Quantum control of gravitationally bound ultracold neutrons

Giovanni Manfredi
CNRS, Institut de Physique et Chimie des Matériaux de Strasbourg

Lazar Friedland
Racah Institute, Hebrew University of Jerusalem

Omar Morandi
University of Florence

Tobias Jenke
Institut Laue Langevin, Grenoble

Hartmut Abele
University of Vienna

Université de Strasbourg

CNRS

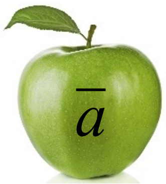
Motivations

- **Ultracold neutron experiments**
  - Can be viewed as a playing ground for quantum gravitational experiments
  - Accurate experimental data available (first realization in 2002)
  - Quantum regime accessible

- **Applications to fundamental physics issues**
  - Deviations from Newton's law at small scales, alternative theories of gravity
  - Tests of the equivalence principle
  - Antimatter and gravity (GBAR experiment)

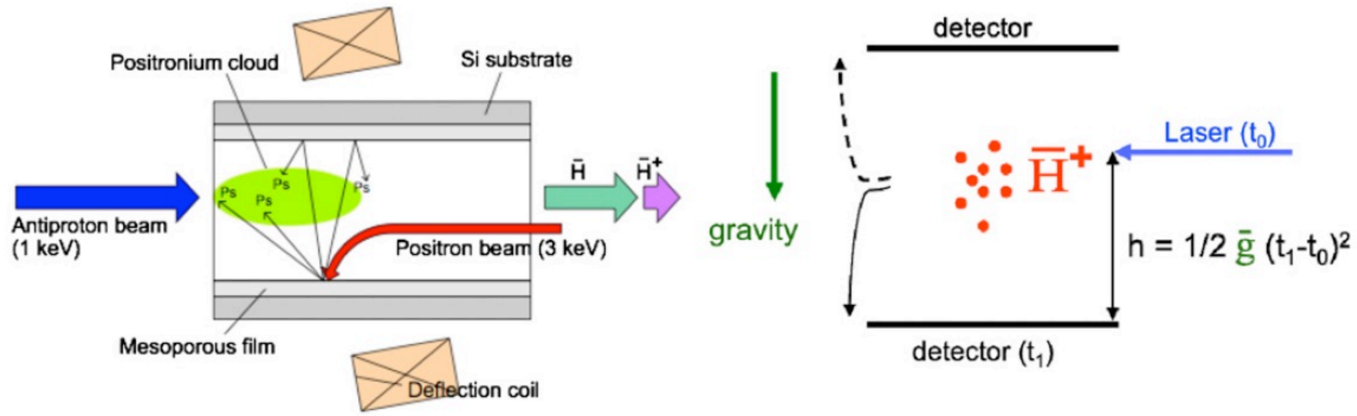
Antimatter and gravity

- **Gravitational behavior of antimatter**
  - Same as matter (attraction)
  - Slightly different (attraction, but different coupling)
  - Matter-antimatter repulsion (anti-gravity)
    - G. Chardin and G. Manfredi. *Gravity, antimatter and the Dirac-Milne universe*, Hyperfine Interact. 239:45 (2018),
    - G. Manfredi et al., Phys. Rev. D **98**, 023514 (2018); **102**, 103518 (2020).



- **Current experimental collaborations (CERN)**
  - **GBAR** (Gravitational Behaviour of Anti-hydrogen at Rest)
    - Free-fall acceleration of antihydrogen atoms in the gravitational field of the Earth
  - **AEGIS**
    - Shift of the interference pattern of free-falling antihydrogen going through gratings
  - **ALPHA-g**
    - Anti-hydrogen is stored in a minimum-B trap

The GBAR experiment



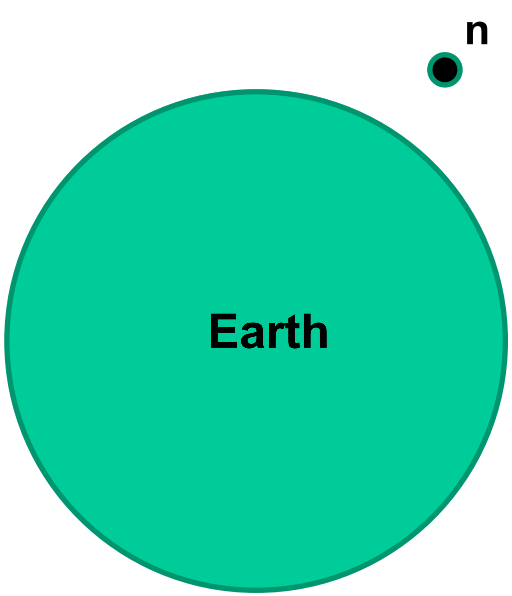
- Slow antiprotons produced at CERN's Antiproton Decelerator (AD) and ELENA ring
- Positively charged anti-hydrogen ions are created via the reaction with positronium (Ps = $e^+ e^-$) atoms :

