Informational steady state and entropy production in continuously monitored systems

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QTeQ QUANTUM TECHNOLOGY at QUEEN'S

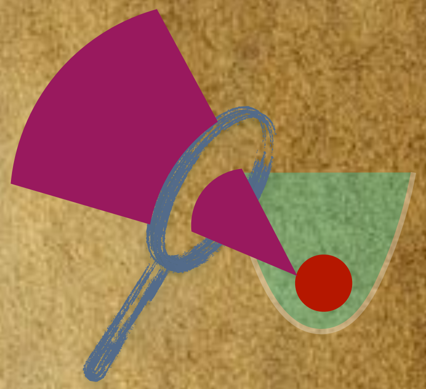
New trends in Quantum Thermodynamics

University of Surrey, 8 July 2024

STRUCTURE OF THE TALK

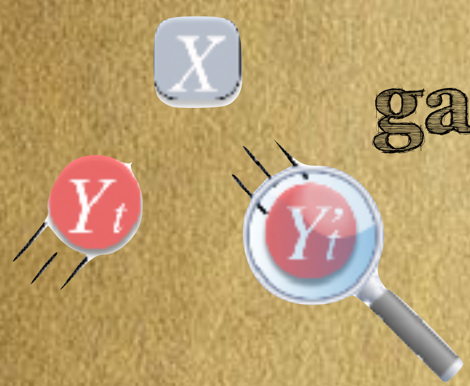
1

Formalism for entropy production in continuously measured quantum systems



Informational steady states: gaining & losing through measurement

2

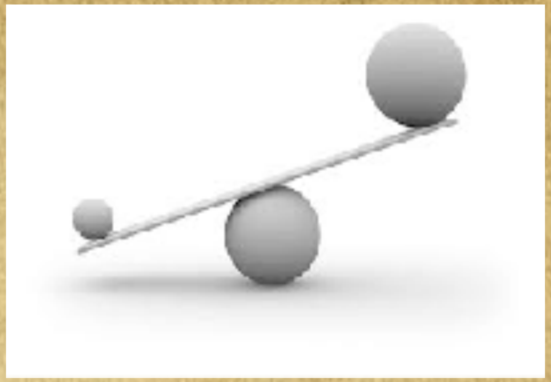


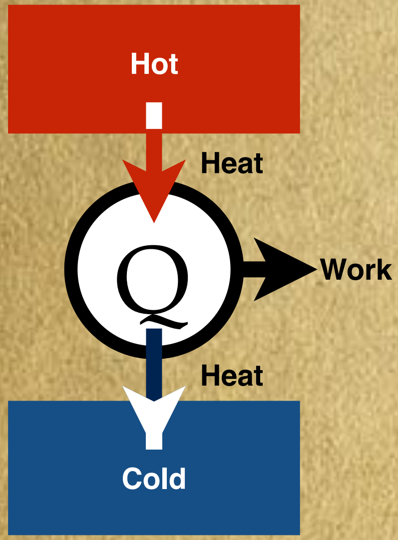
3

Observing irreversible entropy in measured mesoscopic quantum settings

Why entropy production?

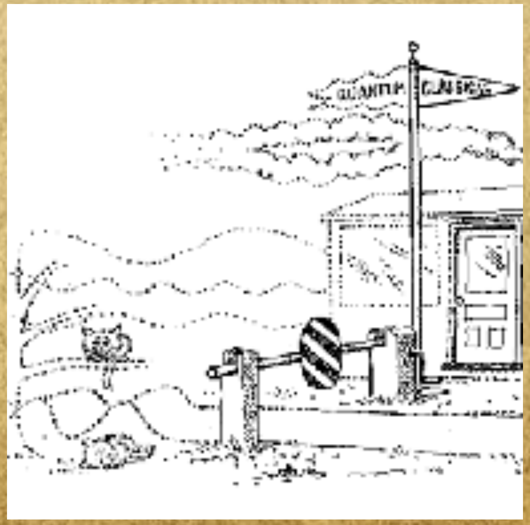
Non-equilibrium processes dissipate energy. This produces irreversible increase of entropy





Entropy production for estimating the performance of devices (exergy is reduced by irreversibility)

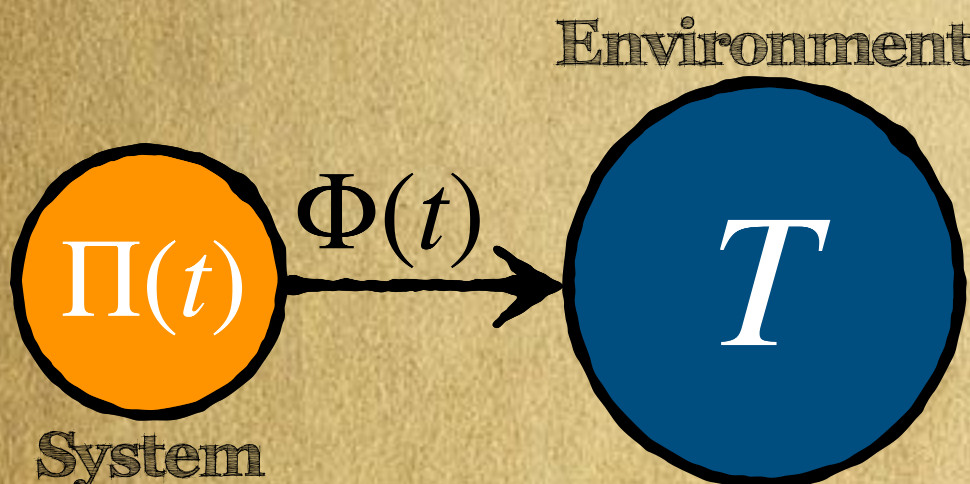
Fantastic framework for pinpointing the quantum-to-classical transition



Entropy production

Second Law: $\Delta S \geq \int \frac{\delta Q}{T} \quad \Rightarrow \quad \Sigma = \Delta S - \int \frac{\delta Q}{T}$

Clausius: "Uncompensated transformation" — Entropy production



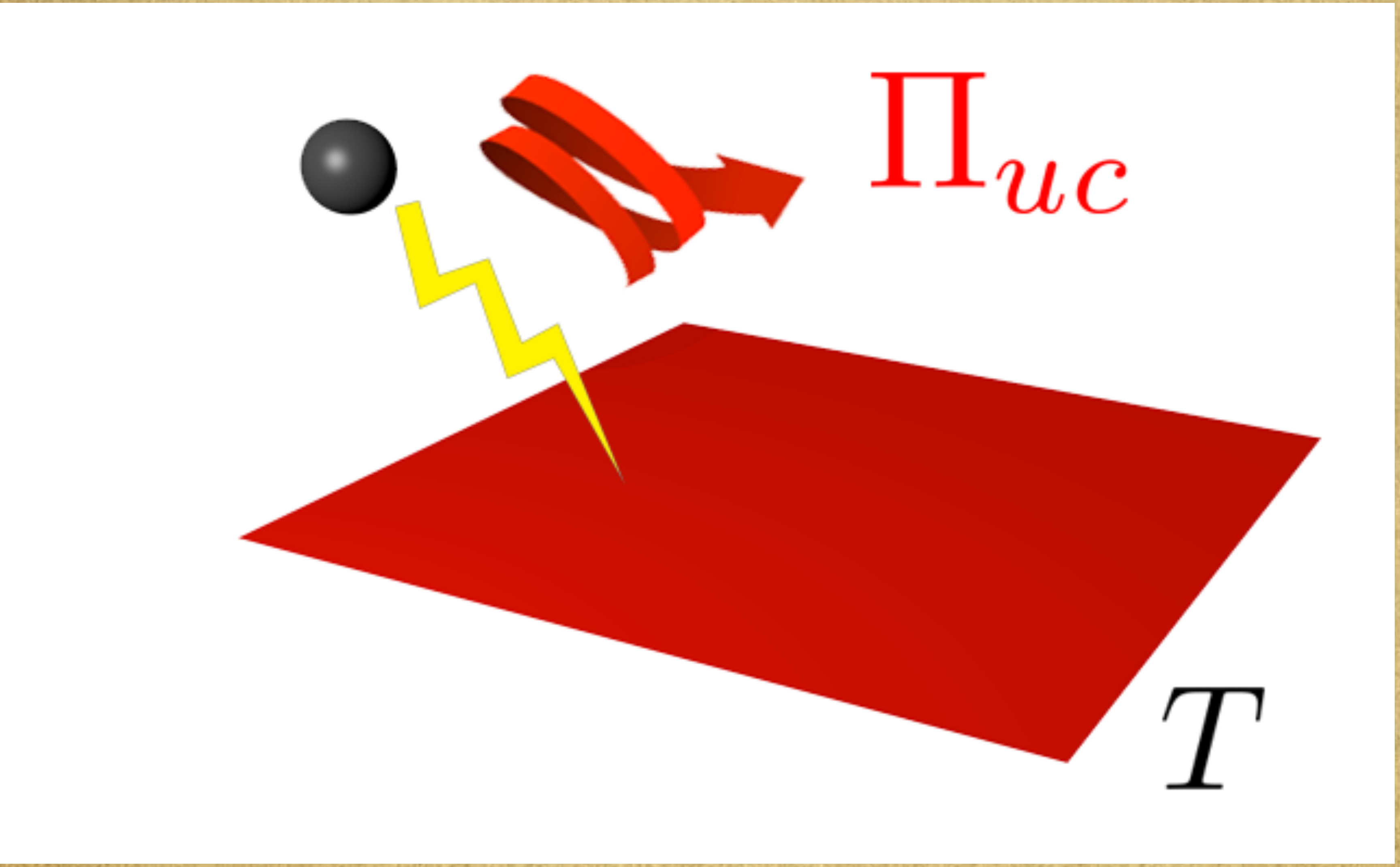
$$\frac{dS}{dt} = \Pi(t) + \Phi(t)$$

Entropy production rate — Entropy flux rate

Which is the role of quantum fluctuations on entropy production?

