we have $\eta_{t+1}(g_L) > 0 > \eta_{t+1}(g_H)$. Thus, (49) implies that the excess burden of taxation increases in good times, $\Phi_{t+1}(g_L) > \Phi_t$. If bad shocks realize, we may have $\Phi_{t+1}(g_H) < \Phi_t$, if the continuation value channel dominates, or $\Phi_{t+1}(g_H) > \Phi_t$, if the period marginal utility channel dominates.

5.3 Recursive utility and incomplete markets

Consider now the case of incomplete markets with recursive utility.[26] Let (b_-, g) denote the state variable, where b_- corresponds to non-contingent debt issued for period t. Let $\mathcal{C}(b, g')$ and $\mathcal{H}(b, g')$ denote the consumption and labor policy functions of the future policymaker at (b, g'), and let $V(b_-, g)$ denote the value function at (b_-, g). The Bellman equation takes the form[27]

$$V(b_-, g) = \max_{c\geq 0, h\in[0,1], b} u(c, 1-h) + \beta H^{-1}\Big(\sum_{g'} \pi(g'|g) H(V(b, g'))\Big) \tag{52}$$

subject to

$$\begin{aligned} u_c(c, 1-h)b_- &= u_c(c, 1-h)c - u_l(c, 1-h)h \\ &\quad + \beta \sum_{g'} \pi(g'|g) m'_{g'} u_c(\mathcal{C}(b, g'), 1 - \mathcal{H}(b, g'))b \qquad (53) \\ c + g &= h, \qquad (54) \end{aligned}$$

[25]The discussion is reversed if the government buys assets, $b'_{g'} < 0$. In that case, the period marginal utility channel makes the government to *front-load* tax distortions, $\Phi'_{g'} < \Phi$. If $\eta'_{g'} > 0$, then the continuation value channel *opposes* the period marginal utility channel, creating incentives to back-load tax distortions; so it is ambiguous if the excess burden of taxation will decrease or increase, $\Phi'_{g'} \lesseqgtr \Phi$. Instead, if $\eta'_{g'} < 0$, the two incentives align with each other and the government has an incentive to increase both future consumption and continuation value, to reduce the price of claims, leading to $\Phi'_{g'} < \Phi$.

[26]See Karantounias and Valaitis (2024) for an extensive analysis of the optimal time-consistent policy in the time-additive case.

[27]In contrast to the case of incomplete markets and commitment, we do not need to capture the promises of the planner across states, so it is not necessary to formulate the problem before the realization of uncertainty at t.

where $m'_{g'} \equiv \frac{H'\left(V(b,g')\right)}{H'(\mu)}$ and $\mu \equiv H^{-1}\left(\sum_{g'} \pi(g'|g) H(V(b,g'))\right)$. The respective revenue is $R(b) \equiv \left(\sum_{g'} \pi(g'|g) m'_{g'} u'_c\right) \cdot b$. The Markov-perfect time-consistency requirement is that $c(b_-, g) = \mathcal{C}(b_-, g)$ and $h(b_-, g) = \mathcal{H}(b_-, g)$ $\forall (b_-, g)$, where c, h the policy functions coming from the above problem.

We proceed again assuming a locally smooth Markov-perfect equilibrium. The optimality condition with respect to debt, that is, the respective GEE, takes again the familiar form of (1),

$$-\sum_{g'} \pi(g'|g) m'_{g'} V_b(b, g') = \Phi \frac{\partial R}{\partial b}. \tag{55}$$

Issuing more non-contingent debt for next period, has a cost in terms of "average" tax distortions, as depicted in the left-hand side of (55). The marginal revenue part on the right-hand side incorporates both the period marginal utility channel and the continuation value channel. To see explicitly these channels, the following proposition rewrites (55) in terms of the excess burden of taxation.

Proposition 7. *("Excess burden of taxation with incomplete markets and no commitment")*

- *The excess burden of taxation without commitment and incomplete markers satisfies*

$$\sum_{g'} \pi(g'|g) n'_{g'} \Phi'_{g'} (1 - \xi'_{g'} b \Phi) = \Phi \big[1 + \sum_{g'} \pi(g'|g) n'_{g'} (\frac{u'_{cc} - u'_{cl}}{u'_c}) \mathcal{C}_b(b, g') \cdot b\big] \tag{56}$$

  where

$$\xi'_{g'} \equiv A(V(b,g')) u'_c - A(\mu) \sum_{g'} \pi(g'|g) m'_{g'} u'_c, \tag{57}$$

  the relative marginal utility position, and $n'_{g'}$ the consumption and continuation-value adjusted change of measure, $n'_{g'} \equiv \frac{m'_{g'} u'_c}{\sum_{g'} \pi(g'|g) m'_{g'} u'_c}$.