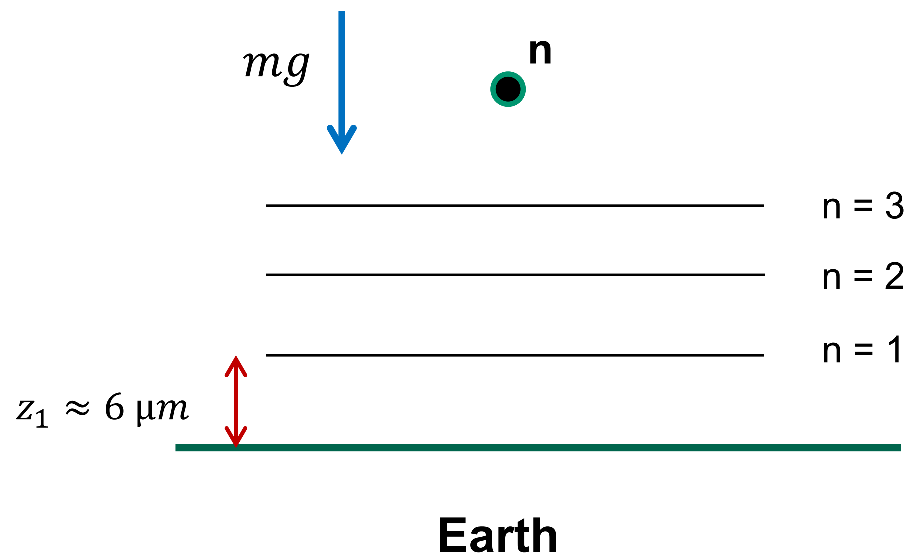
$$\overline{\mathrm{p}} + \mathrm{Ps} \rightarrow \overline{\mathrm{H}} + \mathrm{e}^- \qquad\qquad \overline{\mathrm{H}} + \mathrm{Ps} \rightarrow \overline{\mathrm{H}}^+ + \mathrm{e}^-$$

- The anti-hydrogen ions are cooled sympathetically (with Be^+ ions) in a Paul trap
- The excess positron in $\overline{\mathrm{H}}^+$ is photo-detached just above threshold
- The resulting anti-hydrogen atom falls freely in the gravitational field of the Earth

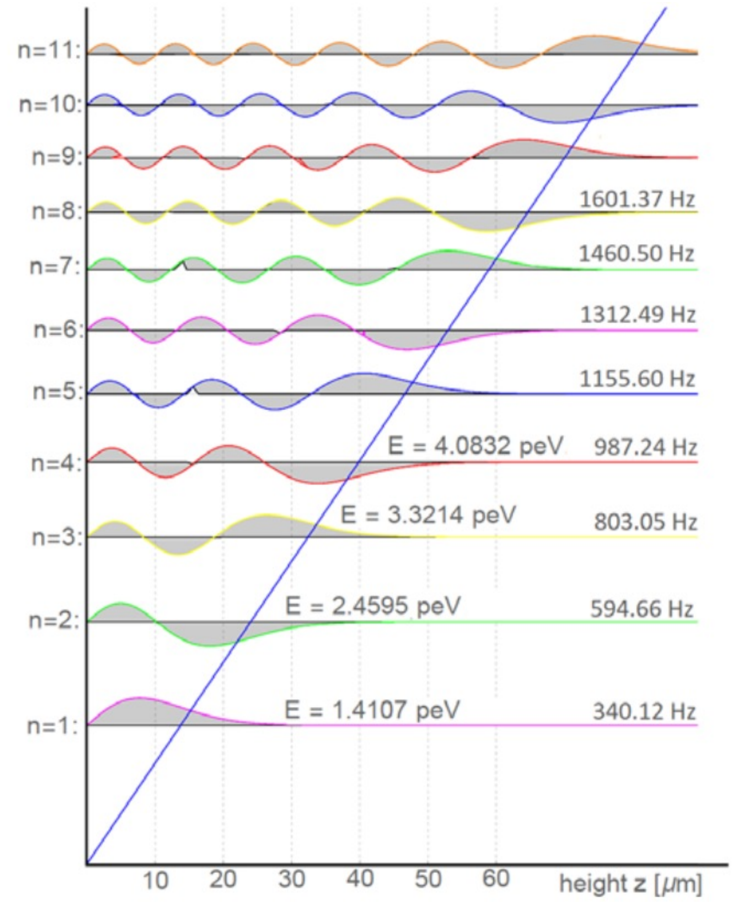
Neutrons in the Earth's gravitational field

- Earth + neutron system behaves as a gravitational "superatom"
- As gravitational force is much weaker, the size of the "superatom" is much larger than that of ordinary atoms.
- Quantum energy levels of the neutrons can be expressed in equivalent classical height: $E_n = mgz_n$
- Then the first few levels have a height of the order of the **micrometer**