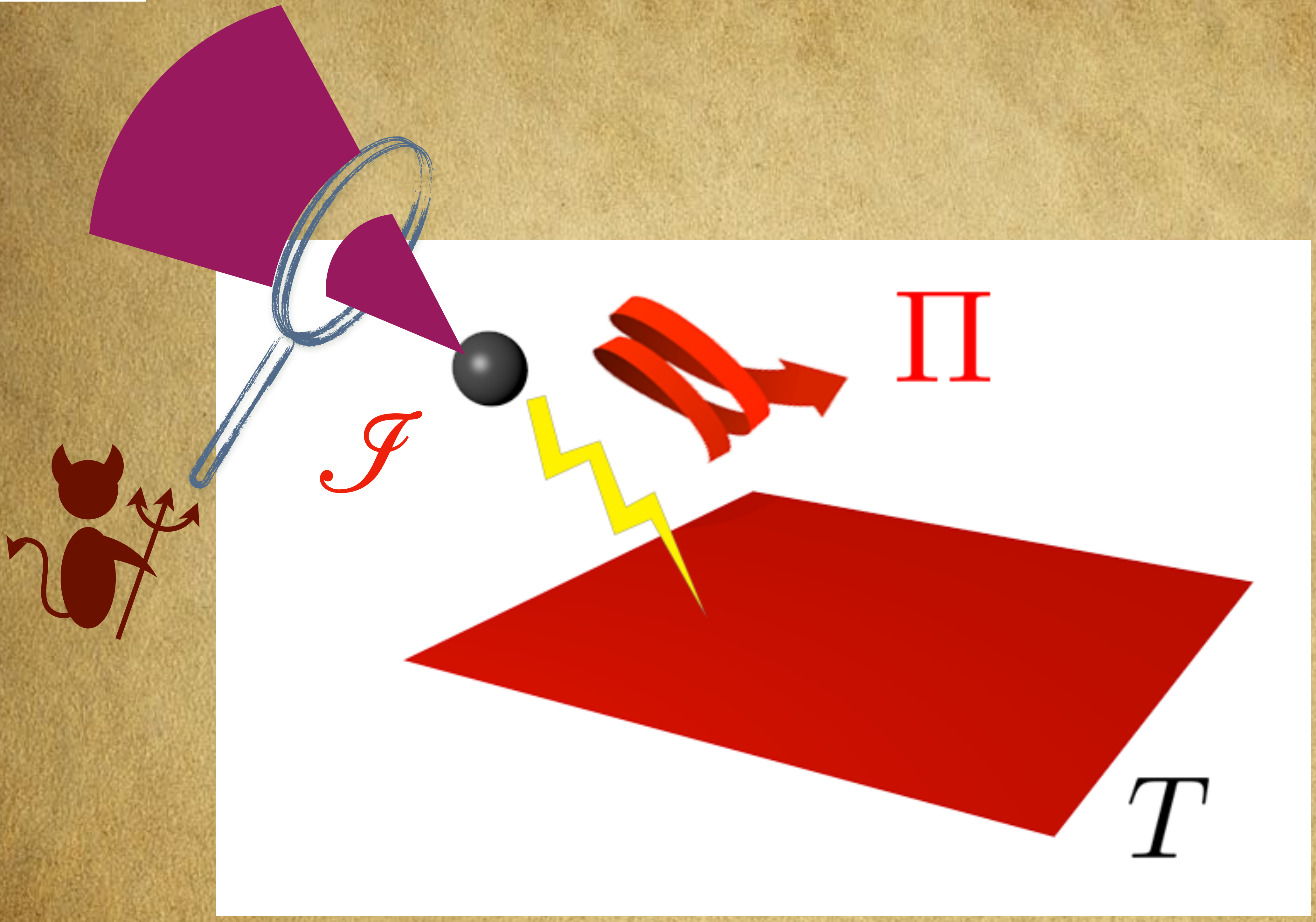
What happens if you plug in the effects of measuring?

Don't look yet!!!



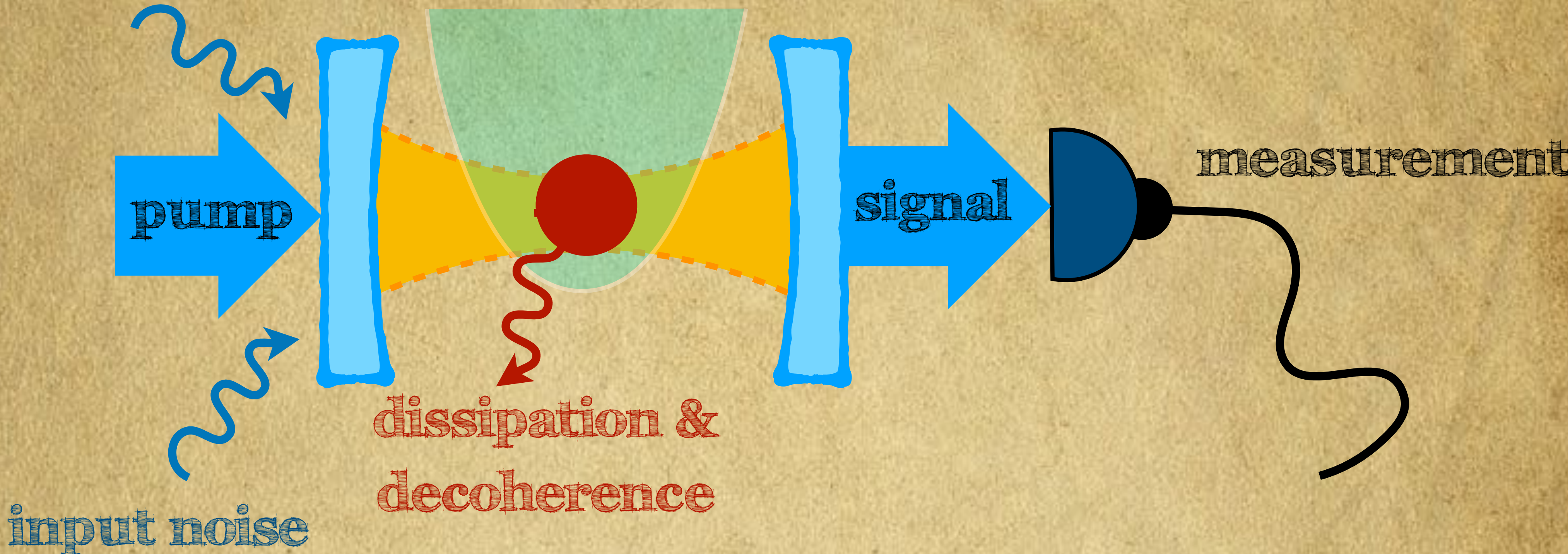
A Belenchia, L Mancino, G T Landi, and M Paternostro,
Nature Quantum Information 6, 97 (2019)

..now open your eyes..



A Belenchia, L Mancino, G T Landi, and M Paternostro,
Nature Quantum Information 6, 97 (2019)

Let's fix the ideas

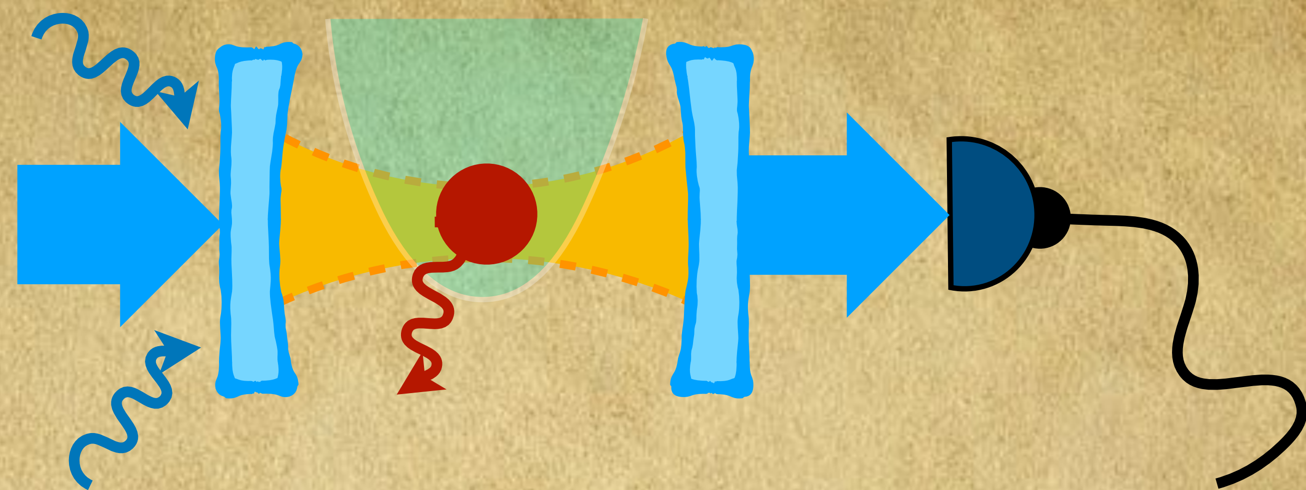


Now restrict the framework to quadratic evolution and Gaussian states & measurements

A Belenchia, M Paternostro, and G T Landi, PRX Quantum 3, 010303 (2022)

A Belenchia, et al., Nature Quantum Information 6, 97 (2019)

General formalism



Stochastic master equation

$$d\rho = -i[\hat{H},\rho]dt + \sum_k \mathscr{D}[\hat{c}_k](\rho)dt + \sum_k \sqrt{\eta_k}\mathscr{H}[\hat{c}_k](\rho)dw_k$$

Deterministic dynamics

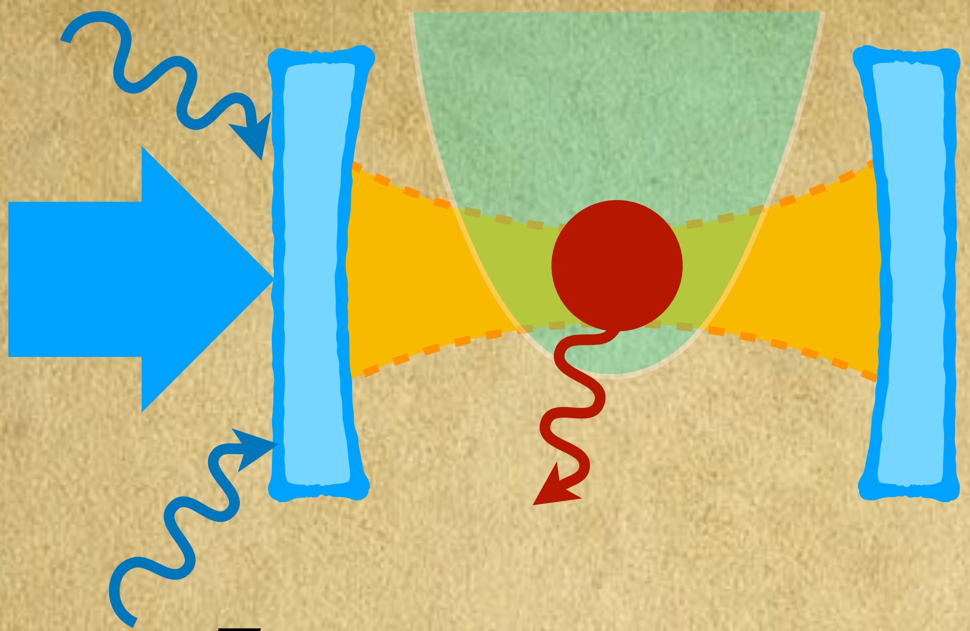
Stochastic terms

$$\mathscr{H}[\hat{c}]\rho = \hat{c}\rho + \rho\hat{c}^\dagger - \langle \hat{c} + \hat{c}^\dagger \rangle \rho$$

A Belenchia, M Paternostro, and G T Landi, PRX Quantum 3, 010303 (2022)