- *The optimal tax rate for $t \geq 0$ is*

$$\tau_t = \frac{\Phi_t \big[(\epsilon_{cc,t} + \epsilon_{hc,t})(1 - \frac{b_{t-1}}{c_t}) + \epsilon_{ch,t} + \epsilon_{hh,t}\big]}{1 + \Phi_t \big(1 + \epsilon_{hh,t} + \epsilon_{hc,t}(1 - \frac{b_{t-1}}{c_t})\big)}. \tag{58}$$

Proof. See the Appendix. □

Several comments are due. Rewrite (56) in sequence notation as

$$E_t n_{t+1}\Phi_{t+1} = \Phi_t\Big(1 + \big[\underbrace{E_t n_{t+1}\frac{u_{cc,t+1} - u_{cl,t+1}}{u_{c,t+1}}\mathcal{C}_b(b_t, g_{t+1})}_{\text{period MU effect (+)}} + \underbrace{E_t n_{t+1}\Phi_{t+1}\xi_{t+1}}_{\text{continuation value effect}}\big]\cdot b_t\Big) \tag{59}$$

where $\xi_{t+1} \equiv A(V_{t+1})u_{c,t+1} - A(\mu_t)E_t m_{t+1}u_{c,t+1}$.[28] The right-hand side in (59) highlights how the period marginal utility channel and the continuation value channel operate with incomplete markets and no commitment. Regarding the period marginal utility channel, an increase in b affects consumption (and therefore period marginal utility) in all states of the worlds next period, since debt is non-contingent. As a result, the *average* reaction $\mathcal{C}_b$ determines the change in prices, increasing the price of non-contingent debt.

From (59) we can see that the continuation value channel is captured by $E_t n_{t+1}\Phi_{t+1}\xi_{t+1}$, that is, the conditional mean (with respect to n) of the product of the excess burden of taxation Φ_{t+1} with the relative marginal utility position ξ_{t+1}. The non-contingency of debt requires to takes an average, since a change in b reduces continuation values for each g', $V_b(b, g')$, increasing therefore the respective excess burden of taxation $\Phi(b, g')$ for all g'.[29] The relative marginal utility position, which we first encountered in the commitment case (see (36)), indicates the benefits of reducing continuation values at states of the world with relatively high marginal utility, to increase the average stochastic discount factor, and therefore, the price of state-contingent debt.

To sign the continuation value channel in (59), note that

$$E_t n_{t+1}\Phi_{t+1}\xi_{t+1} = \underbrace{Cov_t^n(\Phi_{t+1}, \xi_{t+1})}_{(+)} + E_t n_{t+1}\Phi_{t+1} \cdot E_t n_{t+1}\xi_{t+1}, \tag{60}$$

where Cov_t^n refers to the conditional covariance with respect to the consumption-and-continuation value adjusted measure, and $E_t n_{t+1}\Phi_{t+1} > 0$, since the excess burden is positive. Consider the case of constant absolute risk aversion, where ξ_{t+1} reduces to $\xi_{t+1} = A[u_{c,t+1} - E_t m_{t+1}u_{c,t+1}]$, and m_{t+1} the continuation-value adjusted change of measure (6). We expect that the excess burden of taxation increases in bad times of high g, which are also the cases where period marginal utility is high. Consequently, we expect the covariance term in (60) to be positive. Moreover, the conditional mean of the relative marginal utility position with respect to n is positive, $E_t n_{t+1}\xi_{t+1} > 0$.[30] Thus, we expect that $E_t n_{t+1}\Phi_{t+1}\xi_{t+1} > 0$. Therefore, (59) implies that when the government is issuing debt $b_t > 0$, *both* the period marginal utility channel *and*

[28] In contrast to the three previous environments (commitment with complete or incomplete markets, and discretion with complete markets), we cannot rewrite (59) in terms of the *inverse* excess burden of taxation.

[29] The excess burden is related to V_b through the envelope condition $V_b = -\Phi u_c$. See the Appendix for details.

[30] We have $E_t n_{t+1}\xi_{t+1} = AE_t n_{t+1}[u_{c,t+1} - E_t m_{t+1}u_{c,t+1}] = A[E_t n_{t+1}u_{c,t+1} - E_t m_{t+1}u_{c,t+1}]$, since $E_t n_{t+1} = 1$. Recall now that $n_{t+1} = m_{t+1}\frac{u_{c,t+1}}{E_t m_{t+1}u_{c,t+1}}$. Thus,

the continuation value channel generate an incentive to issue more debt and postpone taxes to the future, with average tax distortions increasing relative to today, $E_t n_{t+1}\Phi_{t+1} \geq \Phi_t$.