Gravitational states of ultracold neutrons



- Quantum states are found by solving 1D Schrödinger equation

$$\left(-\frac{\hbar^2}{2m_i}\frac{\partial^2}{\partial z^2} + m_g g z\right)\psi_k = i\hbar\frac{\partial}{\partial t}\psi_k$$

- Eigenfunctions are **Airy functions** Ai(z)
- Typical scales of the ground state are:

$$z_0 = \sqrt[3]{\frac{\hbar^2}{2m_i m_g}} \approx 5.9\ \mu\mathrm{m}, \quad E_0 = m_g g z_0 \approx 0.6\ \mathrm{peV}, \quad t_0 = \frac{\hbar}{E_0} \approx 1.1\ \mathrm{ms}.$$

nature

Explore content · About the journal · Publish with us · Subscribe

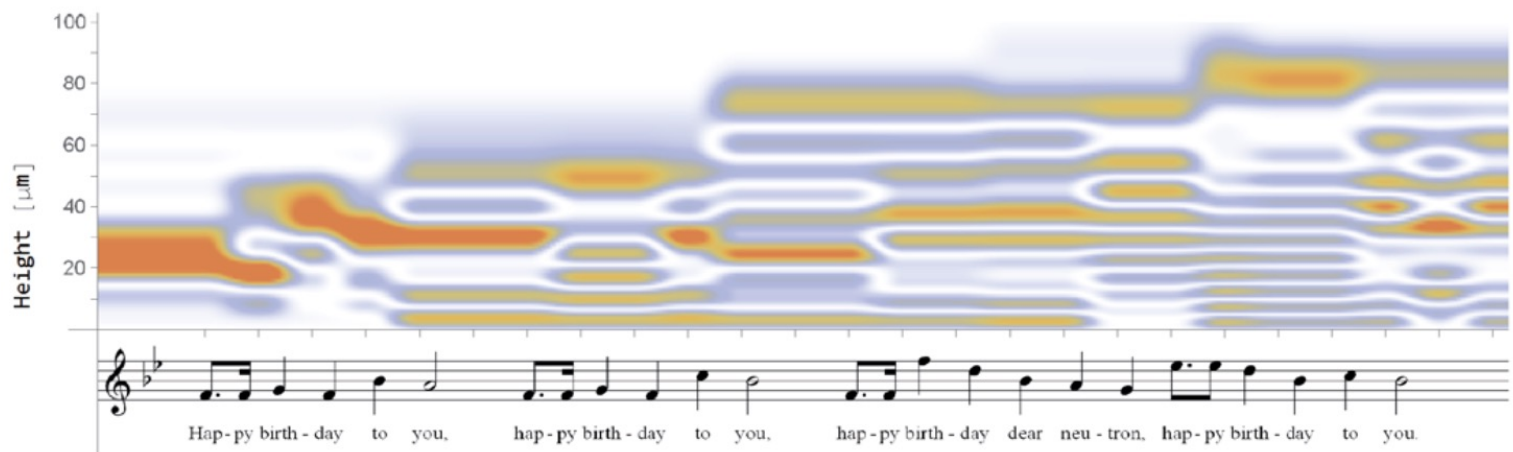
nature > correspondence > article

CORRESPONDENCE | 06 August 2019

Happy birthday, ultracold neutron!

Hartmut Abele, Hartmut Lemmel & Tobias Jenke

This year marks the 50th anniversary of the first production of ultracold neutrons, which could hold the secret to such mysteries as the expansion of the Universe and what dark