M. G. Genoni, L. Lami, and A. Serafini, Contemp. Phys. 57, 331 (2016)

Un-Conditioned Gaussian dynamics



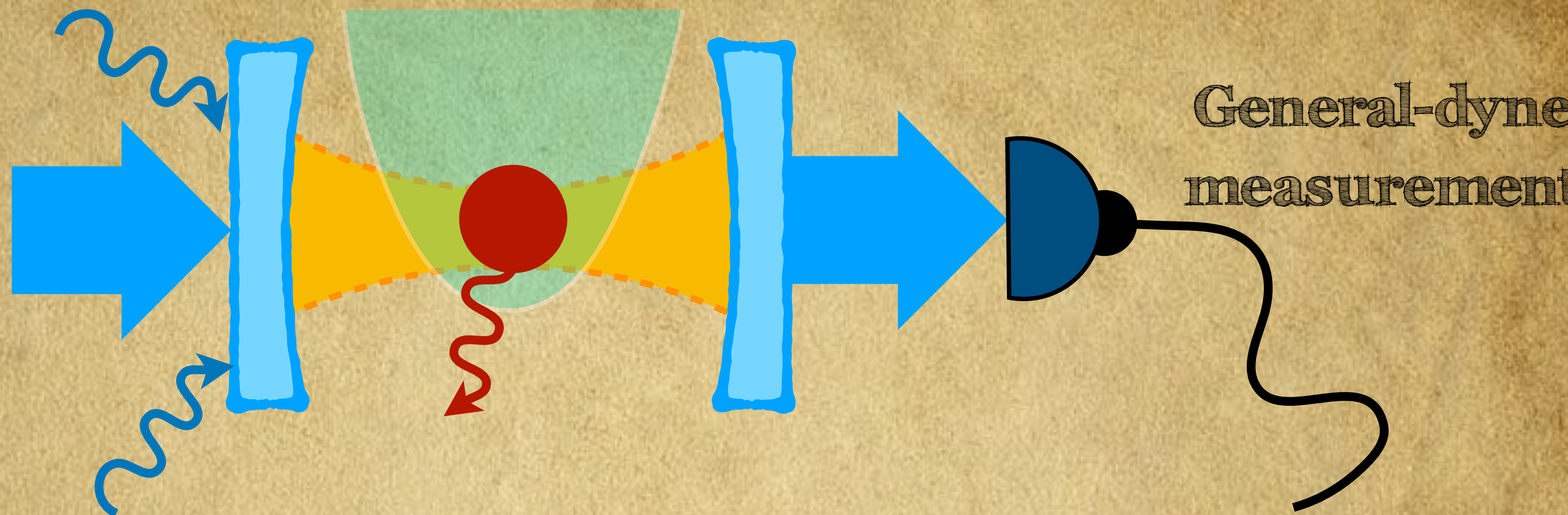
$$\dot{\sigma} = A\sigma + \sigma A^T + D$$

$$d\mathbf{x} = (A\mathbf{x} + \mathbf{b})\, dt$$

A Belenchia, M Paternostro, and G T Landi, PRX Quantum 3, 010303 (2022)

A Belenchia, et al., Nature Quantum Information 6, 97 (2019)

Conditioned Gaussian dynamics



$$\dot{\sigma} = A\sigma + \sigma A^T + D$$

$$\Downarrow$$

$$\dot{\sigma} = \tilde{A}\sigma + \sigma\tilde{A}^T + \tilde{D} - \sigma B B^T \sigma = A\sigma + \sigma A^T + D - \chi(\sigma)$$

contains terms depending on the measurement

$$d\mathbf{x} = (A\mathbf{x} + \mathbf{b})\, dt$$

$$\Downarrow$$

$$d\bar{\mathbf{x}} = (A\bar{\mathbf{x}} + \mathbf{b})dt + \mu(\sigma)d\mathbf{w}$$

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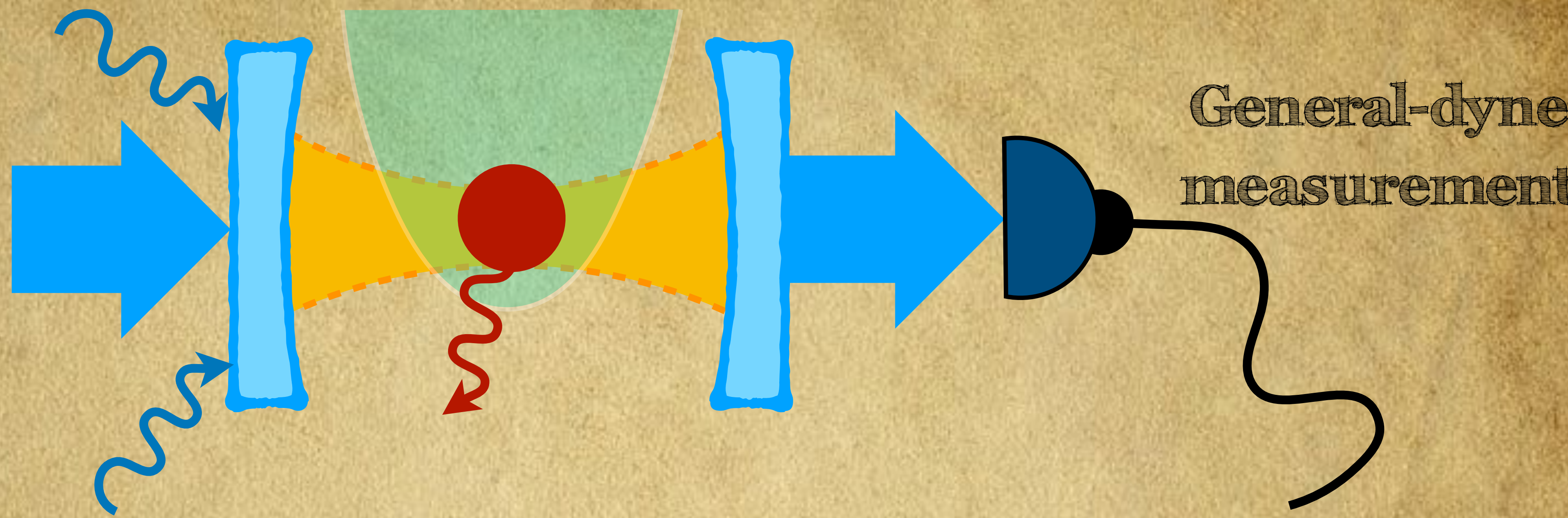
A Belenchia, et al., Nature Quantum Information 6, 97 (2019)

Conditioned Gaussian dynamics



$$dS = d\Phi_{\bar{\mathbf{x}}} + d\Sigma_{\bar{\mathbf{x}}}$$

deterministic (only depends on CM) → dS

stochastic (depend also on 1st moments) → $d\Phi_{\bar{\mathbf{x}}}$, $d\Sigma_{\bar{\mathbf{x}}}$

$$\phi = \mathbb{E}\left[d\Phi_{\bar{\mathbf{x}}}/dt\right]$$

$$\Pi = \mathbb{E}\left[d\Sigma_{\bar{\mathbf{x}}}/dt\right]$$

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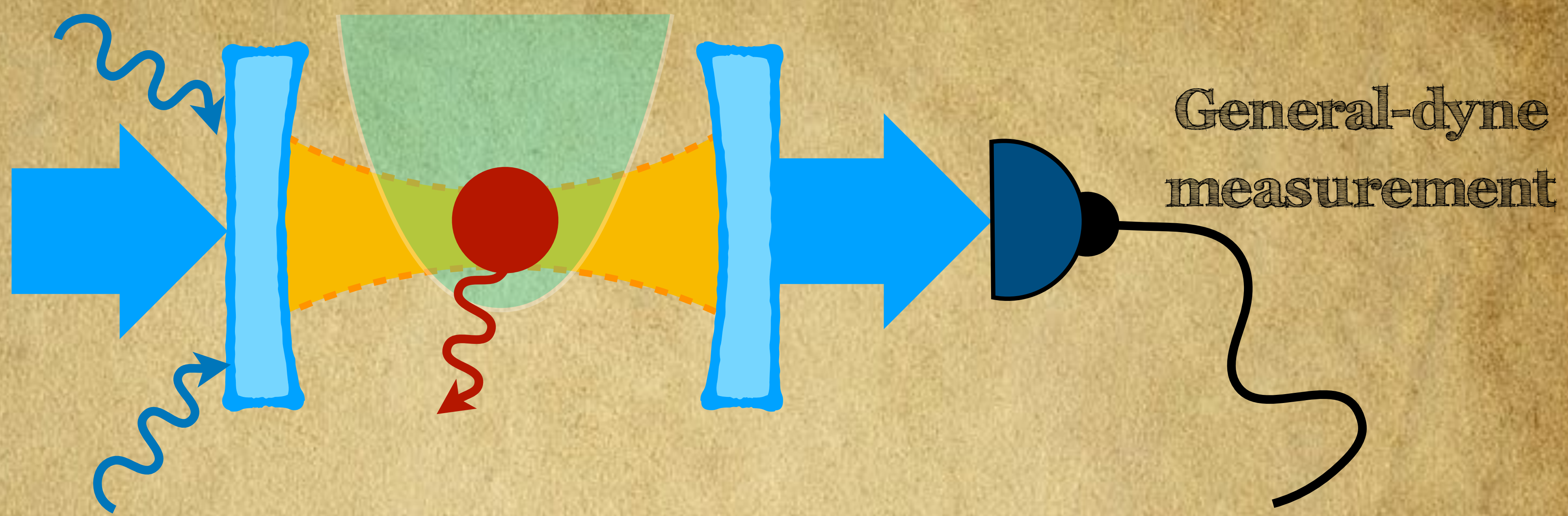
A Belenchia, et al., Nature Quantum Information 6, 97 (2019)

Conditioned Gaussian dynamics



$$dS = dS_{\mathrm{uc}} + \dot{\mathcal{J}}\, dt$$

$$\dot{\mathcal{J}} = \frac{1}{2}\mathrm{Tr}[\sigma^{-1}\mathrm{D} - \sigma^{-1}\chi(\sigma)] - \frac{1}{2}\mathrm{Tr}[\sigma_{\mathrm{uc}}^{-1}\mathrm{D}]$$

$$\phi = \mathbb{E}\left[d\Phi_{\bar{\mathbf{x}}}/dt\right]$$

$$\Pi = \mathbb{E}\left[d\Sigma_{\bar{\mathbf{x}}}/dt\right]$$

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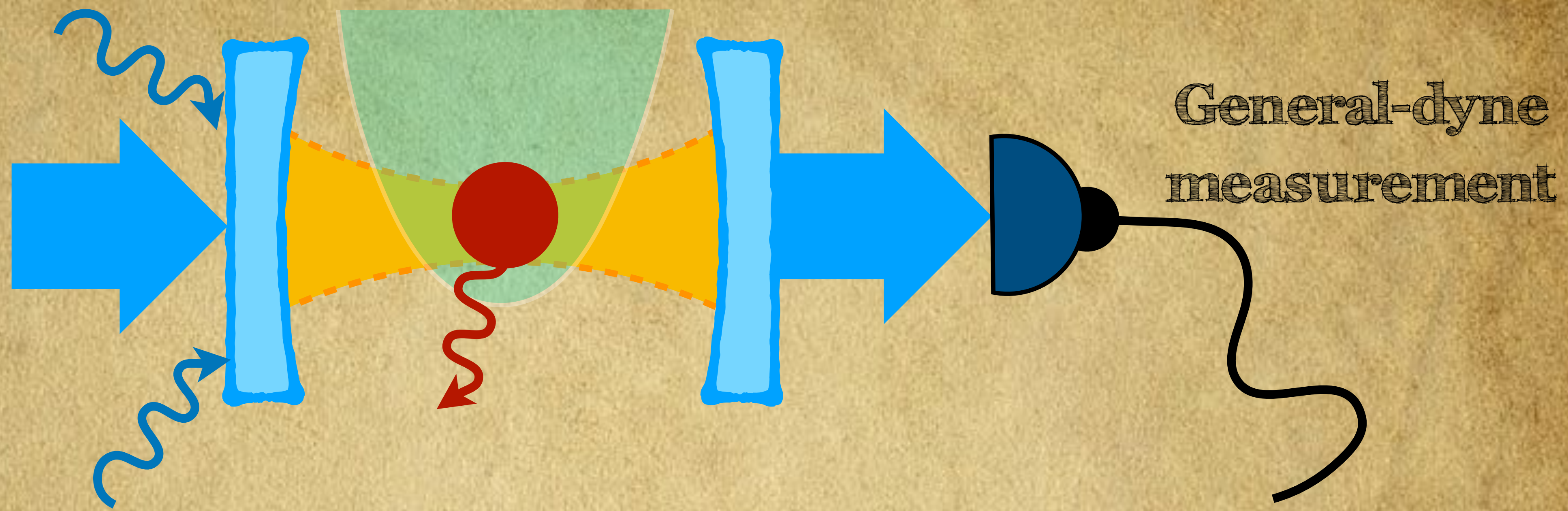
Conditioned Gaussian dynamics