6 Concluding remarks

In this paper I extend the theory of dynamic fiscal policy with a representative agent in several environments using a generalized version of recursive preferences that match better asset pricing facts. The government manipulates the returns of the government portfolio in order to minimize the welfare costs of taxes. The revenue from debt issuance depends on pricing kernels, market structure and the timing protocol, generating differing policy prescriptions on taxes and debt. A common principle though guides the relative costs and benefits of taxes and debt issuance: levy more taxes on a state (date) if it is cheaper to issue debt against this state (date). This simple economic principle goes a long way in generalizing the theory of tax-smoothing.

$$E_t n_{t+1}\xi_{t+1} = AE_t m_{t+1}u_{c,t+1}\big[\frac{u_{c,t+1}}{E_t m_{t+1}u_{c,t+1}} - 1\big] = A\frac{E_t m_{t+1}u_{c,t+1}(u_{c,t+1} - E_t m_{t+1}u_{c,t+1})}{E_t m_{t+1}u_{c,t+1}} = A\frac{Var_t^m(u_{c,t+1})}{E_t m_{t+1}u_{c,t+1}} > 0,$$

where Var_t^m refers to the conditional variance with respect to the continuation-value adjusted measure.

A Complete markets under commitment

A.1 Initial period problem

The $t = 0$ problem determines the initial allocation of consumption and labor and the *optimal* initial value of the pseudo-state variable z_{1,g_1}, that is, the value of state-contingent debt in marginal utility units at $t = 1$, when the shock is g_1. The solution is function of the initial conditions (b_0, g_0). .

The planner chooses $c_0 \geq 0$, $h_0 \in [0, 1]$, and $z_{1,g_1} \in Z(g_1) \forall g_1$, to maximize

$$u(c_0, 1 - h_0) + \beta H^{-1}\Big(\sum_{g_1} \pi(g_1|g_0) H\big(V(z_{1,g_1}, g_1)\big)\Big)$$

subject to

$$u_c(c_0, 1 - h_0)b_0 = u_c(c_0, 1 - h_0)c_0 - u_l(c_0, 1 - h_0)h_0 + \beta \sum_{g_1} \pi(g_1|g_0)m_{1,g_1} z_{1,g_1} \quad \text{(A.1)}$$

$$c_0 + g_0 = h_0 \quad \text{(A.2)}$$

where (b_0, g_0) are given. As usual, m_{1,g_1} is shorthand for the scaled marginal utility of continuation value, $m_{1,g_1} \equiv H'(V(z_{1,g_1}, g_1))/H'(\mu_0)$, where $\mu_0 = H^{-1}(\sum_{g_1} \pi(g_1|g_0)H\big(V(z_{1,g_1}, g_1)\big))$.

A.2 Proof of proposition 1

Proof. To derive (25), collect terms that involve the derivative of the value function V_z in (23), to get $-V_z(z_{g'}, g')[1-\eta'_{g'}\Phi] = \Phi$. The envelope condition is $V_z(z, g) = -\Phi < 0$, so $V_z(z'_{g'}, g') = -\Phi'_{g'}$. Eliminate V_z to get $\Phi'_{g'} = \frac{\Phi}{1-\eta'_{g'}\Phi}$. When $\Phi = 0$, the excess burden remains at zero for every state or date afterwards. Otherwise, take inverses to get (25) for $t \geq 1$.

To find the excess burden of taxation at $t = 0$, we need to solve the initial period problem. Assign multiplier Φ_0 on (A.1). The first-order condition with respect to z_{1,g_1} is

$$-V_z(z_{1,g_1}, g_1) = \Phi_0[1 - V_z(z_{1,g_1}, g_1)\eta_{1,g_1}], \quad \text{(A.3)}$$

where the initial relative debt position is

$$\eta_{1,g_1} \equiv A(V(z_{1,g_1}), g_1)z_{1,g_1} - A(\mu_0) \sum_{g_1} \pi(g_1|g_0)m_{1,g_1} z_{1,g_1}. \quad \text{(A.4)}$$

Collect terms in (A.3), use the envelope condition $V_z(z_{1,g_1}, g_1) = -\Phi_{1,g_1}$ to eliminate V_z, and take inverses to get (25) for $t = 0$. □