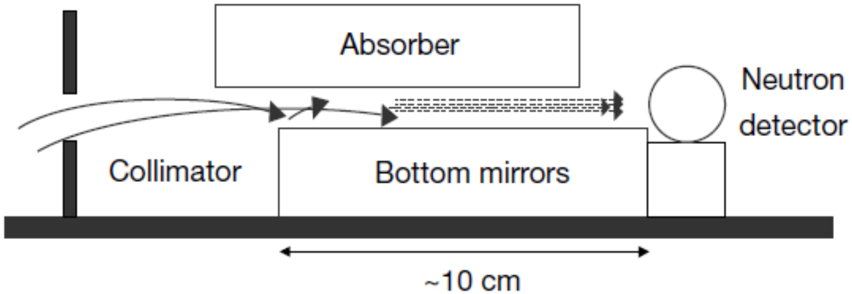


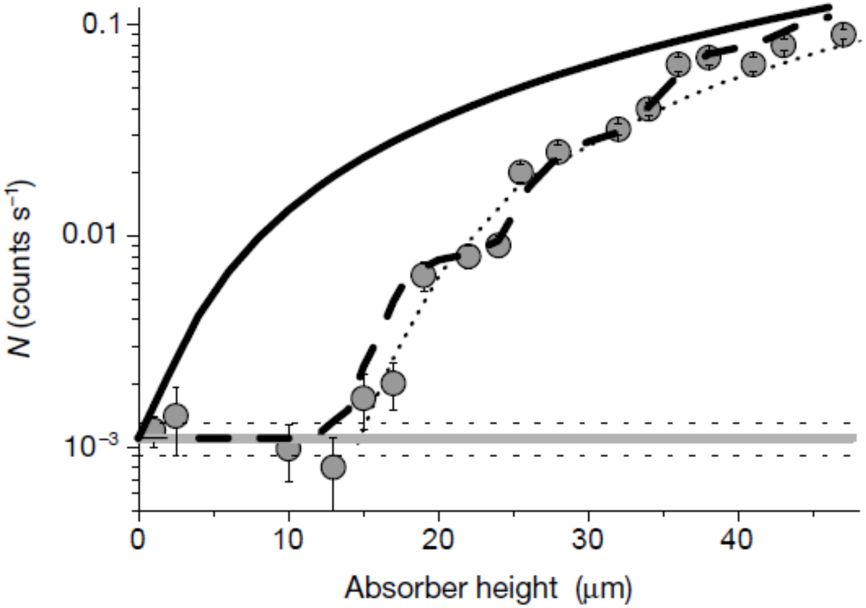
First experimental observation (2002)

Quantum states of neutrons
in the Earth's gravitational field

Valery V. Nesvizhevsky*, Hans G. Börner*, Alexander K. Petukhov*,
Hartmut Abele†, Stefan Baeßler†, Frank J. Rueß†, Thilo Stöferle†,
Alexander Westphal†, Alexei M. Gagarski‡, Guennady A. Petrov‡
& Alexander V. Strelkov§

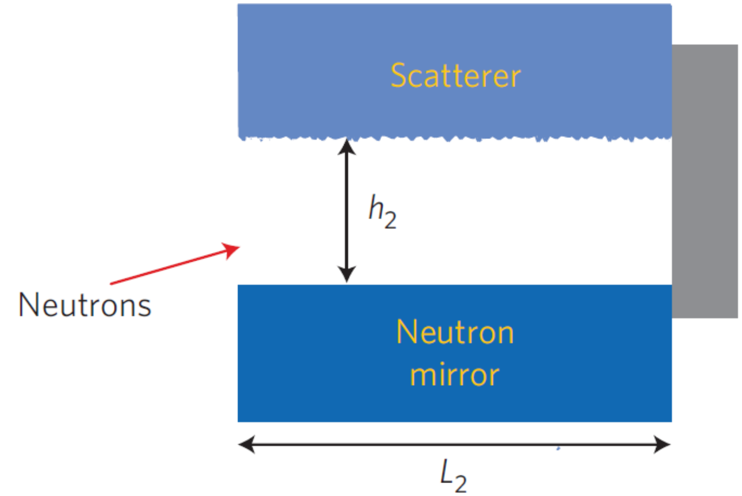
NATURE | VOL 415 | 17 JANUARY 2002 |

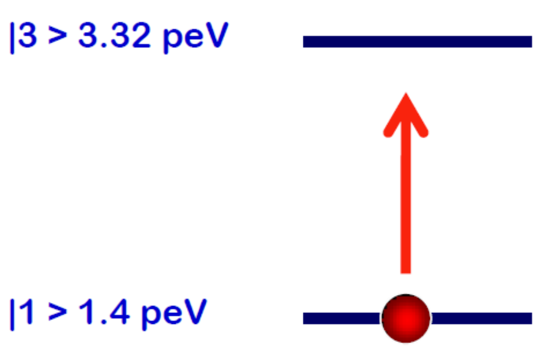




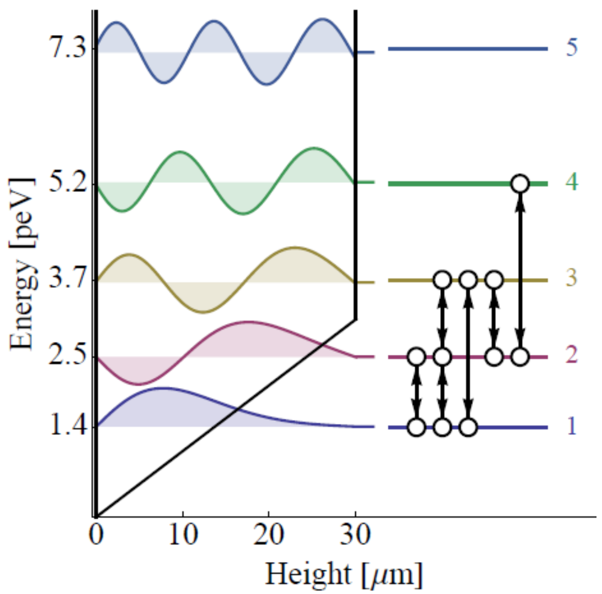
Gravity-resonance spectroscopy

- Quantum gravitational system as a two-level system
- **Transitions between two quantum states are driven by oscillating mirror**
- **All-mechanical coupling**
- Q-bounce experiment