$$dS = dS_{\rm uc} + \dot{\mathcal{I}}\, dt$$

$$\Pi_c(t) = \Pi_{uc}(t) + \dot{\mathcal{I}}$$

$$\phi = \mathbb{E}\left[d\Phi_{\bar{\mathbf{x}}}/dt\right]$$

$$\Pi = \mathbb{E}\left[d\Sigma_{\bar{\mathbf{x}}}/dt\right]$$

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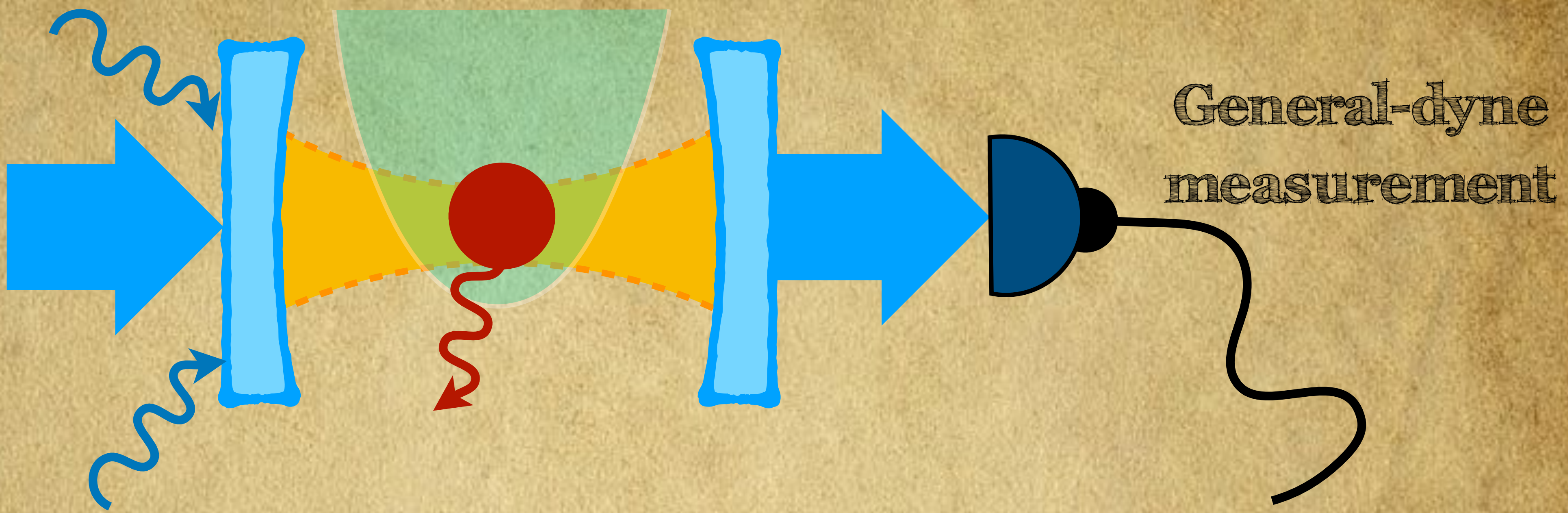
Conditioned Gaussian dynamics

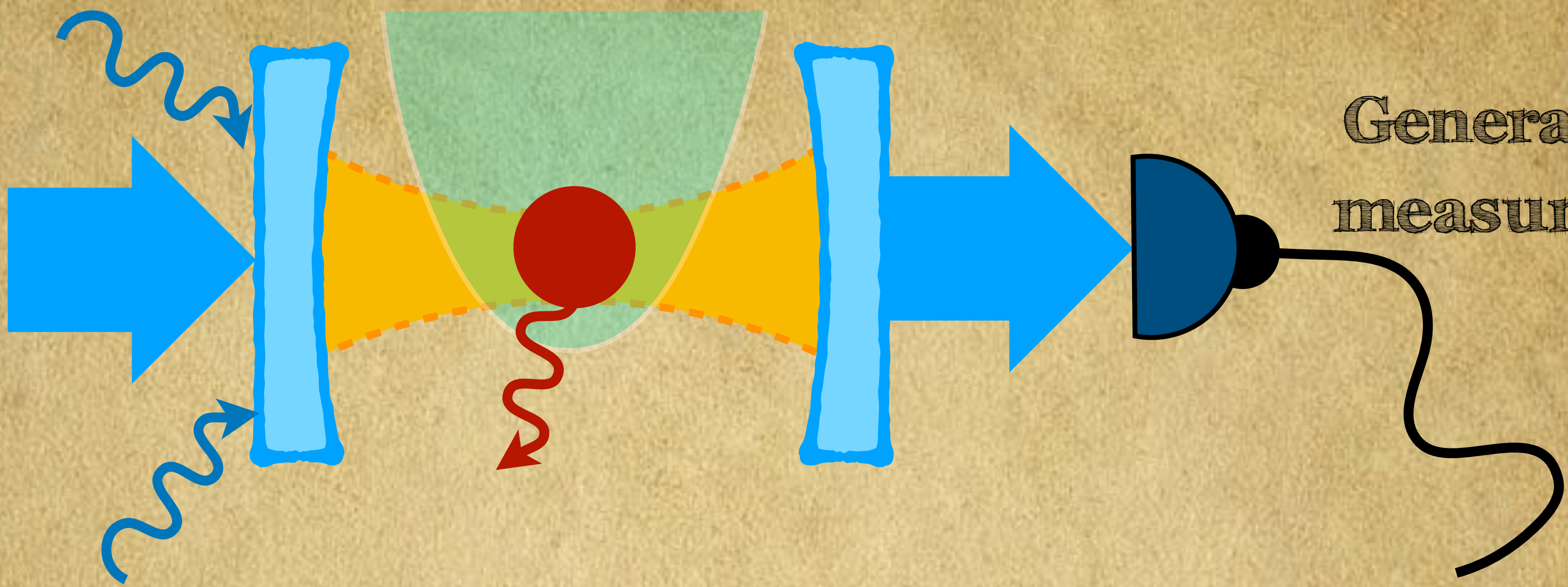


$\Pi_{uc}(t) \geq 0$ **second law for un-conditioned dynamics**

$$\Pi_c(t) = \Pi_{uc}(t) + \dot{\mathcal{J}}$$

A Belenchia, M Paternostro, and G T Landi, PRX Quantum 3, 010303 (2022)
A Belenchia, et al., Nature Quantum Information 6, 97 (2019)

Conditioned Gaussian dynamics



$\Pi_c(t) \geq \dot{\mathcal{J}}$ second law for conditioned dynamics

$$\Pi_c(t) = \Pi_{uc}(t) + \dot{\mathcal{J}}$$

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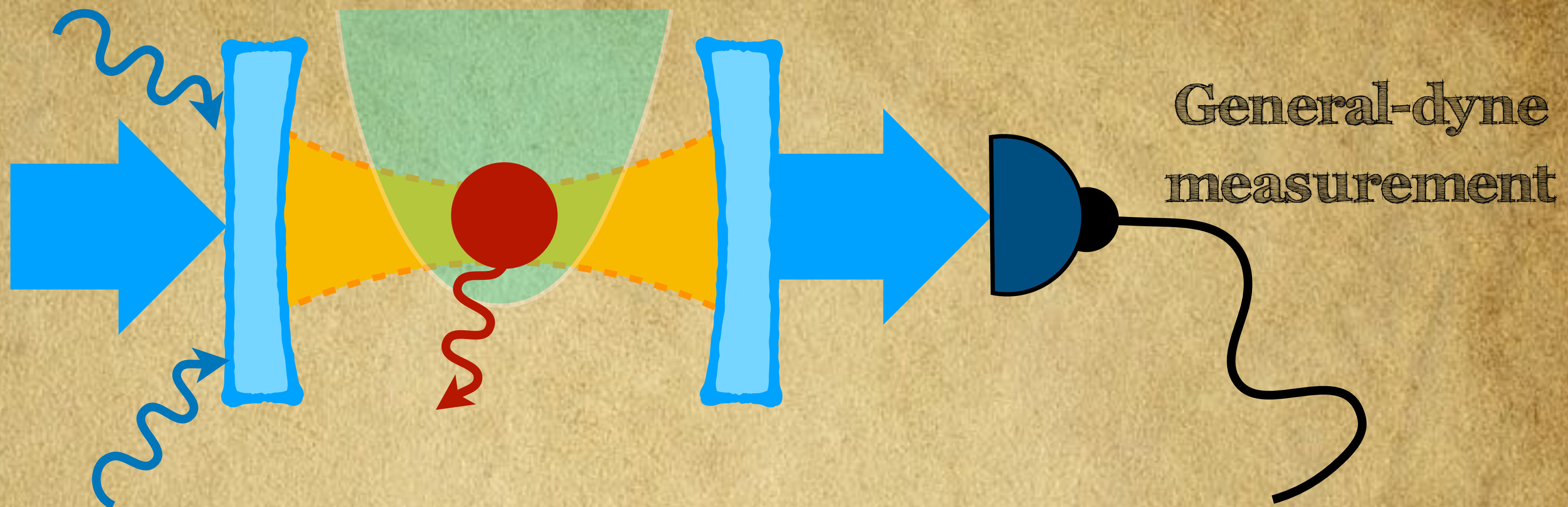
A Belenchia, et al., Nature Quantum Information 6, 97 (2019)

