UNIVERSITY OF SURREY

QUANTUM-CLASSICAL INTERFACE IN CLOSED AND OPEN SYSTEMS
June 21, 2023

General robust preparation of maximally entangled states via identical particle interferometry

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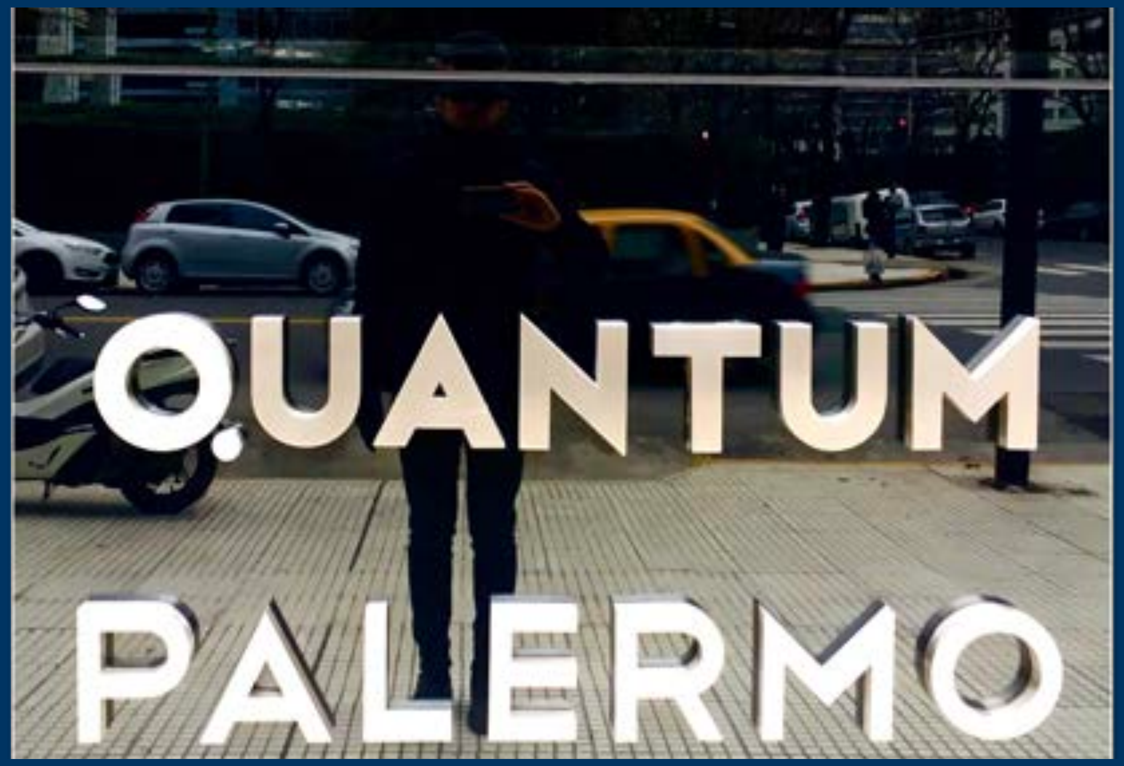

[arXiv:2303.11484]

Asymptotically-deterministic robust preparation of maximally entangled bosonic states

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[arXiv:2305.14285]

Robust engineering of maximally entangled states by identical particle interferometry

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The problem with decoherence

Strategies to protect quantum states from detrimental effects of noise

Portrait of an Artist (Pool with Two Figures)
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Use particle indistinguishability in interferometric setups

Outline

- **Scenario:** two identical qubits subject to arbitrary local noise
- **Objective:** preparing maximally entangled states in a robust way with high success rate
- **Resource:** particle indistinguishability in interferometric setups
- **Key elements:** Linear optics devices | Externally-activated noise | Non-absorbing parity check detector
- **Applicability:** bosons or fermions in arbitrary initial states

- **Results:**
- *Asymptotically-deterministic robust preparation of maximally entangled states*
- *Independent of both initial state and type of noise*
- *Preparing any maximally entangled two-qubit state through passive optical equivalences*

Two qubits in two spatial modes: maximally entangled states

- Separated spatial modes L, R ➟ Hilbert space with **two bases**

Bell states

$$\mathcal{B}_{\mathrm{LR}} := \begin{cases} |1_\pm\rangle_{\mathrm{LR}} := \frac{1}{\sqrt{2}}\Big(|L\uparrow, R\downarrow\rangle \pm |L\downarrow, R\uparrow\rangle\Big) \\ |2_\pm\rangle_{\mathrm{LR}} := \frac{1}{\sqrt{2}}\Big(|L\uparrow, R\uparrow\rangle \pm |L\downarrow, R\downarrow\rangle\Big) \end{cases}$$

Particle number on one spatial mode:
odd parity (1)

NOON states

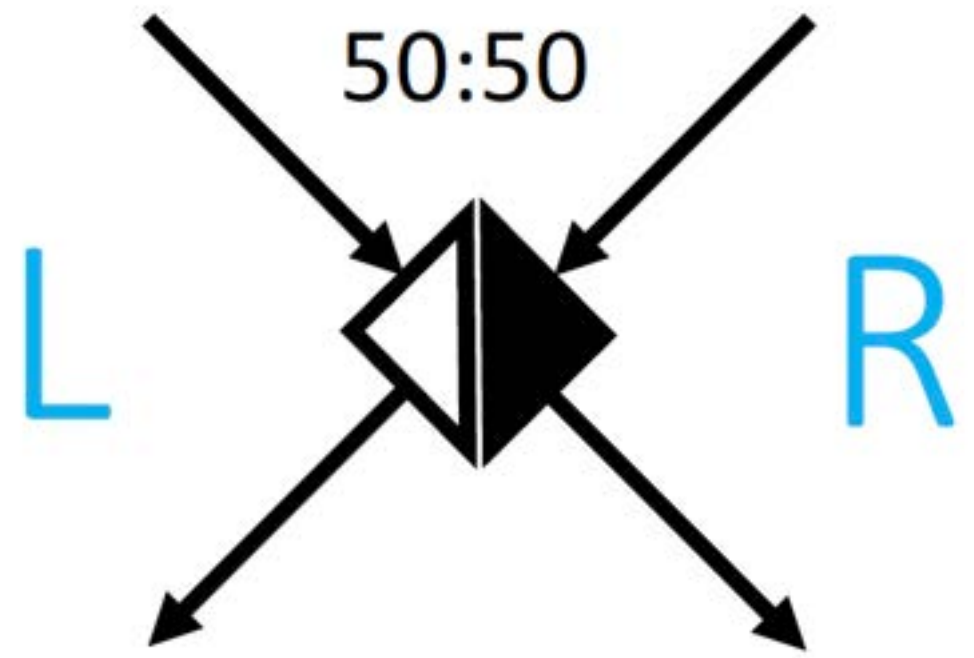
$$\mathcal{B}_{\mathrm{NO}} := \begin{cases} |1_\pm\rangle_{\mathrm{NO}} := \frac{1}{\sqrt{2}}\Big(|L\uparrow, L\downarrow\rangle \pm |R\uparrow, R\downarrow\rangle\Big), \\ |U_\pm\rangle_{\mathrm{NO}} := \frac{1}{2}\Big(|L\uparrow, L\uparrow\rangle \pm |R\uparrow, R\uparrow\rangle\Big), \\ |D_\pm\rangle_{\mathrm{NO}} := \frac{1}{2}\Big(|L\downarrow, L\downarrow\rangle \pm |R\downarrow, R\downarrow\rangle\Big). \end{cases}$$