A.3 Proof of proposition 2

Proof. Consider the CARA case. The relative debt position simplifies to $\eta_{t+1} = A \cdot [z_{t+1} - E_t m_{t+1} z_{t+1}]$, so $E_t m_{t+1} \eta_{t+1} = A[E_t m_{t+1} z_{t+1} - E_t m_{t+1} E_t m_{t+1} z_{t+1}] = 0$, since $E_t m_{t+1} = 1$. Take conditional expectations in (25) with respect to the change of measure m_{t+1}, to get $E_t m_{t+1} \frac{1}{\Phi_{t+1}} = \frac{1}{\Phi_t} E_t m_{t+1} + E_t m_{t+1} \eta_{t+1} = \frac{1}{\Phi_t}$ since $E_t m_{t+1} = 1$ and $E_t m_{t+1} \eta_{t+1} = 0$. Consequently, $1/\Phi_t$ is a martingale with respect to $\pi \cdot M$. For the submartingale result, note that the function $f(x) \equiv 1/x, x > 0$, is convex. Thus, from Jensen's inequality, and given that $1/\Phi_t$ is a martingale, we have $E_t m_{t+1} \Phi_{t+1} = E_t m_{t+1} f(1/\Phi_{t+1}) \geq f(E_t m_{t+1} \frac{1}{\Phi_{t+1}}) = f(1/\Phi_t) = \Phi_t$.

For the CRRA case, the relative debt position becomes $\eta_{t+1} = \alpha \cdot [V_{t+1}^{-1} z_{t+1} - \mu_t^{-1} E_t \left(\frac{V_{t+1}}{\mu_t}\right)^{-\alpha} z_{t+1}] = \alpha \cdot [V_{t+1}^{-1} z_{t+1} - E_t \left(\frac{V_{t+1}}{\mu_t}\right)^{1-\alpha} V_{t+1}^{-1} z_{t+1}] = \alpha \cdot [V_{t+1}^{-1} z_{t+1} - E_t \kappa_{t+1} V_{t+1}^{-1} z_{t+1}]$, since $V_{t+1}^{1-\alpha}/\mu_t^{1-\alpha} = V_{t+1}^{1-\alpha}/E_t V_{t+1}^{1-\alpha} \equiv \kappa_{t+1}$. As a result, we have $E_t \kappa_{t+1} \eta_{t+1} = 0$, and since $E_t \kappa_{t+1} = 1$, we can repeat the same steps as in the CARA case, to get the martingale and submartingale results with respect to the measure $\pi \cdot K$. For the logarithmic case, $\alpha = 1$, κ_{t+1} becomes identically equal to unity, $\kappa_{t+1} \equiv 1$, so the relative debt position simplifies to $\eta_{t+1} = V_{t+1}^{-1} z_{t+1} - E_t V_{t+1}^{-1} z_{t+1}$. The martingale and the submartingale results with respect to π follow.

□

A.4 Proof of proposition 3

Proof. The derivation of the tax rate in (26) uses the first-order conditions with respect to (c, h) and is provided here for completeness. The derivation in the incomplete markets case of proposition 5 follows similar steps, so the proof for the complete markets case serves also as a roadmap. Let

$$\Omega(c, h) \equiv u_c(c, 1-h)c - u_l(c, 1-h)h. \tag{A.5}$$

Ω stands for the government surplus in marginal utility units as a function of the allocation (c, h). Let $\Omega_i, i = c, h$ stand for the respective partial derivative and let λ denote the multiplier on the resource constraint (21). The first-order conditions with respect to consumption and labor (which hold for $t \geq 1$) are

$$c: \quad u_c + \Phi\Omega_c = \lambda \tag{A.6}$$

$$h: \quad u_l - \Phi\Omega_h = \lambda \tag{A.7}$$

Combine (A.6) and (A.7) in order to eliminate λ to get $\frac{u_l}{u_c} \cdot \frac{1-\Phi\Omega_h/u_l}{1-\Phi\Omega_c/u_c} = 1$. Note that

$$\frac{\Omega_c}{u_c} = 1 - \epsilon_{cc} - \epsilon_{ch} \tag{A.8}$$

$$\frac{\Omega_h}{u_l} = -1 - \epsilon_{hh} - \epsilon_{hc}, \tag{A.9}$$

where the elasticities defined in proposition 1. Use the fact that $u_l/u_c = 1 - \tau$ and rewrite the combined condition in terms of the tax rate in order to get (26).