Conditioned Gaussian dynamics



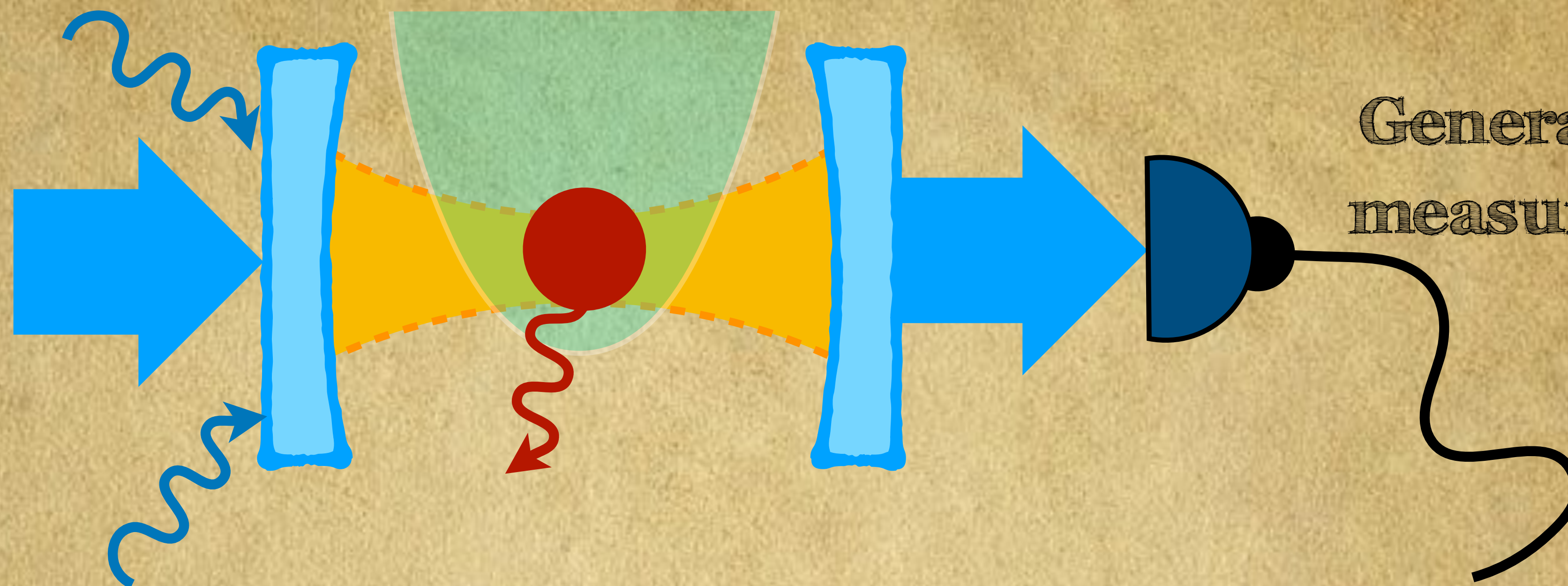
$\Sigma_c \geq \mathcal{J}$ integral second law for conditioned dynamics

$$\Pi_c(t) = \Pi_{uc}(t) + \dot{\mathcal{J}}$$

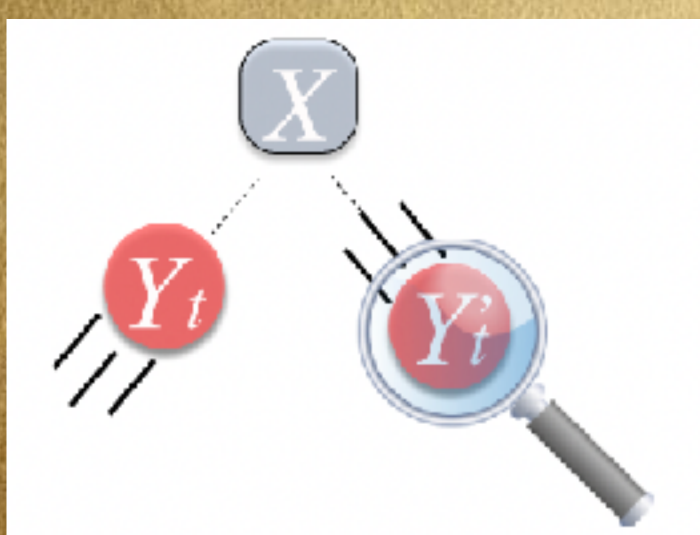
Observable (SEE PART 3)

A Belenchia, M Paternostro, and G T Landi, PRX Quantum 3, 010303 (2022)
A Belenchia, et al., Nature Quantum Information 6, 97 (2019)

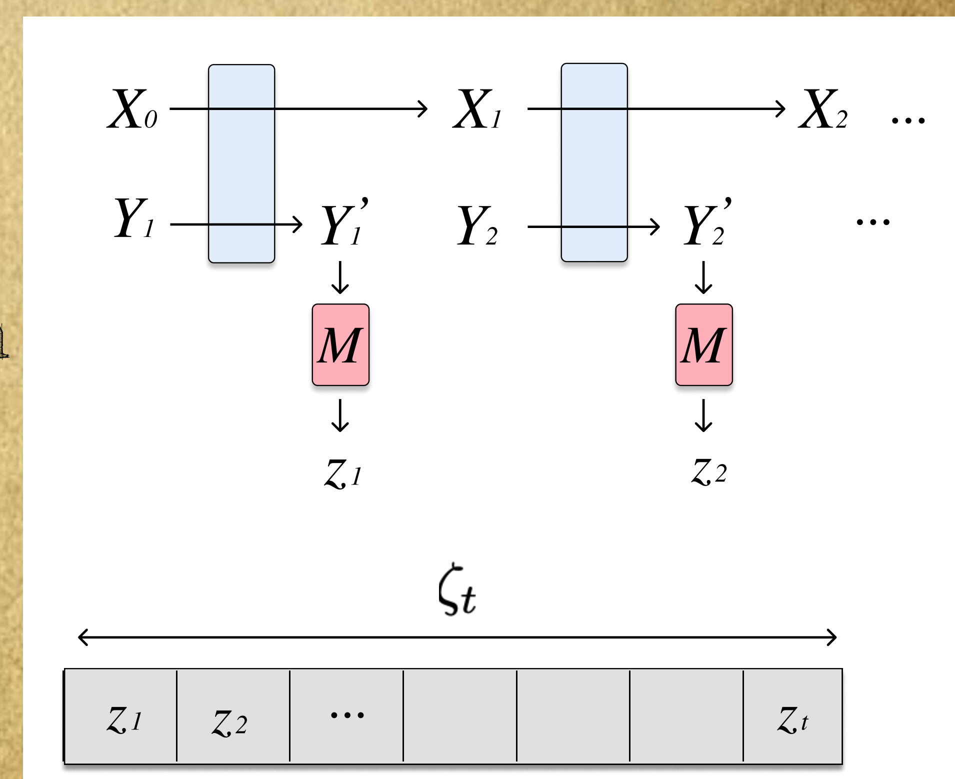
Generalising it



General-dyne measurement



Interpretation through collisional model



G T Landi, M Paternostro, and A Belenchia, PRX Quantum 3, 010303 (2022)

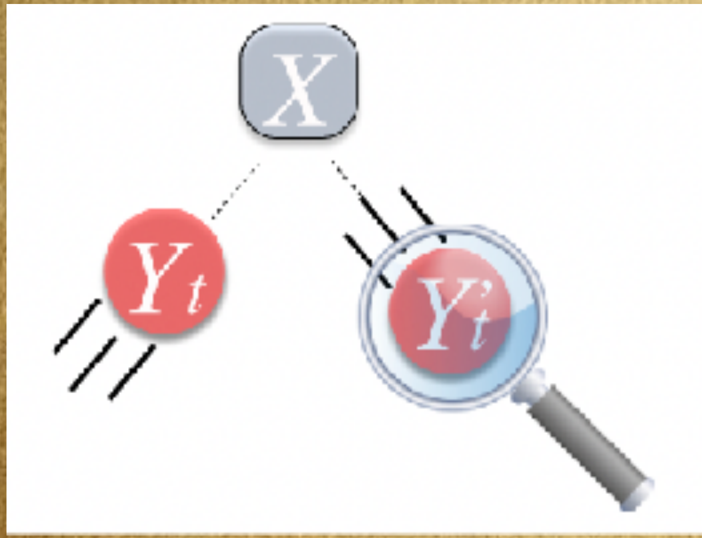
Can we generalise?

$$P(\zeta_t) = \mathrm{tr}_{XY_1\ldots Y_t}\left\{M_{z_t}\ldots M_{z_1}\rho_{XY_1\ldots Y_t}M_{z_1}^\dagger\ldots M_{z_t}^\dagger\right\}$$

$$\rho_{XY_1\ldots Y_t} = \left(\Pi_{k=1}^t U_k\right)\left(\rho_{X_0}\bigotimes_{j=1}^t \rho_{Y_j}\right)\left(\Pi_{k=1}^t U_k\right)^\dagger$$

$$\rho_{X_t|\zeta_t} = \frac{1}{P(\zeta_t)}\mathrm{tr}_{Y_1\ldots Y_t}\left\{\left(\Pi_{k=1}^t M_{z_k}\right)\rho_{XY_1\ldots Y_t}\left(\Pi_{k=1}^t M_{z_k}\right)^\dagger\right\}$$

Conditional state



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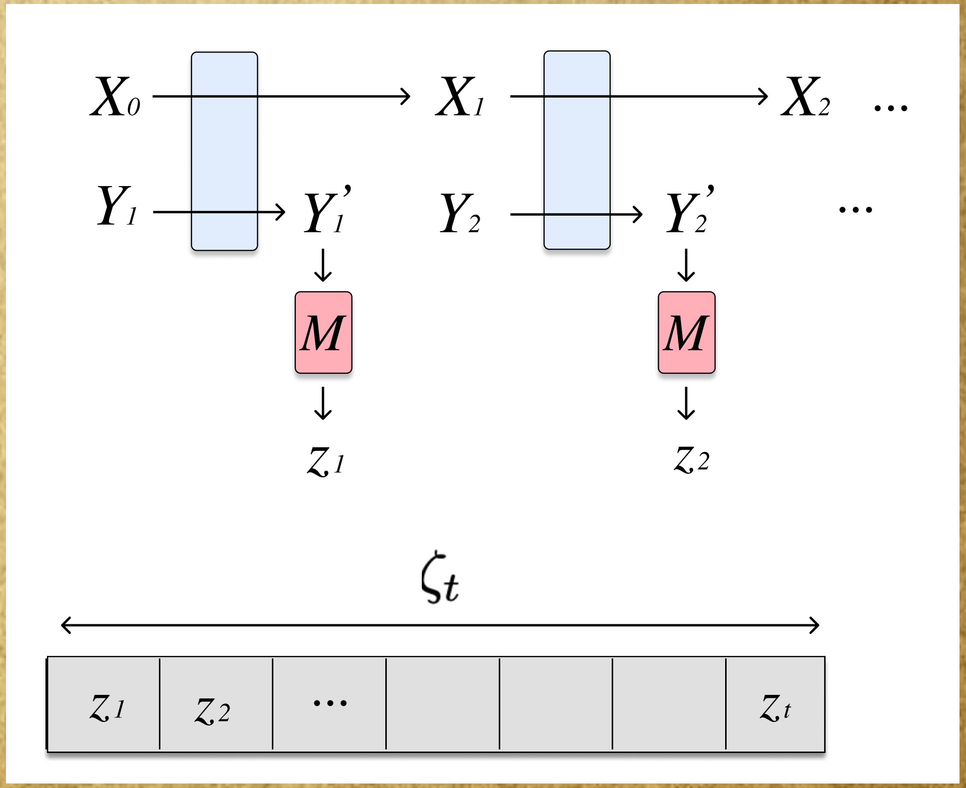


G T Landi, M Paternostro, and A Belenchia, PRX Quantum 3, 010303 (2022)

Can we generalise?