Particle number on one spatial mode:
even parity (0 or 2)

Bosons: **10-dimensional**

Fermions: **6-dimensional**

Beam splitting action on Bell states



$|L\rangle \longrightarrow (|L\rangle + |R\rangle)/\sqrt{2} \qquad |R\rangle \longrightarrow (|L\rangle - |R\rangle)/\sqrt{2}$

Bosons

$$\begin{cases} |1_-\rangle_{\mathrm{LR}} \longleftrightarrow -|1_-\rangle_{\mathrm{LR}}\,, \\ |1_+\rangle_{\mathrm{LR}} \longleftrightarrow |1_-\rangle_{\mathrm{NO}}\,, \\ |2_-\rangle_{\mathrm{LR}} \longleftrightarrow (|U_-\rangle_{\mathrm{NO}} - |D_-\rangle_{\mathrm{NO}})/\sqrt{2}, \\ |2_+\rangle_{\mathrm{LR}} \longleftrightarrow (|U_-\rangle_{\mathrm{NO}} + |D_-\rangle_{\mathrm{NO}})/\sqrt{2}. \end{cases}$$

antibunching (first line); bunching (remaining lines)

Fermions

$$\begin{cases} |1_-\rangle_{\mathrm{LR}} \longleftrightarrow |1_-\rangle_{\mathrm{NO}}\,, \\ |1_+\rangle_{\mathrm{LR}} \longleftrightarrow -|1_+\rangle_{\mathrm{LR}} \\ |2_-\rangle_{\mathrm{LR}} \longleftrightarrow -|2_-\rangle_{\mathrm{LR}} \\ |2_+\rangle_{\mathrm{LR}} \longleftrightarrow -|2_+\rangle_{\mathrm{LR}} \end{cases}$$

bunching (first line); antibunching (remaining lines)

- **Interference** occurring due to **indistinguishability** of identical particles
- For both bosons and fermions: the **singlet state assumes a different parity** from the other Bell states after the action of a 50:50 beam splitter (BS)

Externally-activated depolarizing channel: resetting the system

- Complete local depolarizing noise on each qubit (whatever initial state and noisy environment)

maximally mixed state

$$\rho_{\text{dep}} := \frac{1}{4}\,\Pi_{\text{LR}} = \frac{1}{4}\sum_{|v\rangle\in\mathcal{B}_{\text{LR}}} |v\rangle\langle v|$$

Action of a 50:50 BS

Bosons

$$\rho_{\text{BS}} = \frac{1}{4}\,|1_-\rangle_{\text{LR}}\langle 1_-|_{\text{LR}} + \frac{3}{4}\rho_{\text{NO}}$$

$$\rho_{\text{NO}} := \frac{1}{3}\Big(|1_-\rangle_{\text{NO}}\langle 1_-|_{\text{NO}} + |U_-\rangle_{\text{NO}}\langle U_-|_{\text{NO}} + |D_-\rangle_{\text{NO}}\langle D_-|_{\text{NO}}\Big)$$

Fermions

$$\rho_{\text{BS}} = \frac{1}{4}\,|1_-\rangle_{\text{NO}}\langle 1_-|_{\text{NO}} + \frac{3}{4}\rho_{\text{LR}}$$

$$\rho_{\text{LR}} := \frac{1}{3}\Big(|1_+\rangle_{\text{LR}}\langle 1_+|_{\text{LR}} + |2_+\rangle_{\text{LR}}\langle 2_+|_{\text{LR}} + |2_-\rangle_{\text{LR}}\langle 2_-|_{\text{LR}}\Big)$$

Parity check detector: distilling maximally entangled state

Bosons

$$\rho_{\mathrm{BS}} = \frac{1}{4} |1_-\rangle_{\mathrm{LR}} \langle 1_-|_{\mathrm{LR}} + \frac{3}{4}\rho_{\mathrm{NO}}$$

Fermions

$$\rho_{\mathrm{BS}} = \frac{1}{4} |1_-\rangle_{\mathrm{NO}} \langle 1_-|_{\mathrm{NO}} + \frac{3}{4}\rho_{\mathrm{LR}}$$

singlet component assumes a parity different from the others

- A *pseudospin-independent parity check detector* can discriminate it probabilistically

POVM: $\{\Pi_{\mathrm{LR}}, \Pi_{\mathrm{NO}}\}$

$\Pi_{\mathrm{LR}} = \sum_{|v\rangle \in \mathcal{B}_{\mathrm{LR}}} |v\rangle\langle v|$ **(odd parity)**

$\Pi_{\mathrm{NO}} = \sum_{|k\rangle \in \mathcal{B}_{\mathrm{NO}}} |k\rangle\langle k|$ **(even parity)**

$$\Pi_{\mathrm{LR}} + \Pi_{\mathrm{NO}} = \mathbf{I}$$

Iterating the process: asymptotically-deterministic preparation

- Assuming the parity-check detector is ***non-absorbing***, the process can be iterated until it succeeds