Jenke T *et al* 2011 *Nature Phys.* **7** 468

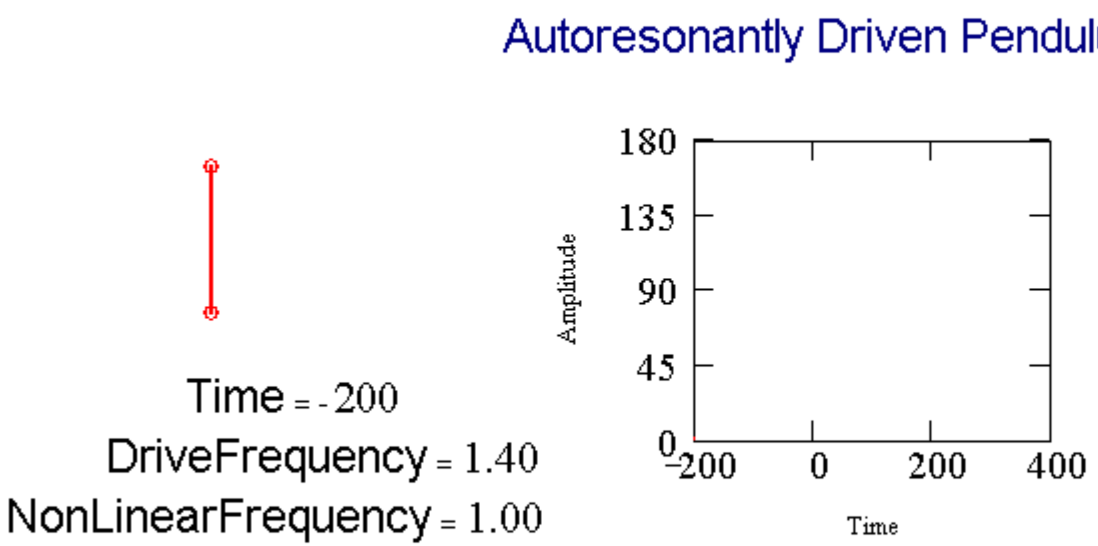


Chirped-frequency excitation □ Classical autoresonance

- **Nonlinear pendulum excited by a force with slowly varying frequency**

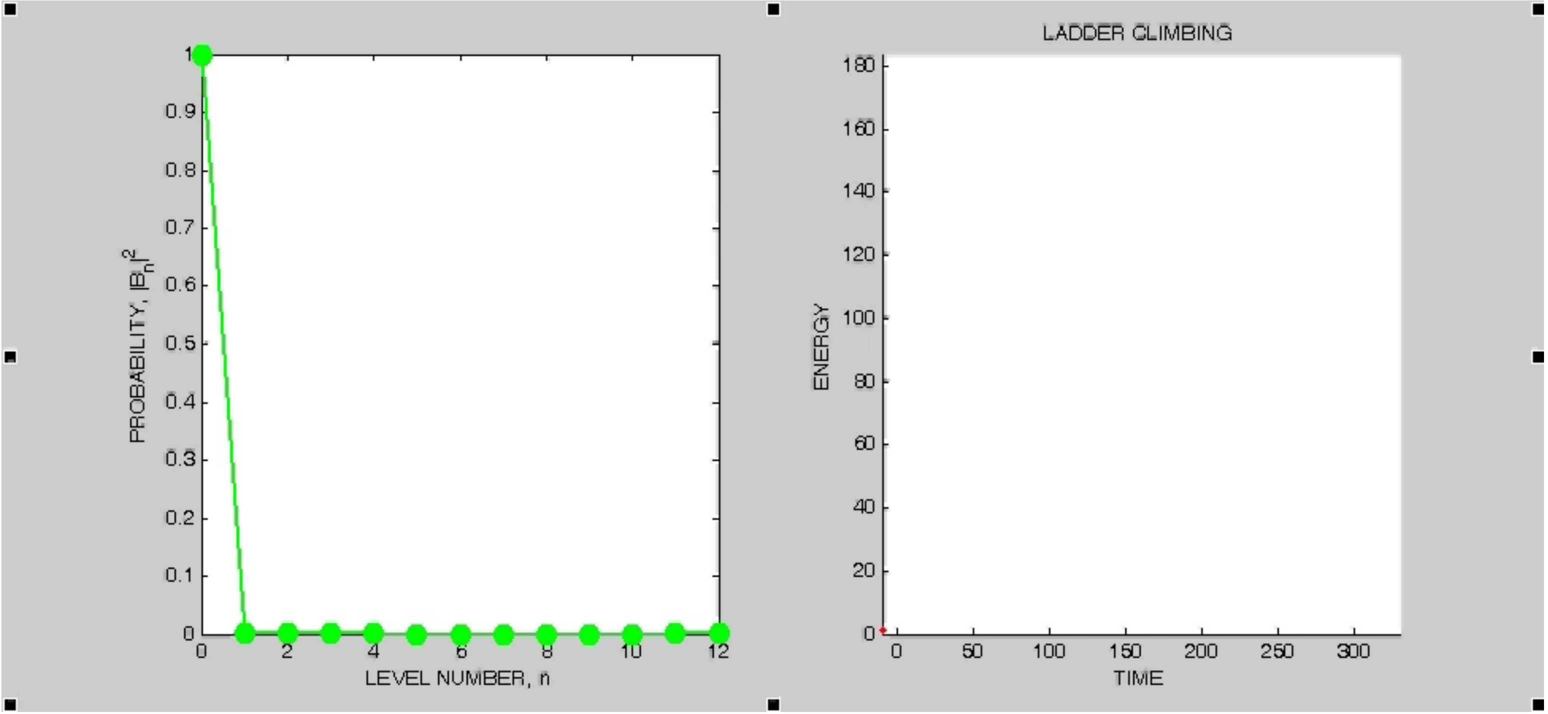
$$x_{tt} + \sin x = \varepsilon \cos \int \omega(t) dt \, ; \qquad \omega(t) = 1 - \alpha t \qquad (\varepsilon = 0.02, \quad \alpha = 0.001)$$