Period utility function (27)**.** The proof follows the same steps as Karantounias (2018), who dealt with EZW preferences. It is assumed that parameters are such that $u > 0$, if the power or logarithmic function is used for H. The formula for the tax rate (26) reduces to

$$\tau_t = \frac{\Phi_t(\rho + \phi_h)}{1 + \Phi_t(1 + \phi_h)}, t \geq 1. \tag{A.10}$$

where $1/\phi_h$ captures the Frisch elasticity of labor supply. Invert τ_t to get $\frac{1}{\tau_t} = \frac{1}{(\rho+\phi_h)\Phi_t} + \frac{1+\phi_h}{\rho+\phi_h}$. Thus,

$$\frac{1}{\tau_{t+1}} - \frac{1}{\tau_t} = \frac{1}{\rho + \phi_h}\Big[\frac{1}{\Phi_{t+1}} - \frac{1}{\Phi_t}\Big] \overset{(25)}{=} -\frac{1}{\rho + \phi_h}\eta_{t+1}, \tag{A.11}$$

which delivers (28). Note that expression (28) holds for $t \geq 1$ and not for $t = 0$, since the optimal tax rate at $t = 0$ is not given by (A.10). Besides the multiplication of the increment η_{t+1} with $1/(\rho + \phi_h)$, the law of motion for the tax rate (28) is the same as the law of motion for Φ_t in (25). For all three parametric examples, η_{t+1} has zero conditional mean (according to the respective measure), so we can repeat the same calculations as in proposition 2 and get that $1/\tau_t$ is a martingale, and τ_t a submartingale, according to the respective measure of each case.

□

B Incomplete markets under commitment

Initial period problem. The initial period problem determines (c_0, h_0) and the optimal value of the pseudo-state variable B_0 as functions of the initial conditions (b_{-1}, g_0). The planner chooses $c_0 \geq 0$, $h_0 \in [0,1]$ and $B_0 \in [\underline{B}_0, \bar{B}_0]$ to maximize

$$u(c_0, 1-h_0) + \beta W(B_0, g_0)$$

subject to

$$u_c(c_0, 1-h_0) b_{-1} = u_c(c_0, 1-h_0) c_0 - u_l(c_0, 1-h_0) h_0 + \beta B_0 \tag{B.1}$$

$$c_0 + g_0 = h_0, \tag{B.2}$$

where b_{-1} and the initial shock g_0 are given.

Lagrangian. To derive (35) and the results of proposition 4, it is convenient to keep the random variable m_g and treat (34) as an additional constraint. Assign multipliers $\pi(g|g_-)\tilde{\Phi}_g$, $\pi(g|g_-)\tilde{\lambda}_g$ and $\pi(g|g_-)\zeta_g$ on constraints (32), (33) and (34) respectively and recall the definition of Ω in (A.5). The Lagrangian takes the form

$$\begin{aligned} L = {} & H^{-1}\Big(\sum_g \pi(g|g_-) H\big(u(c_g, 1-h_g) + \beta W(B_g, g)\big)\Big) \\ & - \sum_g \pi(g|g_-)\tilde{\Phi}_g \Big[\frac{u_c(c_g, 1-h_g)}{\sum_g \pi(g|g_-) m_g u_c(c_g, 1-h_g)} B_- - \Omega(c_g, h_g) - \beta B_g\Big] \\ & - \sum_g \pi(g|g_-)\tilde{\lambda}_g [c_g + g - h_g] \\ & - \sum_g \pi(g|g_-)\zeta_g \Big[m_g - \frac{H'(u(c_g, 1-h_g) + \beta W(B_g, g))}{H'\Big(H^{-1}\big(\sum_g \pi(g|g_-) H(u(c_g, 1-h_g) + \beta W(B_g, g))\big)\Big)}\Big]. \end{aligned}$$

Define for convenience

$$K \equiv \frac{\sum_g \pi(g|g_-) u_c(c_g, 1-h_g)\tilde{\Phi}_g}{\sum_g m_g u_c(c_g, 1-h_g)} \quad \text{and} \quad I \equiv \frac{\sum_g \pi(g|g_-) H'\big(u(c_g, 1-h_g) + \beta W(B_g, g)\big)\zeta_g}{H'\Big(H^{-1}\big(\sum_g \pi(g|g_-) H(u(c_g, 1-h_g) + \beta W(B_g, g))\big)\Big)}.$$

B.1 First-order conditions

The first-order necessary conditions for an interior solution take the form:

$$c_g: \quad \pi(g|g_-)m_g u_c(c_g, 1-h_g) + \pi(g|g_-)\tilde{\Phi}_g \Omega_c(c_g, h_g) + \frac{\partial I}{\partial c_g} - \frac{\partial K}{\partial c_g} B_- = \pi(g|g_-)\tilde{\lambda}_g \tag{B.3}$$

$$h_g: \quad -\pi(g|g_-)m_g u_l(c_g, 1-h_g) + \pi(g|g_-)\tilde{\Phi}_g \Omega_h(c_g, h_g) + \frac{\partial I}{\partial h_g} - \frac{\partial K}{\partial h_g} B_- = -\pi(g|g_-)\tilde{\lambda}_g \tag{B.4}$$

$$m_g: \quad \pi(g|g_-)\zeta_g = -\frac{\partial K}{\partial m_g} B_- \tag{B.5}$$

$$B_g: \quad -\beta\pi(g|g_-)m_g W_B(B_g, g) = \beta\pi(g|g_-)\tilde{\Phi}_g + \frac{\partial I}{\partial B_g} \tag{B.6}$$