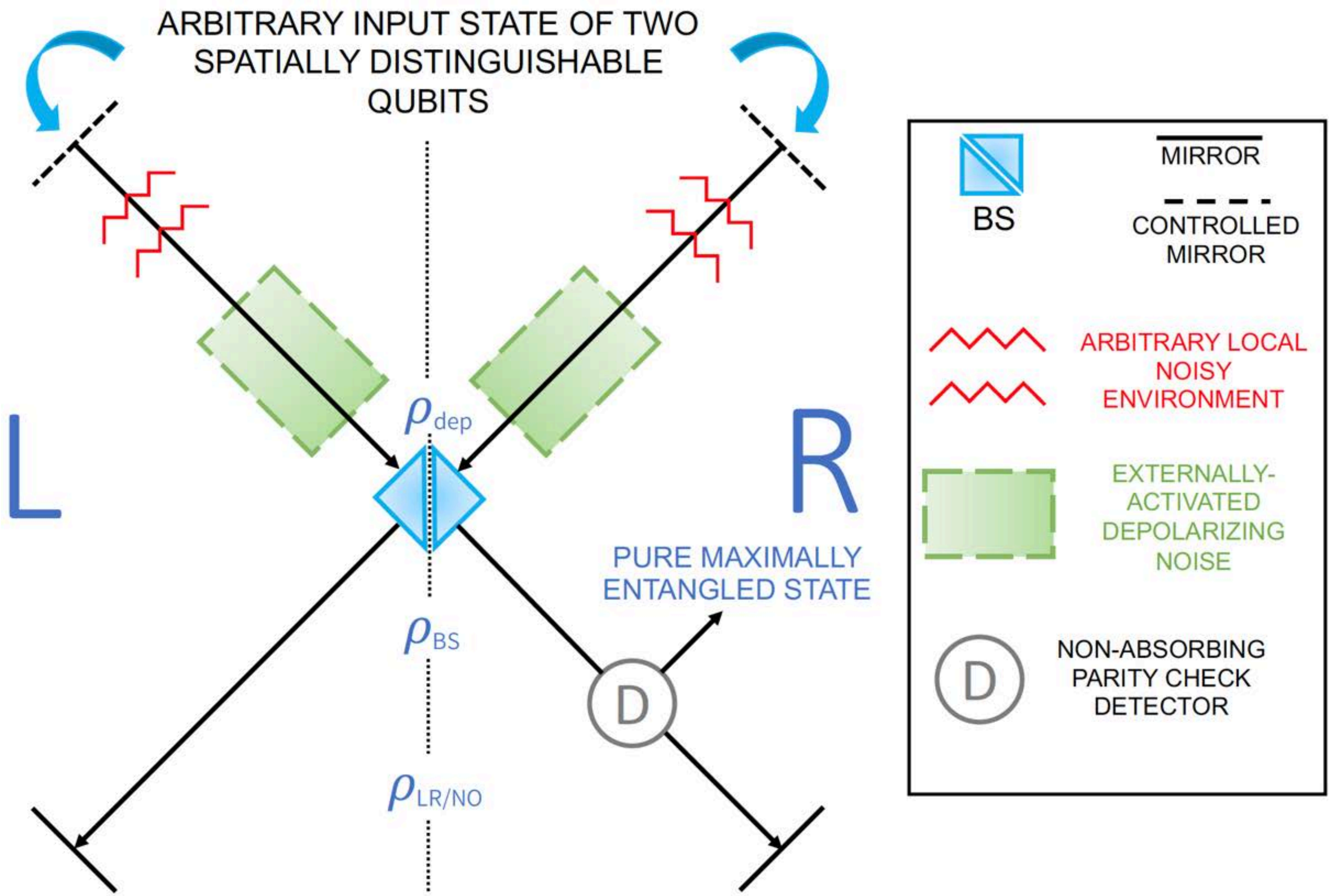
The undistilled pair of particles impinge on a BS once again, re-acquiring original odd parity (one qubit per spatial mode)

Resetting the system: each qubit is again depolarized, leading to a classical mixture of Bell states

A new BS operation is applied

A new parity detection is performed

Theoretical scheme: closed implementation



Distillation probability at the first iteration:

$$p_{\mathrm{bos}}^{(1)} = \mathrm{Tr}[\Pi_{\mathrm{LR}}\, \rho_{\mathrm{BS}}] = 1/4$$

$$p_{\mathrm{fer}}^{(1)} = \mathrm{Tr}[\Pi_{\mathrm{NO}}\, \rho_{\mathrm{BS}}] = 1/4$$

Distillation probability at the *j*-th iteration:

$$p_{\mathrm{bos}}^{(j)} = p_{fer}^{(j)} = \sum_{n=1}^{j} \left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^{n-1}$$

converges to 1 exponentially for $j \to +\infty$

Can we practically connect all the different maximally entangled states?