- The drive frequency crosses the linear frequency of the pendulum at t = 0.
- The oscillator is caught in resonance and the amplitude grows indefinitely



Quantum ladder climbing

$$H = \frac{p^2}{2m} + m\omega_0^2\left(\frac{1}{2}x^2 - \frac{\beta}{4}x^4\right) + \varepsilon\, x \cos\varphi_d$$



K. W. Murch et al, Nature Physics (2011)

Autoresonant excitation of UCN □ Semiclassical theory

- Hamiltonian: $H = p_z^2/2m + mgz.$
- The oscillating plate is located at:

$$z = L(t) = L_0 \cos \phi_d(t) \qquad \omega_d(t) = \dot{\phi}_d$$

- We transform to the reference frame of the oscillating plate
- The tra $x = z - L(t)$ onian is:
- Note th $H = p_x^2/2m + mgx - mL_0\omega_d^2 x \cos \phi_d$

$$\dot{\omega}_d \ll \omega_d^2 \text{ (adiabatic regime)}$$

Autoresonant excitation of UCN Semiclassical theory

- One can show that the drive frequency must satisfy the following:

$$|\dot{\omega}_d| < \frac{\epsilon}{2}\,|\Omega'(\bar{I})|\,\frac{\omega_d^2}{\Omega^2(\bar{I})} \approx \frac{\epsilon}{6}\,\frac{\omega_d^4}{b^3},$$

$b = (\pi^2/12)^{1/3}$

$\epsilon = L_0/a$

- Taking the limit case (equality sign) with a “safety factor” q<1, we get the optimal chirp

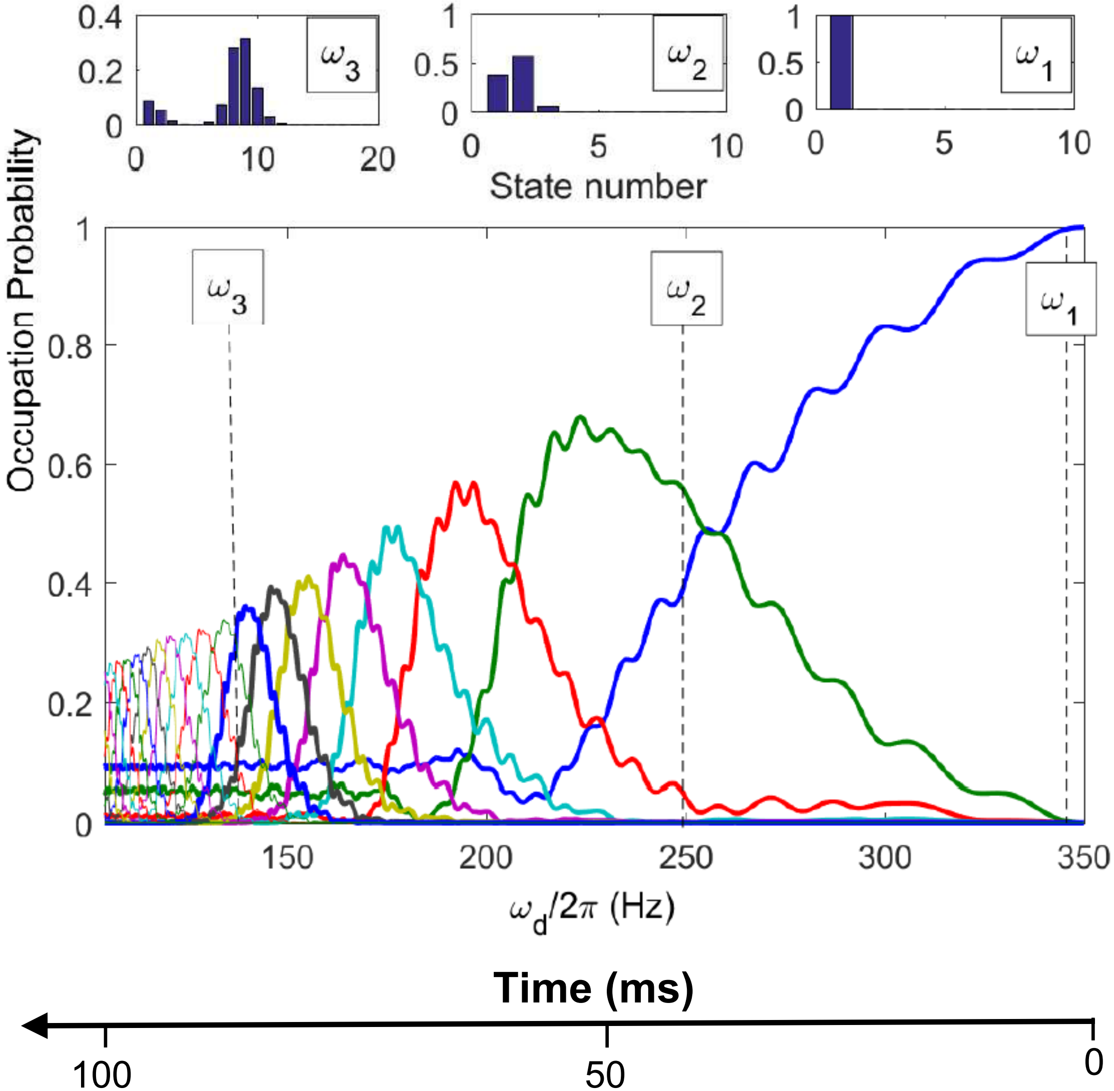
$$\frac{1}{\omega_d^3(t)} = \frac{1}{\omega_d^3(0)} + q\,\frac{\epsilon t}{2b^3}.$$