Information rate

$$\Delta I_t := I(X_t:\zeta_t) - I(X_{t-1}:\zeta_{t-1})$$

can take any sign

Holevo information: info on X contained in ζ_t

$$I(X_t:\zeta_t) := S(X_t) - S(X_t|\zeta_t)$$

strictly positive

$$\Delta I_t = G_t - L_t$$

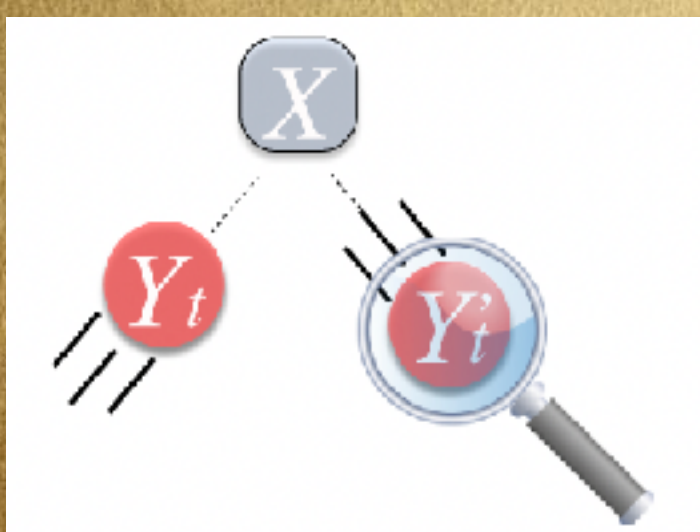
differential information gain

$$G_t = I(X_t:\zeta_t) - I(X_t:\zeta_{t-1}) = S(X_t|\zeta_{t-1}) - S(X_t|\zeta_t)$$

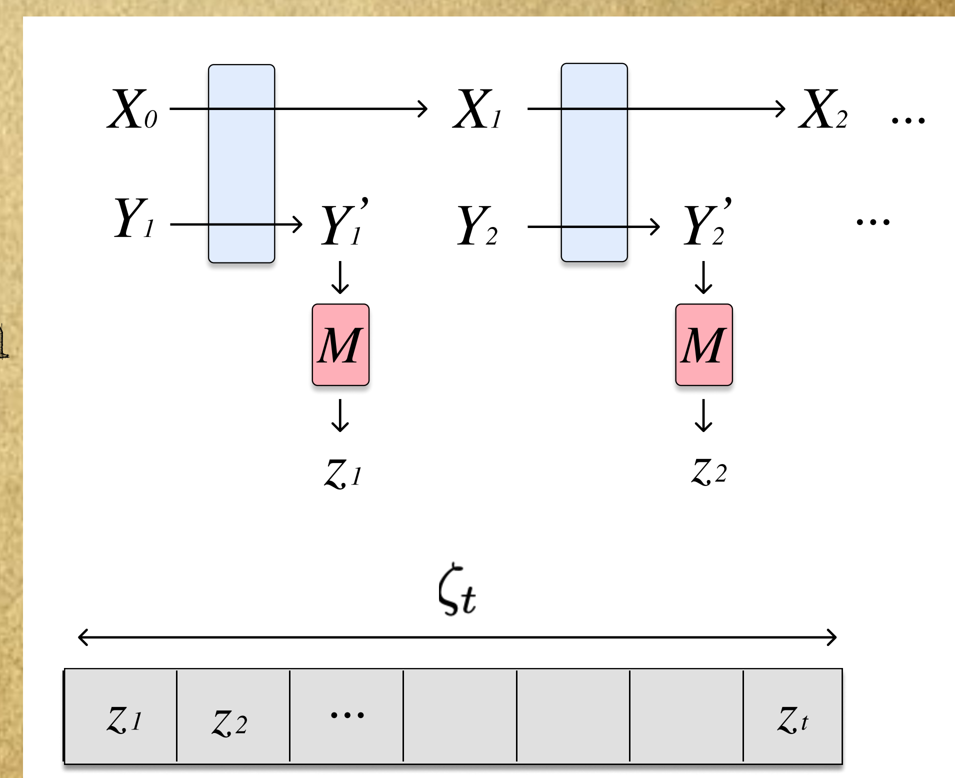
information loss

$$L_t := I(X_{t-1}:\zeta_{t-1}) - I(X_t:\zeta_{t-1})$$

non-negative



Interpretation through collisional model



G T Landi, M Paternostro, and A Belenchia, PRX Quantum 3, 010303 (2022)

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can take any sign

Holevo information: info on X contained in ζ_t

$$I(X_t : \zeta_t) := S(X_t) - S(X_t|\zeta_t)$$

strictly positive

$$\Delta I_t = G_t - L_t \xrightarrow{\text{steady state}} \Delta I_\infty = 0 \quad \text{with} \quad G_\infty = L_\infty \neq 0$$

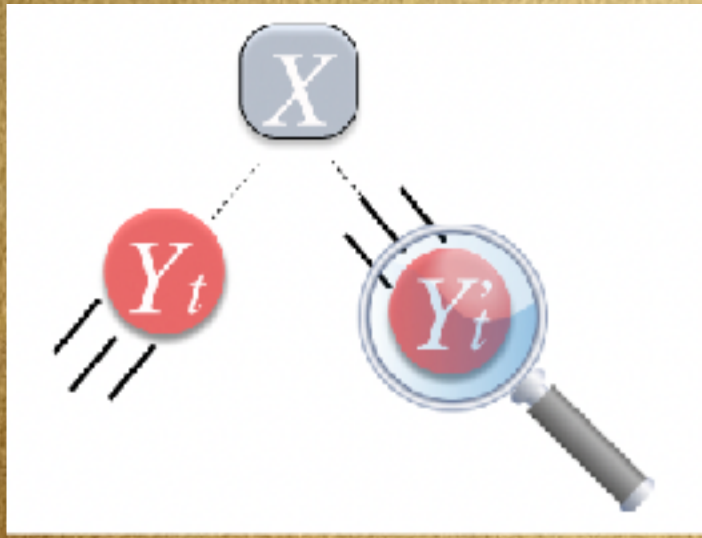
Informational steady state:

balance between gain & loss

measurements sustain the state

$$\Delta\Sigma_t^u = S(X_t) - S(X_{t-1}) + \Delta\Phi_t^u$$

$$\Delta\Sigma_t^c = S(X_t|\zeta_t) - S(X_{t-1}|\zeta_{t-1}) + \Delta\Phi_t^c$$



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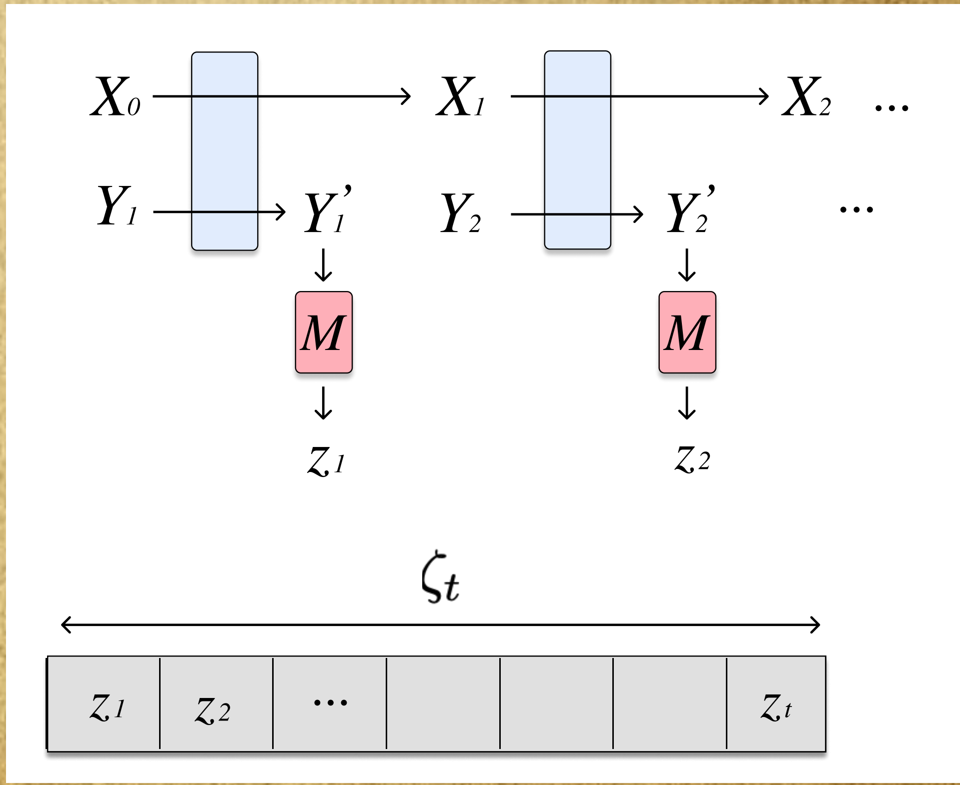

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Can we generalise?

Conditioning on the outcomes is subjective (I decide to read outcomes or not...)

No influence on flux of entropy into the ancillae

$$\Delta\Phi_t^c = \Delta\Phi_t^u$$

$$\Delta\Sigma_t^c = \Delta\Sigma_t^u + \Delta I_t$$

$$\Delta\Sigma_t^u = S(X_t) - S(X_{t-1}) + \Delta\Phi_t^u$$

$$\Delta\Sigma_t^c = S(X_t|\zeta_t) - S(X_{t-1}|\zeta_{t-1}) + \Delta\Phi_t^c$$

Informational steady state:

balance between gain & loss

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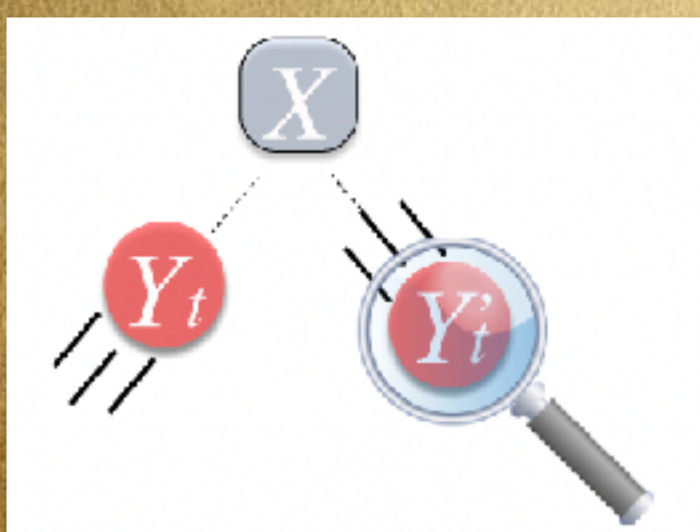