The derivatives of expression K and I are as follows.

$$\frac{\partial K}{\partial m_g} = -\pi(g|g_-)x_g \sum_g \pi(g|g_-)x_g\tilde{\Phi}_g \tag{B.7}$$

$$\frac{\partial K}{\partial c_g} = \pi(g|g_-)\frac{u_{cc}(c_g, 1-h_g)}{\sum_g \pi(g|g_-)m_g u_c(c_g, 1-h_g)}\left[\tilde{\Phi}_g - m_g \sum_g \pi(g|g_-)x_g\tilde{\Phi}_g\right] \tag{B.8}$$

$$\frac{\partial K}{\partial h_g} = -\pi(g|g_-)\frac{u_{cl}(c_g, 1-h_g)}{\sum_g \pi(g|g_-)m_g u_c(c_g, 1-h_g)}\left[\tilde{\Phi}_g - m_g \sum_g \pi(g|g_-)x_g\tilde{\Phi}_g\right] \tag{B.9}$$

and

$$\frac{\partial I}{\partial c_g} = -\pi(g|g_-)m_g u_c(c_g, 1-h_g)\nu_g \tag{B.10}$$

$$\frac{\partial I}{\partial h_g} = \pi(g|g_-)m_g u_l(c_g, 1-h_g)\nu_g \tag{B.11}$$

$$\frac{\partial I}{\partial B_g} = -\beta\pi(g|g_-)m_g W_B(B_g, g)\nu_g \tag{B.12}$$

where

$$\nu_g \equiv A(V_g)\zeta_g - A(\mu_-)\sum_g \pi(g|g_-)m_g\zeta_g, \tag{B.13}$$

the "innovation" (adjusted properly by absolute risk aversion) in the multiplier ζ_g, which captures the shadow value of increasing m_g. V_g is shorthand for value at g, $V_g = u(c_g, 1-h_g) + \beta W(B_g, g)$ and μ_- denotes the certainty equivalent at $t-1$, so at the optimum it is equal to $W(B_-, g_-)$. Recall also that x_g is the consumption-risk adjustment, defined in the text,

$$x_g \equiv \frac{u_c(c_g, 1-h_g)}{\sum_g \pi(g|g_-) m_g u_c(c_g, 1-h_g)}. \tag{B.14}$$

Use now (B.7) in (B.5) to get

$$\zeta_g = x_g \big(\sum_g \pi(g|g_-) x_g \tilde{\Phi}_g \big) B_- = x_g \big(\sum_g \pi(g|g_-) n_g \Phi_g \big) B_-, \tag{B.15}$$

where we used the *normalized* multiplier $\Phi_g \equiv \tilde{\Phi}_g / m_g$, and the definition of the change of measure n_g from the text, $n_g \equiv m_g x_g$, with $\sum_g \pi(g|g_-) n_g = 1$. Use (B.15) to get

$$\sum_g \pi(g|g_-) m_g \zeta_g = \big(\sum_g \pi(g|g_-) n_g \Phi_g \big) B_-. \tag{B.16}$$

Thus, we can eliminate the multiplier ζ_g from the innovation ν_g in (B.13) to get

$$\begin{aligned} \nu_g &= \big[A(V_g) x_g - A(\mu_-) \big] \big(\sum_g \pi(g|g_-) n_g \Phi_g \big) B_- \\ &\overset{(B.14)}{=} \xi_g \cdot \big(\sum_g \pi(g|g_-) n_g \Phi_g \big) \frac{B_-}{\sum_g \pi(g|g_-) m_g u_c(c_g, 1-h_g)} \\ &= \xi_g \cdot \big(\sum_g \pi(g|g_-) n_g \Phi_g \big) b_- \end{aligned} \tag{B.17}$$

where ξ_g the relative marginal utility position defined in (36), and b_- non-contingent debt, $b_- = B_- / \sum_g \pi(g|g_-) m_g u_c(c_g, 1-h_g)$.