- Then we solve the Schrödinger equation: $i\hbar\frac{\partial\psi}{\partial t} = \mathrm{H}\,\psi$

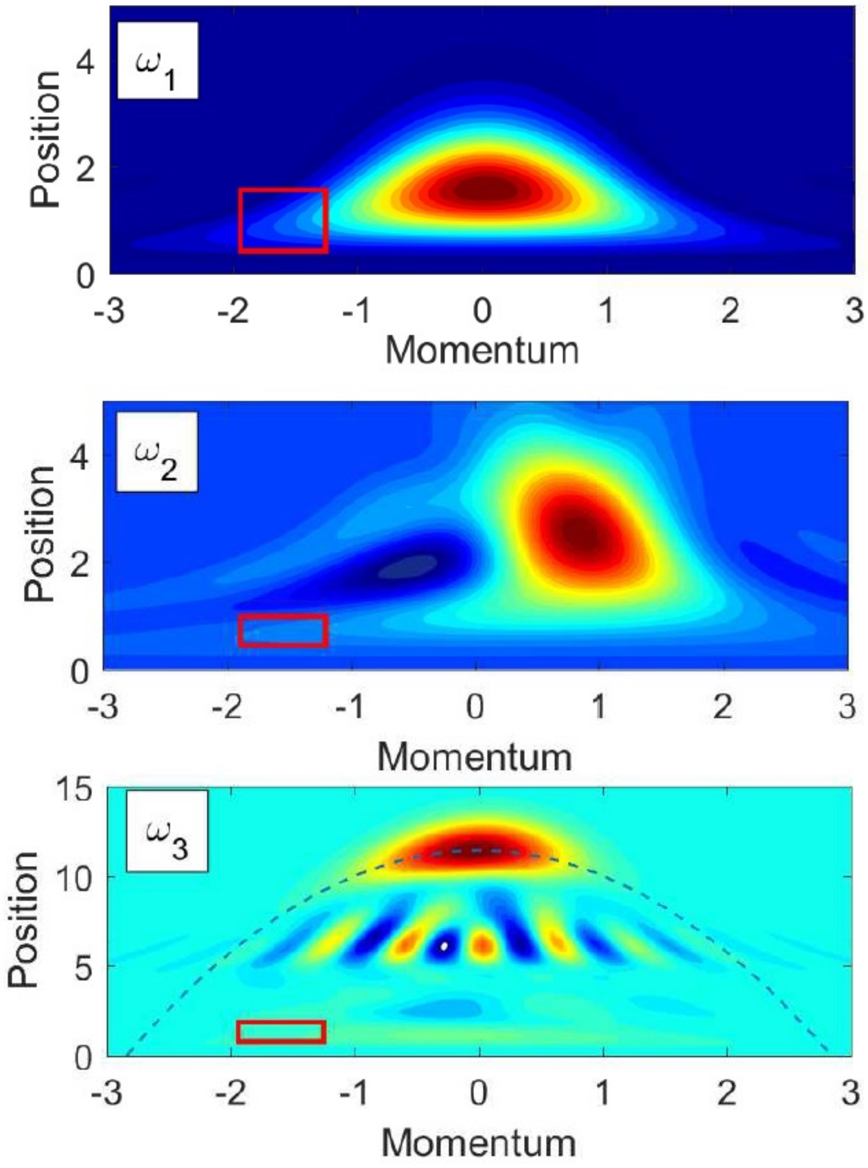
  with fixed boundary conditions: $\psi(x=0,t) = \psi(x=\infty,t) = 0$

  and initial condition: $\psi(x,t=0) = ground\ state$.