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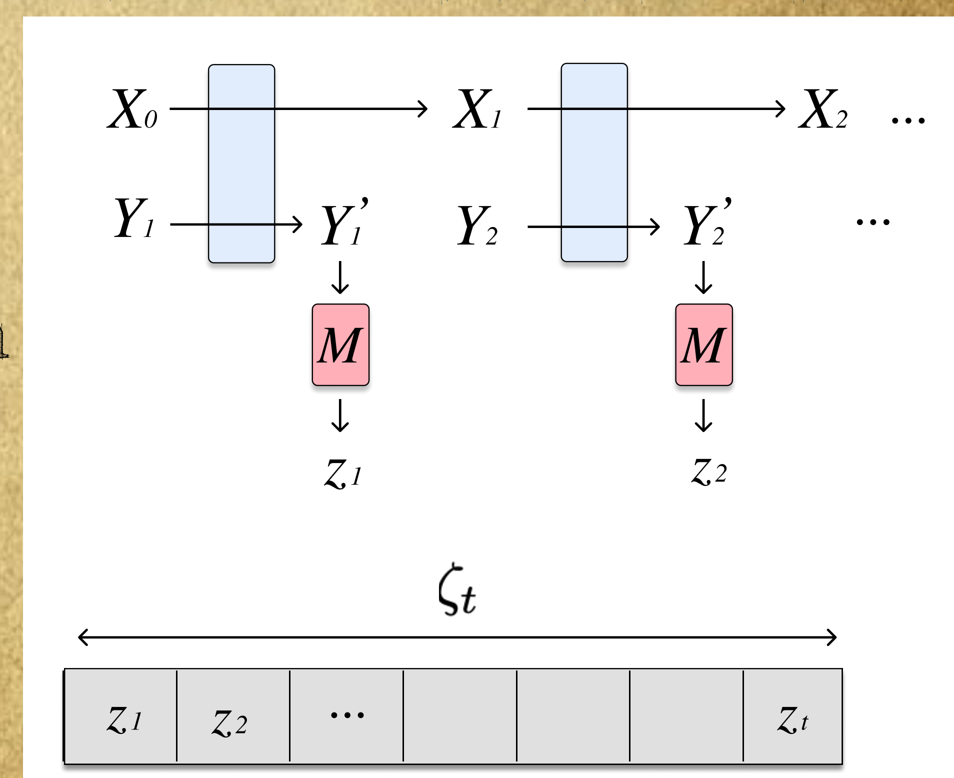
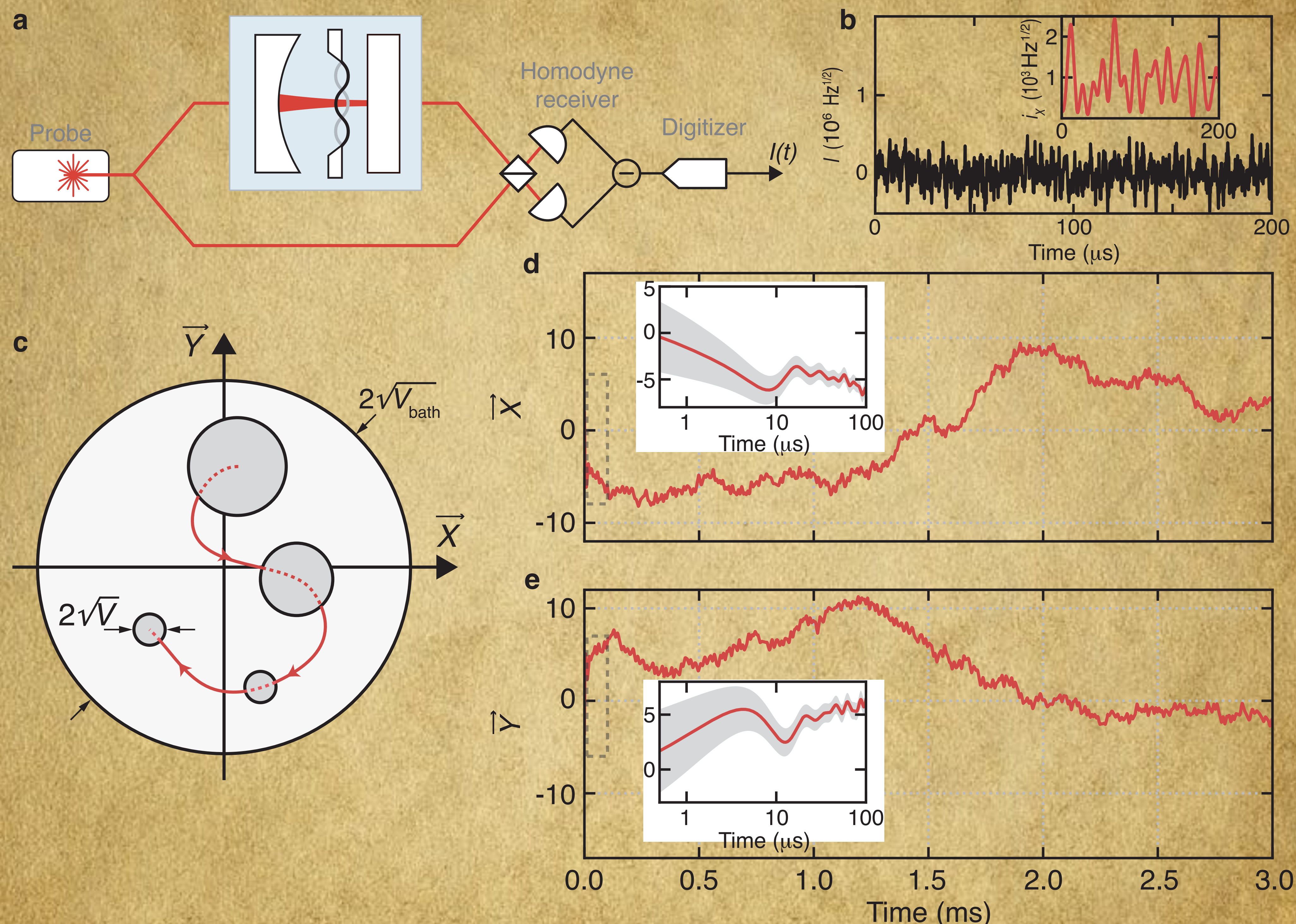


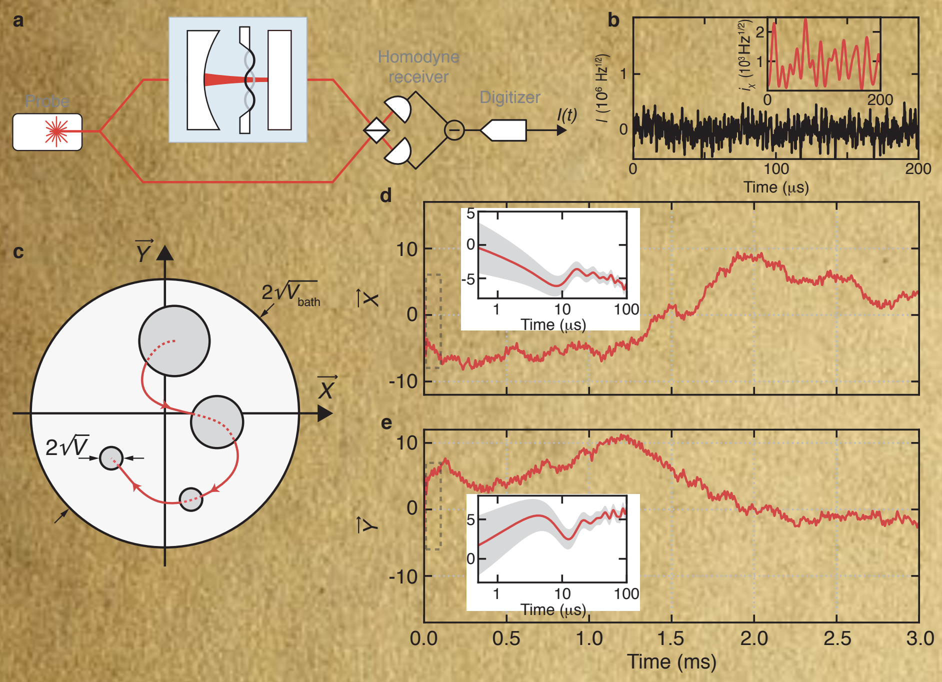
G T Landi, M Paternostro, and A Belenchia, PRX Quantum 3, 010303 (2022)

Observing trajectories of mechanical systems



M. Rossi, D. Mason, J. Chen, and A. Schliesser, Phys. Rev. Lett. 123, 163601 (2019)

Observing trajectories of mechanical systems



$$d\mathbf{r}(t) = -\frac{\Gamma_m}{2}\mathbf{r}dt + \sqrt{4\eta_{\rm det}\Gamma_{\rm qba}V(t)}d\mathbf{W},$$

$$\dot{V}(t) = \Gamma_m(V_{\rm uc} - V(t)) - 4\eta_{\rm det}\Gamma_{qba}V(t)^2$$

dynamics of the experiment

$$d\mathbf{r}(t) = A\mathbf{r}(t)dt + (V(t)C^T + \Gamma^T)d\mathbf{W},$$

$$\dot{V}(t) = AV(t) + V(t)A^T + D - \underbrace{(V(t)C^T + \Gamma^T)(CV(t) + \Gamma^T)}_{\chi(V(t))}$$

theoretical counterpart

M. Rossi, L. Mancino, G. T. Landi, M. Paternostro, A. Schliesser, and A. Belenchia, Phys. Rev. Lett. 125, 080601 (2020)

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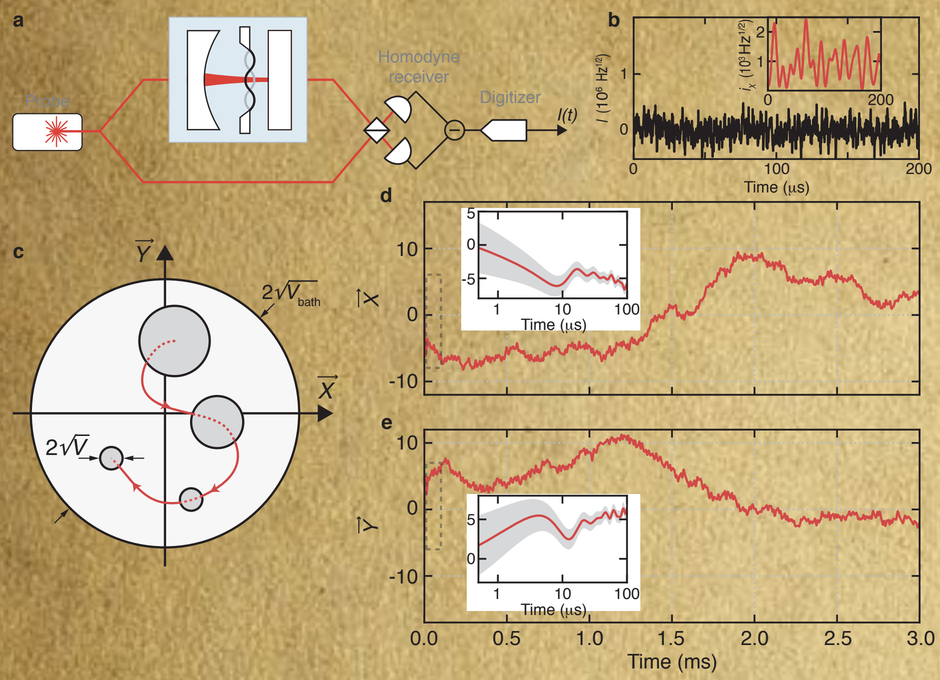
Observing trajectories of mechanical systems