$|1_-\rangle_{\mathrm{LR}}$ distilled for bosons

$|1_-\rangle_{\mathrm{NO}}$ distilled for fermions

The Splash © *David Hockney, 1966*

Passive Optical (PO) operations

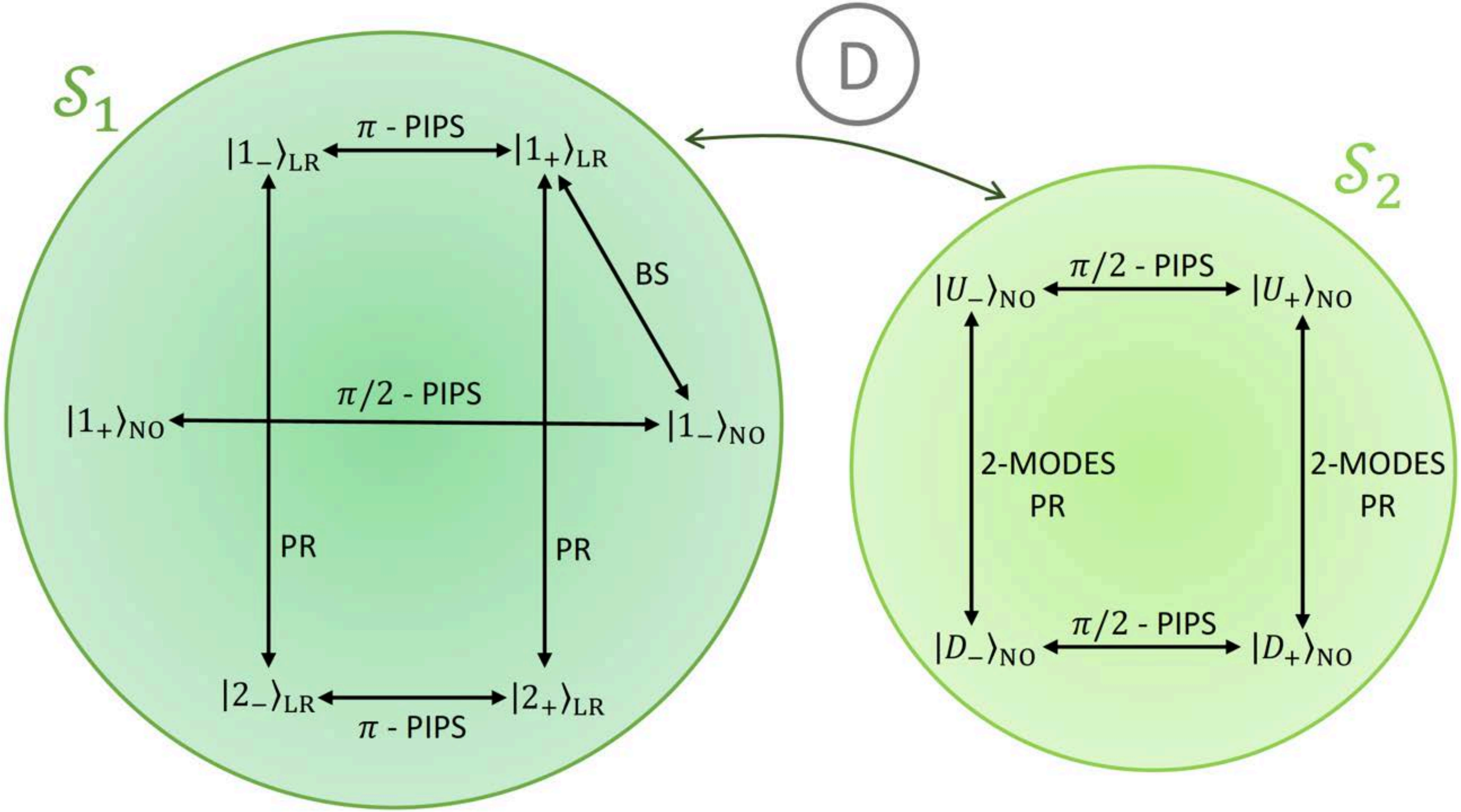
- **PO operations:** extension to generic bosons and fermions of the set of transformations which, in a photonic implementation, can be obtained by a proper sequence of
  - Beam splitters (BSs)
  - Polarization BSs (PBSs)
  - Local polarization-dependent or -independent phase shifters (PDPSs/PIPSs)
  - Local polarization rotators (PRs): $|\uparrow\rangle \leftrightarrow |\downarrow\rangle$

- Two or more states are *PO equivalent* if they can be obtained from one another **by means of PO operations**

- Extending previously introduced local operations connecting only Bell states
  [C. H. Bennett, G. Brassard, S. Popescu, B. Schumacher, J. A. Smolin, and W. K. Wootters, Phys. Rev. Lett. **76**, 722 (1996)]

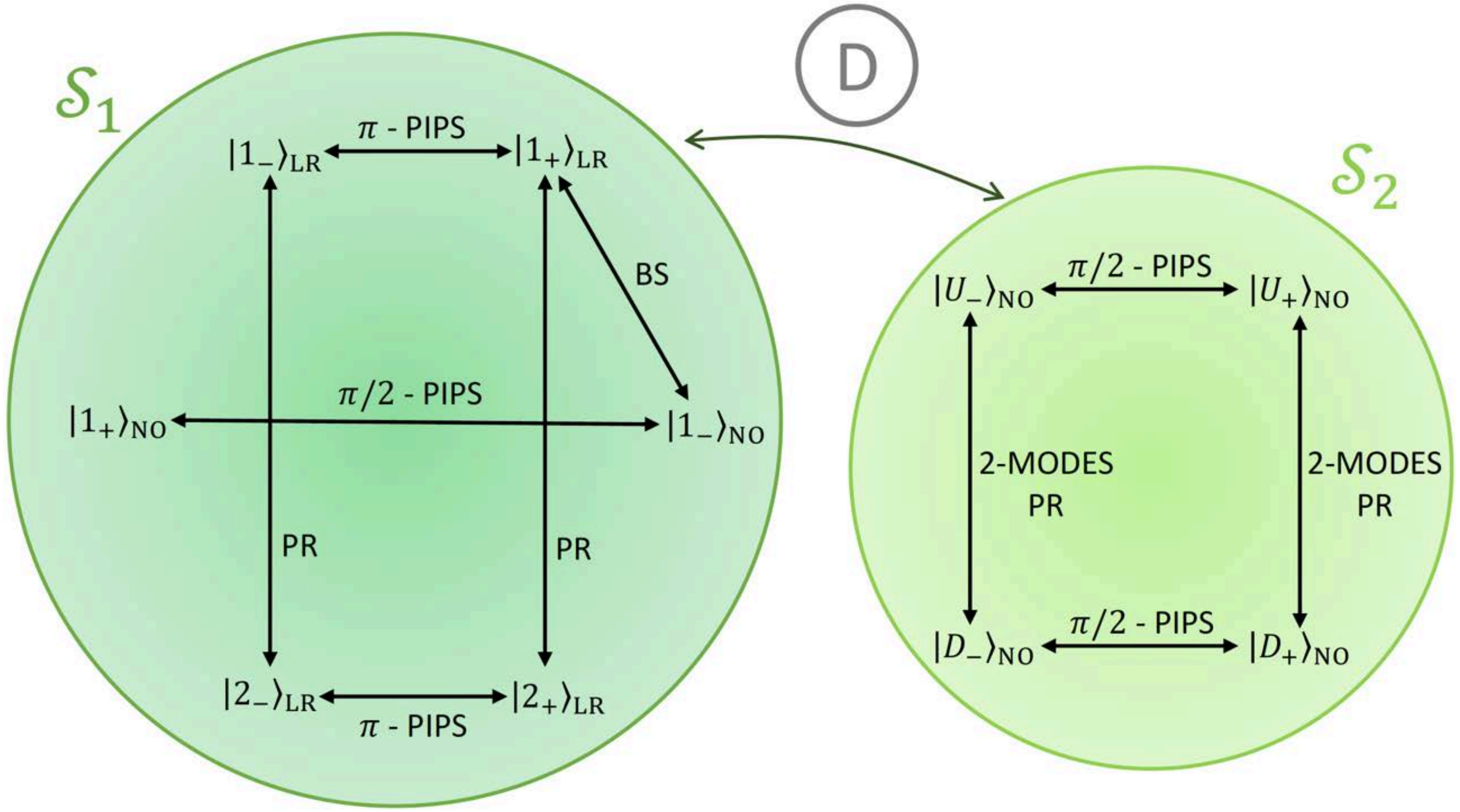
Passive Optical (PO) operations - Bosons