B.2 Excess burden of taxation

Use now (B.12) in (B.6), simplify, and use (B.17) to get (35) in the text. Moreover, we can collect terms to get

$$-W_B(B_g, g)(1-\nu_g) = \Phi_g \Rightarrow \frac{1}{-W_B(B_g, g)} = \frac{1}{\Phi_g} - \frac{\nu_g}{\Phi_g} \tag{B.18}$$

Use now (B.17) to finally get

$$\frac{1}{-W_B(B_g,g)} = \frac{1}{\Phi_g} - \frac{\sum_g \pi(g|g_-)n_g\Phi_g}{\Phi_g}\xi_g b_-. \tag{B.19}$$

The envelope condition is

$$W_B(B_-,g_-) = -\sum_g \pi(g|g_-)x_g\tilde{\Phi}_g = -\sum_g \pi(g|g_-)n_g\Phi_g. \tag{B.20}$$

Update one period, turn into sequence notation and replace $W_B(B_g,g)$ in (B.19) to get the law of motion (37) in proposition 4.

Initial period. Note that (B.18) holds from period one onward, i.e. $-W_B(B_t,g_t)(1-\nu_t) = \Phi_t$ for $t \geq 1$. Assign multiplier Φ_0 on the implementability constraint (B.1) to get the first-order condition $-W_B(B_0,g_0) = \Phi_0$. From the envelope condition of the problem from period one onward in (B.20) we have $W_B(B_0,g_0) = -E_0 n_1\Phi_1$. Combining these two conditions we get $E_0n_1\Phi_1 = \Phi_0$. In the law of motion (37), this is achieved by setting $\xi_0 \equiv 0$. The rest of the results in the proposition is straightforward.

B.3 Proof of proposition 5

Proof. Use (B.8) and (B.10) in (B.3), define the normalized multiplier $\lambda_g \equiv \tilde{\lambda}_g/m_g$, simplify and collect terms to get the final version of the optimality condition with respect to c:

$$u_c(c_g,1-h_g)(1-\nu_g) + \Phi_g\Omega_c(c_g,h_g) - u_{cc}(c_g,1-h_g)\big[\Phi_g - \sum_g \pi(g|g_-)n_g\Phi_g\big]b_- = \lambda_g \tag{B.21}$$

Similarly, use (B.9) and (B.11) in (B.4) to get

$$u_l(c_g,1-h_g)(1-\nu_g) - \Phi_g\Omega_h(c_g,h_g) - u_{cl}(c_g,1-h_g)\big[\Phi_g - \sum_g \pi(g|g_-)n_g\Phi_g\big]b_- = \lambda_g. \tag{B.22}$$

Combine now (B.21) and (B.22) and eliminate λ_g to get

$$\frac{u_l}{u_c}\cdot\frac{1-\nu_g-\Phi_g\frac{\Omega_h}{u_l}-\frac{u_{cl}\cdot c}{u_l}(\Phi_g-\sum_g\pi(g|g_-)n_g\Phi_g)\frac{b_-}{c}}{1-\nu_g+\Phi_g\frac{\Omega_c}{u_c}-\frac{u_{cc}\cdot c}{u_c}(\Phi_g-\sum_g\pi(g|g_-)n_g\Phi_g)\frac{b_-}{c}} = 1, \tag{B.23}$$

where I suppress the arguments of the various functions in order to ease notation. Recall that $u_l/u_c = 1-\tau$. Use the elasticity expressions in (A.8) and (A.9), use (B.17) to eliminate ν_g, and rewrite (B.23) in terms of τ, to get expression (38) in sequence notation. To get (39), just use $\epsilon_{cc} = \rho, \epsilon_{hh} = \phi_h, \epsilon_{ch} = \epsilon_{hc} = 0$ in (38).

□

C No commitment

C.1 Proof of proposition 6

Proof. Assign multipliers Φ and λ on (45) and (46) respectively and recall the definition of (A.5). First-order necessary conditions for an interior solution are as follows.

$$c: \quad u_c + \Phi[\Omega_c - u_{cc} b] = \lambda \tag{C.1}$$

$$h: \quad u_l - \Phi[\Omega_h + u_{cl} b] = \lambda \tag{C.2}$$

$$b'_{g'}: \quad \pi(g'|g) m'_{g'} V_b(b'_{g'}, g') + \Phi \frac{\partial R}{\partial b'_{g'}} = 0, \tag{C.3}$$

where the revenue and marginal revenue are respectively,

$$R(\{b'_{g'}\}_{g'}) \equiv \sum_{g'} \pi(g'|g) m'_{g'} u_c(\mathcal{C}(b'_{g'}, g'), 1 - \mathcal{H}(b'_{g'}, g')) b'_{g'} \tag{C.4}$$

and

$$\frac{\partial R}{\partial b'_{g'}} = \pi(g'|g) m'_{g'} \left[u'_c + (u'_{cc} - u'_{cl}) \mathcal{C}_b(b'_{g'}, g') b'_{g'} - V_b(b_{g'}, g') \eta'_{g'} \right], \tag{C.5}$$

with $\eta'_{g'}$ the relative debt position in marginal utility units, defined in (48). Use (C.5) in (C.3) and simplify to get (47). Rewrite (47) as

$$-V_b(b'_{g'}, g')[1 - \eta'_{g'} \Phi] = \Phi \left[u'_c + (u'_{cc} - u'_{cl}) \mathcal{C}_b(b'_{g'}, g') b'_{g'} \right]. \tag{C.6}$$