Initial state: equilibrium state at environment temperature

↓

Steady state of the unconditional dynamics: NESS very close to equilibrium

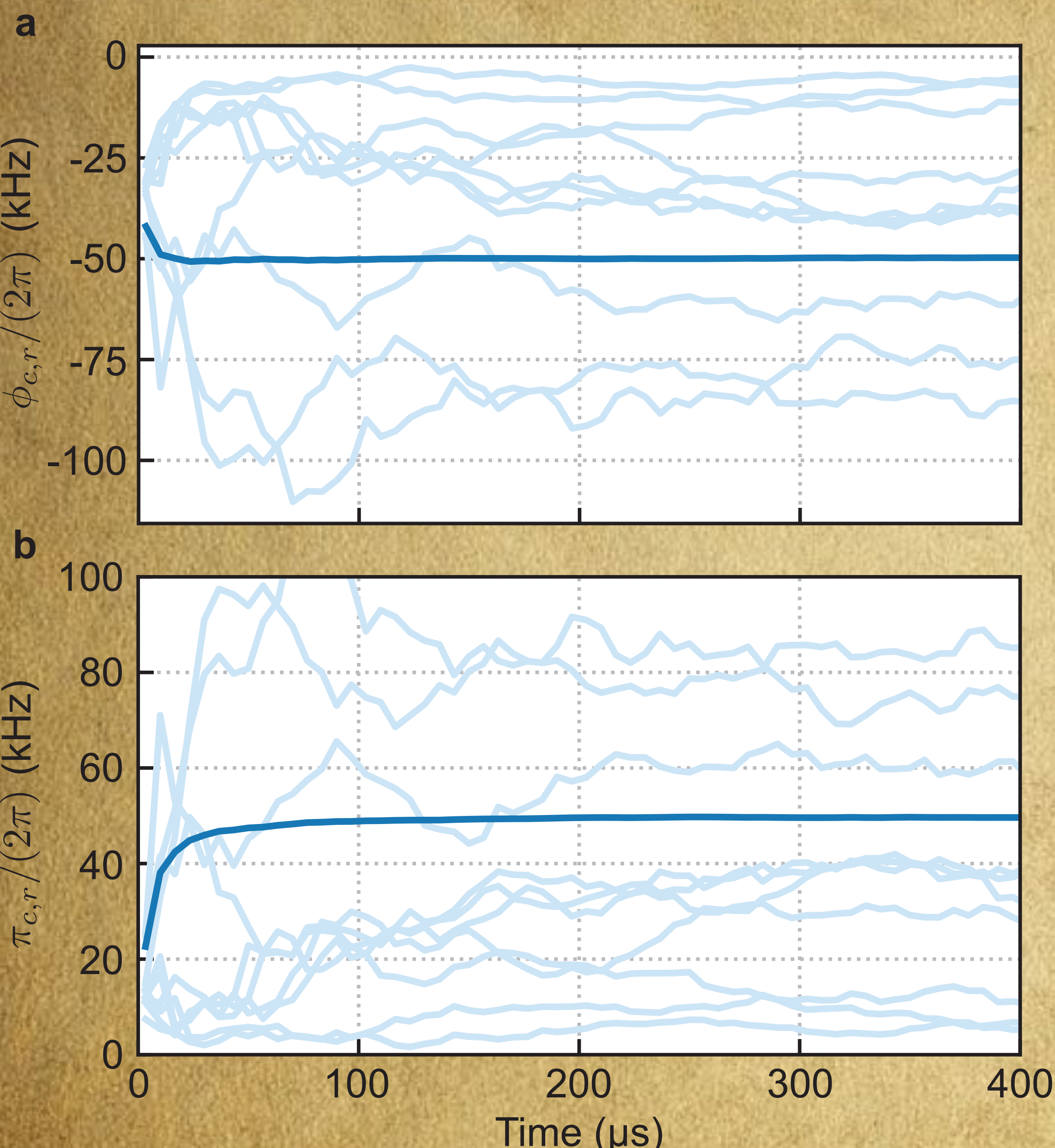
$$\Pi_{uc}(t) = \Gamma_m \left| V_{\rm uc}/(n_{\rm th}+1/2)-1\right| + 4\Gamma_{\rm qba} V_{\rm uc}$$

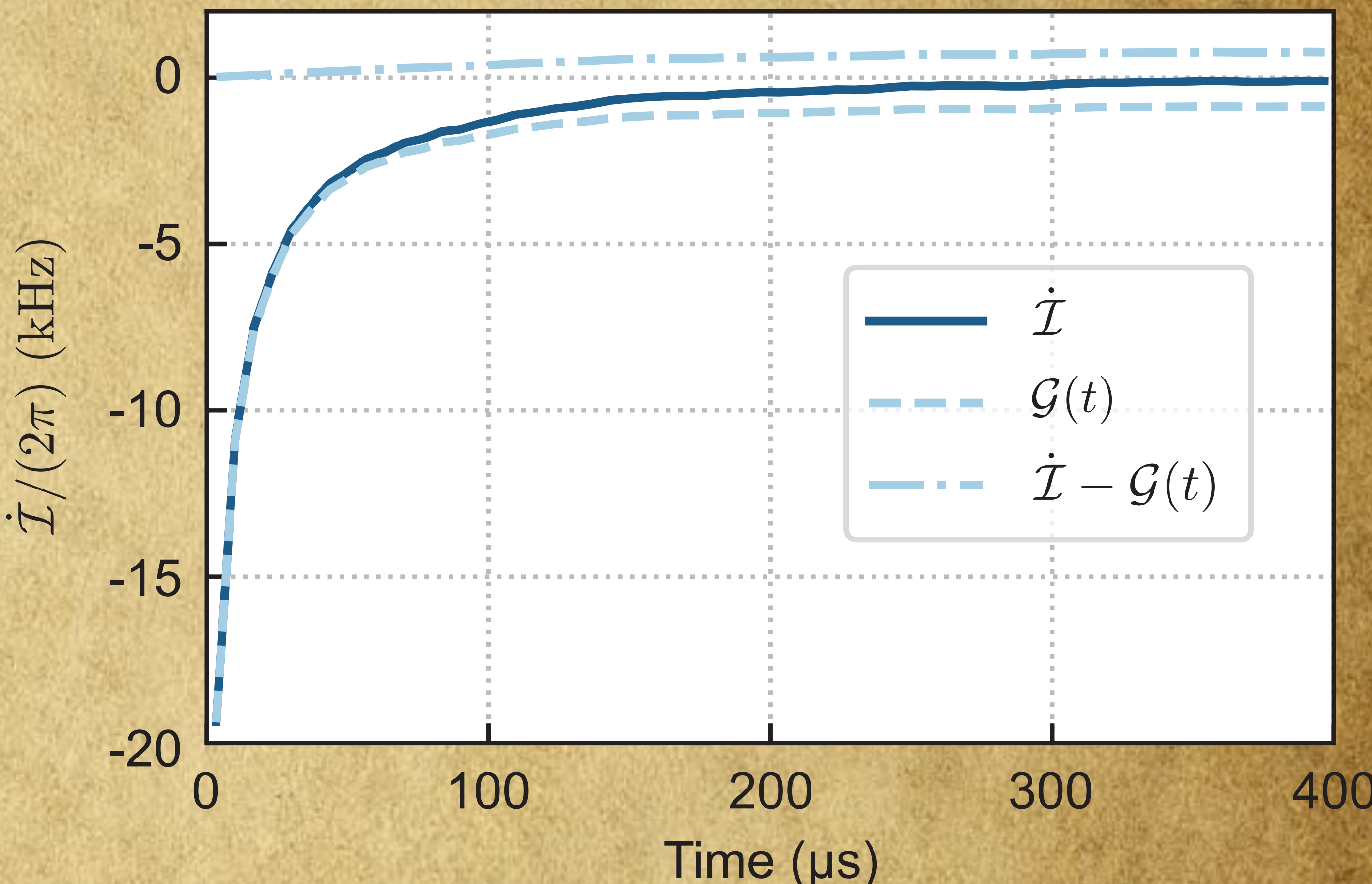
$\Pi_{uc}(t) = \text{const}\,.$ and $\Pi_c(t) = \dot{\mathcal{J}} + \text{const}\,.$

$$\dot{\mathcal{J}} = \Gamma_{\rm m}\left(V_{\rm uc}/V(t) - 1\right) - 4\eta_{\rm det}\Gamma_{\rm qba} V(t)$$

M. Rossi, L. Mancino, G. T. Landi, M. Paternostro, A. Schliesser, and A. Belenchia, Phys. Rev. Lett. 125, 080601 (2020)

Observing entropy production rates of a measured system





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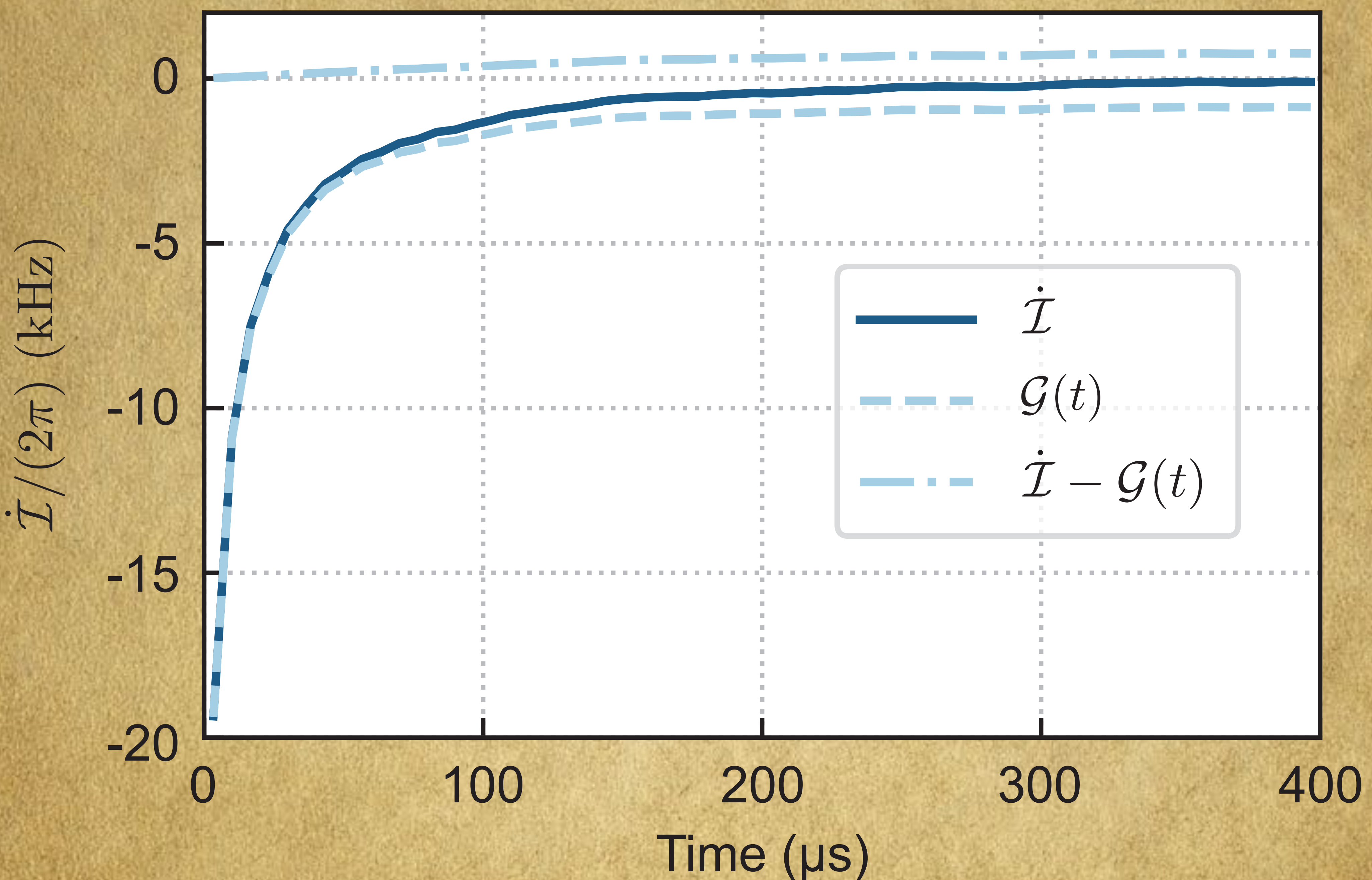

M. Rossi, L. Mancino, G. T. Landi, M. Paternostro, A. Schliesser, and A. Belenchia, Phys. Rev. Lett. 125, 080601 (2020)

Bread on tables..

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The crews

in (very) warm places..

..& (slightly) colder ones.

Shameless advertisement

At least 1 PhD position

Several Postdoc positions soon to be open to work on quantum thermodynamics for quantum computation

m.paternostro@qub.ac.uk mauro.paternostro@unipa.it

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