Bosons: two sets $\mathcal{S}_1$ and $\mathcal{S}_2$ of PO equivalent states, linkable by the non-absorbing parity check detector D, such that $\mathcal{S}_1 \cup \mathcal{S}_2 = \mathcal{B}_{\mathrm{LR}} \cup \mathcal{B}_{\mathrm{NO}}$



Passive Optical (PO) operations - Bosons

$\mathcal{S}_1 \rightarrow \mathcal{S}_2$



Mach-Zehnder interferometer as a *polarization sensitive detector*

$|U_-\rangle_{NO}$

↑ BS

$|L\uparrow, R\uparrow\rangle$

↑ PBS on mode R_1

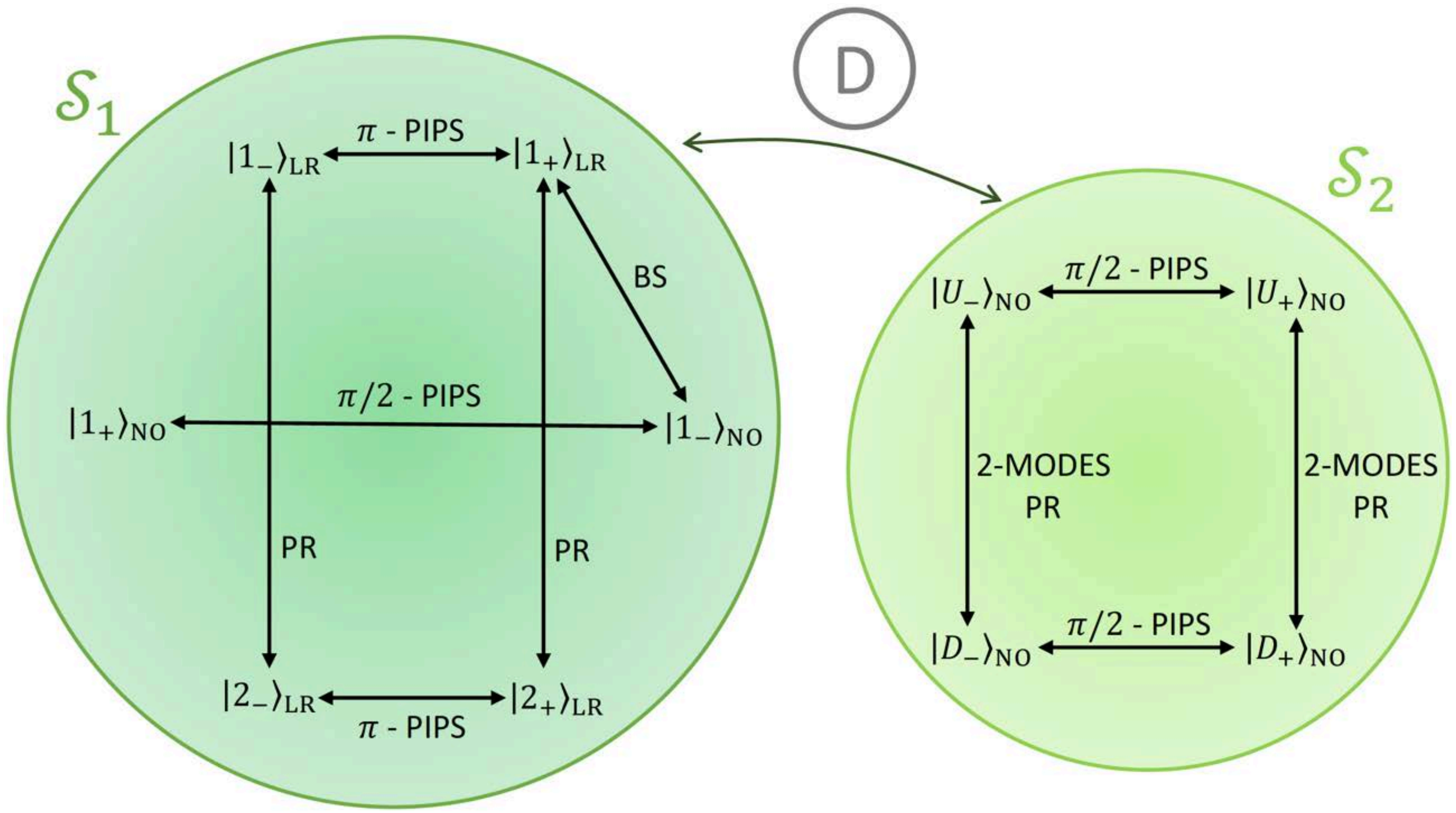
$|2_+\rangle_{LR}$ → PBS on mode R → $|L\uparrow, R_1\uparrow\rangle + |L\downarrow, R_2\downarrow\rangle$ → D on mode R_1 → $|L\uparrow, R_1\uparrow\rangle$

or

$|L\downarrow, R_2\downarrow\rangle$ → PBS on mode R_2 → $|L\downarrow, R\downarrow\rangle$ → BS → $|D_-\rangle_{NO}$