Use the envelope condition $V_b(b, g) = -\Phi u_c$ and update it one period to get $V_b(b', g') = -\Phi(b'_{g'}, g') u'_c$. Eliminate the derivative of the value function in (C.6) and rewrite to get (49) in the proposition.

Tax rate. To get the optimal tax rate, eliminate λ from first-order conditions (C.1) and (C.2) and rewrite to get

$$\frac{u_l}{u_c} \cdot \frac{1 - \Phi\left[\frac{\Omega_h}{u_l} + \frac{u_{cl}}{u_l} b\right]}{1 + \Phi\left[\frac{\Omega_c}{u_c} - \frac{u_{cc}}{u_c} b\right]} = 1. \tag{C.7}$$

As in the proof of proposition 3, use the definition of the elasticities, the formulas (A.8) and (A.9) and $u_l/u_c = 1 - \tau$, to rewrite (C.7) as (51).

□

C.2 Proof of proposition 7

Proof. The revenue from debt issuance is

$$R(b) = \frac{\sum_{g'} \pi(g'|g) H'(V(b, g')) u_c(\mathcal{C}(b, g'), 1 - \mathcal{H}(b, g'))}{H'(H^{-1}(\sum_{g'} \pi(g'|g) H(V(b, g'))))} \cdot b \tag{C.8}$$

Differentiate with respect to non-contingent debt b to get

$$\frac{\partial R(b)}{\partial b} = \sum_{g'} \pi(g'|g) m'_{g'} u'_c + \Big\{ \sum_{g'} \pi(g'|g) m'_{g'} (u'_{cc} - u'_{cl}) \mathcal{C}_b(b, g') - \sum_{g'} \pi(g'|g) m'_{g'} V_b(b, g') \xi'_{g'} \Big\} \cdot b \tag{C.9}$$

where $\xi'_{g'}$ is defined in (57). Use (C.9) in (55) and collect the terms involving V_b to get

$$\begin{aligned} -\sum_{g'} \pi(g'|g) m'_{g'} V_b(b, g') \left[1 - \xi'_{g'} \Phi b\right] = \Phi \Big[& \sum_{g'} \pi(g'|g) m'_{g'} u'_c \\ & + \sum_{g'} \pi(g'|g) m'_{g'} (u'_{cc} - u'_{cl}) \mathcal{C}_b(b, g) \cdot b \Big] \end{aligned} \tag{C.10}$$

The envelope condition is $V_b(b_-, g) = -\Phi u_c$, so $V_b(b, g') = -\Phi'_{g'} u'_c$, where the excess burden of taxation next period is a function of the respective state, $\Phi'_{g'} = \Phi(b, g')$. Eliminate V_b from (C.10) to get

$$\sum_{g'} \pi(g'|g) m'_{g'} u'_c \Phi'_{g'} (1 - \xi'_{g'} b \Phi) = \Phi \Big[\sum_{g'} \pi(g'|g) m_{g'} u'_c + \sum_{g'} \pi(g'|g) m'_{g'} (u'_{cc} - u'_{cl}) \mathcal{C}_b(b, g') \cdot b \Big] \tag{C.11}$$

Divide both sides of (C.11) over $\sum_{g'} \pi(g'|g) m_{g'} u'_c$ and express the resulting expression in terms of the change of measure $n'_{g'} \equiv \frac{m'_{g'} u'_c}{\sum_{g'} \pi(g'|g) m_{g'} u'_c}$ to get (56).

Tax rate. To derive the optimal tax rate, note that the first-order necessary conditions for consumption and labor are

$$c: \qquad u_c + \Phi[\Omega_c - u_{cc} b_-] = \lambda \tag{C.12}$$

$$h: \qquad u_l - \Phi[\Omega_h + u_{cl} b_-] = \lambda, \tag{C.13}$$

where λ the multiplier on (54). The only difference from the respective first-order conditions (C.1) and (C.2) of the complete markets case is the fact that debt is non-contingent. Follow the same steps as in the proof of proposition 6 to get the optimal tax rate in (58).

□

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