Passive Optical (PO) operations - Bosons

$\mathcal{S}_2 \rightarrow \mathcal{S}_1$



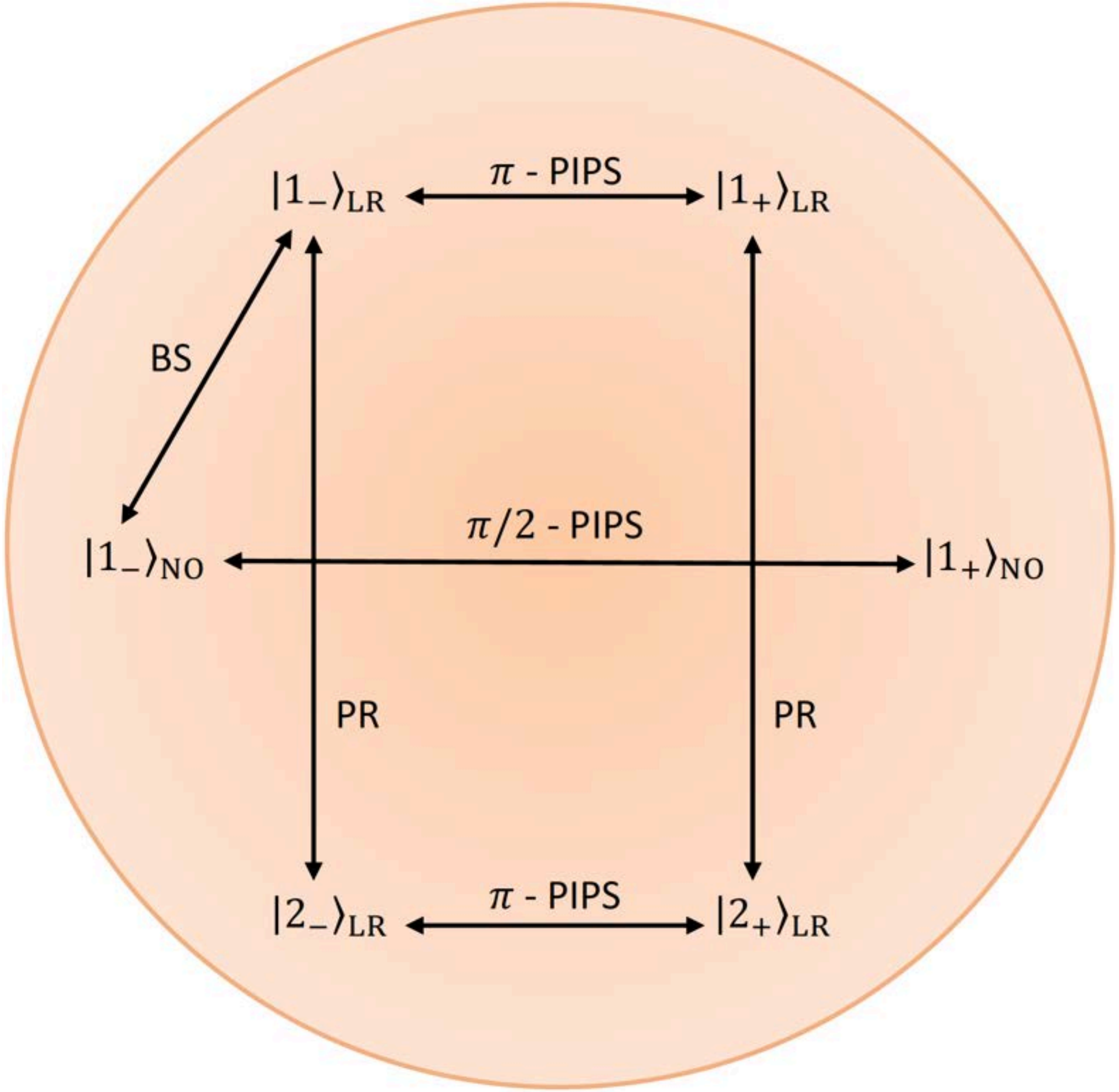
$$|U_-\rangle_{\mathrm{NO}} \xrightarrow{\text{BS}} (|2_+\rangle_{\mathrm{LR}} + |2_-\rangle_{\mathrm{LR}})/\sqrt{2} \xrightarrow{\text{PR on mode R}} (|1_+\rangle_{\mathrm{LR}} + |1_-\rangle_{\mathrm{LR}})/\sqrt{2}$$

$$\downarrow \text{BS}$$

$$|1_-\rangle_{\mathrm{LR}} \textbf{ or } |1_-\rangle_{\mathrm{NO}} \xleftarrow{\text{parity-check D}} (|1_-\rangle_{\mathrm{NO}} - |1_-\rangle_{\mathrm{LR}})/\sqrt{2}$$

Passive Optical (PO) operations - Fermions

Fermions: all the maximally entangled states in $\mathcal{B}_{\mathrm{LR}} \cup \mathcal{B}_{\mathrm{NO}}$ are PO equivalent



The indistinguishability-based distillation protocol prepares any maximally entangled state of two identical qubits

thanks to the connections allowed by PO operations and D

What about the comparison with LOCC-based distillation protocol?

[1] C. H. Bennett, G. Brassard, S. Popescu, B. Schumacher, J. A. Smolin, and W. K. Wootters, Phys. Rev. Lett. **76**, 722 (1996).

[2] C. H. Bennett, H. Bernstein, S. Popescu, and B. Schumacher, Phys. Rev. A. **53**, 2046 (1996).

[3] M. Horodecki, P. Horodecki, and R. Horodecki, Phys. Rev. Lett. **78**, 574 (1997).

[4] M. Horodecki, P. Horodecki, and R. Horodecki, Phys. Rev. Lett. **80**, 5239 (1998).

«The Entrance», David Hockney (detail). © Galerie Lelong

	LOCC-based Distillation Protocol (LDP)	Indistinguishability-based Distillation Protocol (IDP)
SIMILARITIES	1. Employs externally activated depolarization. Fidelity of the resulting Werner state with $\|2_+\rangle$ must be $F > \frac{1}{2}$ 2. Involves LOCC on the two qubits	1. Employs externally activated depolarization. Resulting Werner state must be maximally mixed 2. Involves PO operations, which include LOCC
PROs	1. Requires entangled initial states * 2. Requires n pairs of qubits as input 3. Succeeds probabilistically $\rightarrow m < n$ bipartite entangled states distilled 4. Prepares Bell states only	1. Works for any arbitrary initial state of two identical qubits * 2. Requires one pair of qubits as input 3. Succeeds with asymptotic determinism 4. Prepares both Bell states and NOON states
CONs	1. Does not require exotic devices 2. Allows to **distribute** entangled pairs to remote parties *	1. Requires exotic non-absorbing, parity check detectors 2. Allows to **prepare** entangled pairs, but noiseless channels are assumed after the BS *

Possible joint implementation:

entangled states robustly prepared by IDP and distributed by LDP

«Santa Monica Boulevard», © David Hockney, 1979

What happens in the case of a faulty parity check detector ?

What happens for a faulty parity check detector

$\Pi_{\mathrm{LR}} = \sum_{|v\rangle \in \mathcal{B}_{\mathrm{LR}}} |v\rangle\langle v|$ **(odd parity)**

$\Pi_{\mathrm{NO}} = \sum_{|k\rangle \in \mathcal{B}_{\mathrm{NO}}} |k\rangle\langle k|$ **(even parity)**

$$\Pi'_{\mathrm{LR}} := (1-\epsilon)\,\Pi_{\mathrm{LR}} + \epsilon'\,\Pi_{\mathrm{NO}}$$

$$\Pi'_{\mathrm{NO}} := (1-\epsilon')\,\Pi_{\mathrm{NO}} + \epsilon\,\Pi_{\mathrm{LR}}$$

bosonic qubits
projected into

$$\rho'_{\mathrm{LR}} = \frac{1}{4}\Big[(1-\epsilon)\,|1_-\rangle_{\mathrm{LR}}\langle 1_-|_{\mathrm{LR}} + 3\,\epsilon'\,\rho_{\mathrm{NO}}\Big]/p'_{\mathrm{LR}}$$

$$p'_{\mathrm{LR}} = (1-\epsilon)/4 + 3\epsilon'/4$$

$$\rho'_{\mathrm{NO}} = \frac{1}{4}\Big[3\,(1-\epsilon')\,\rho_{\mathrm{NO}} + \epsilon\,|1_-\rangle_{\mathrm{LR}}\langle 1_-|_{\mathrm{LR}}\Big]/p'_{\mathrm{NO}}$$

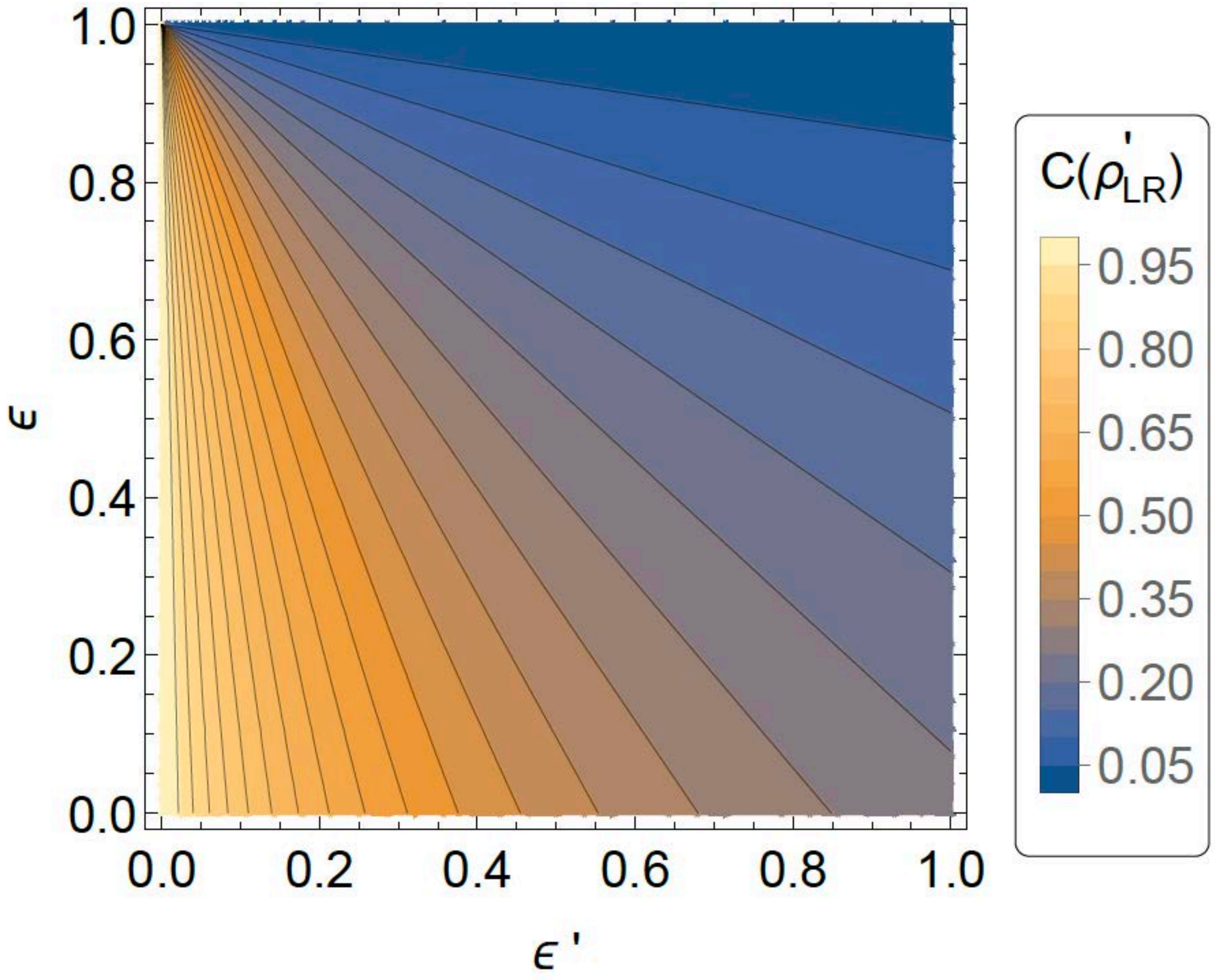
$$p'_{\mathrm{NO}} = 3(1-\epsilon')/4 + \epsilon/4 = 1 - p'_{\mathrm{LR}}$$

What happens for a faulty parity check detector

Concurrence of the prepared (faulty) state
(for bosons): ρ'_{LR}

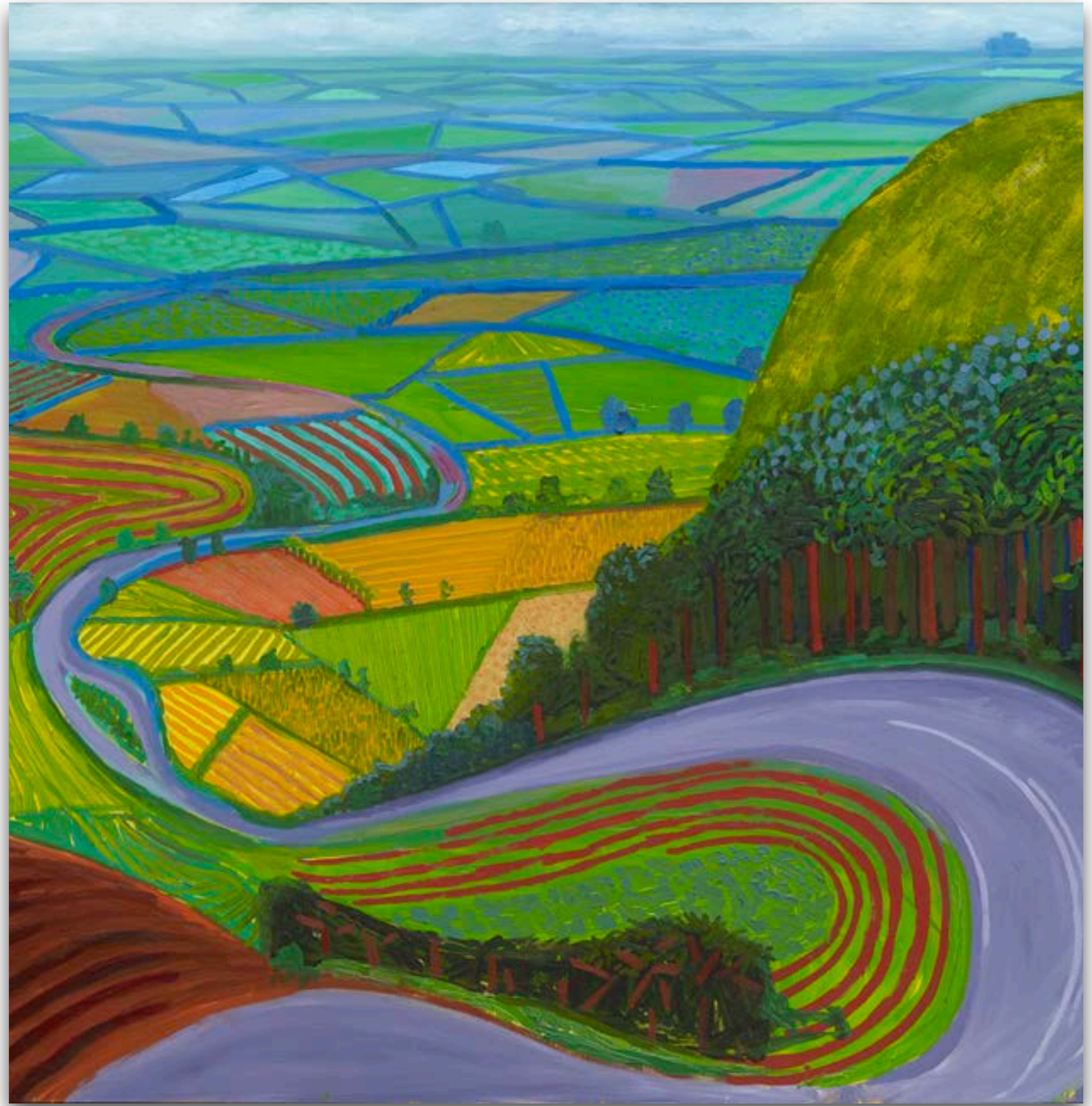
polarization entanglement

$$C(\rho'_{\mathrm{LR}}) = \frac{1-\epsilon}{1-\epsilon+3\,\epsilon'}$$



«Garrowby Hill», © David Hockney, 1998

Conclusions & Prospects



[1] M. Piccolini, V. Giovannetti, R. Lo Franco, arXiv:2303.11484

[2] M. Piccolini, V. Giovannetti, R. Lo Franco, arXiv:2305.14285

Conclusions

- **Preparation of a pure, maximally entangled Bell state (bosons) or NOON state (fermions) which can be transformed into any other maximally entangled state via PO operations**
  - Applicable to both bosons and fermions
  - Independent of the initial state
  - Robust to arbitrary local noise prior to the externally-activated depolarization
  - Succeeds with asymptotic certainty
  - Employs externally activated noise as an advantage

- **Quantum resource:** interference due to the indistinguishability of identical particles (*different 2-particle probability amplitudes indistinguishable when the qubits are collected*)

- **Key device:** Polarization-insensitive non-absorbing parity check detector

Conclusions

- **Insights on the relevance of a parity check detector** for the preparation of pure, maximally entangled states within noisy environments
  What if it is not used? ➟ *conditional robust preparation (resort to deferred measurements)*

- Externally induced depolarization resets the state to a maximally mixed one.
  What if it is not implemented? ➟ *no iteration*

- Conditional scheme, in general, to get a desired maximally entangled state
- Dependence on the characteristics of the environments
- Dependence on the initial state
- Dependence on the system-environment interaction time

[see: M. Piccolini, V. Giovannetti, R. Lo Franco, arXiv:2305.14285;
F. Nosrati *et al.*, arXiv:2305.11964]

Prospects

- **Results valid for any platform implementing linear optics operations and parity-check detection**
  - Motivate design of experimental implementation

- **Extending the analysis** of PO transformations to systems of $N > 2$ particles ➟ suitable generalization of the protocol to prepare robust multipartite entangled states

«The arrival of spring in Woldgate East Yorkshire in 2011», © David Hockney, 2011

Thank you!