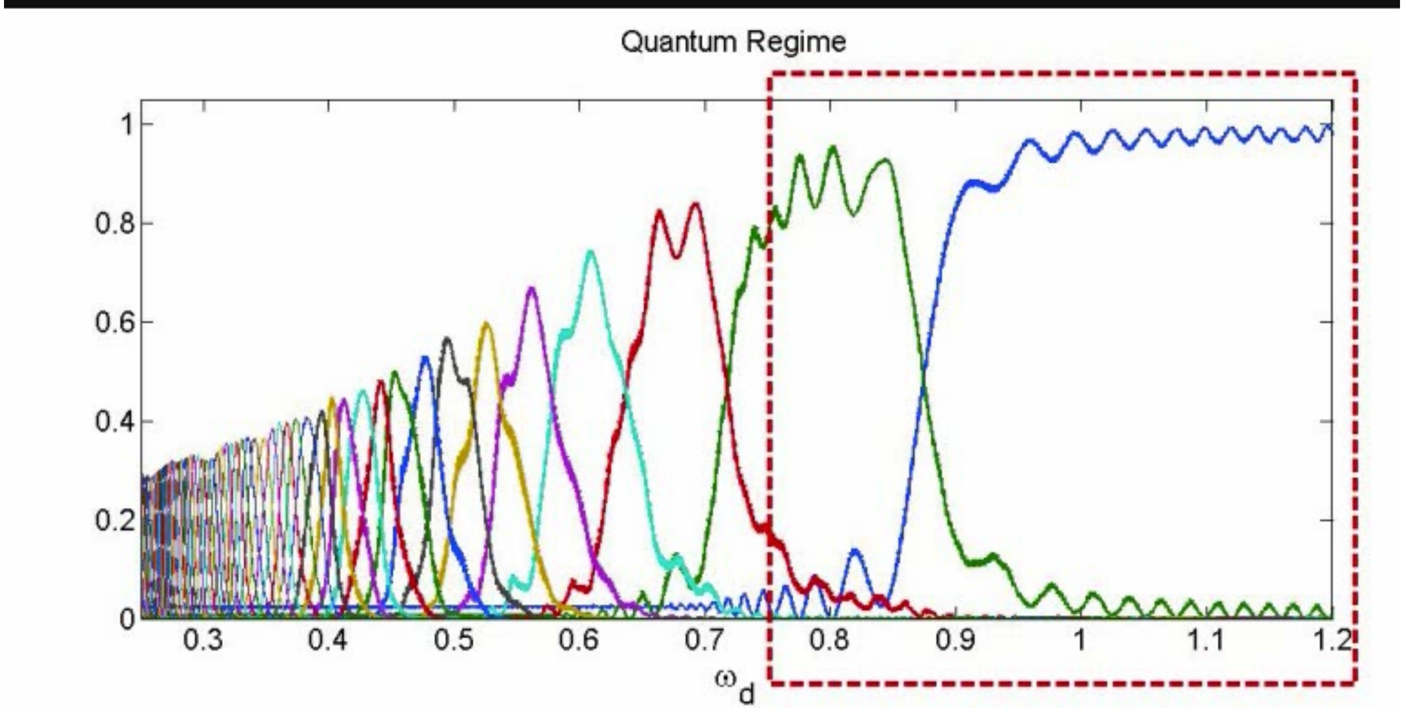
Quantum results Occupation probabilities



Quantum results Wigner functions



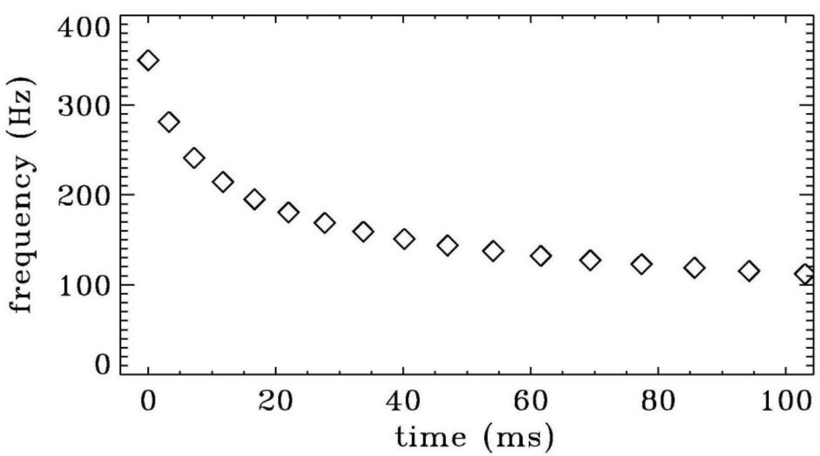
Quantum results Wigner functions

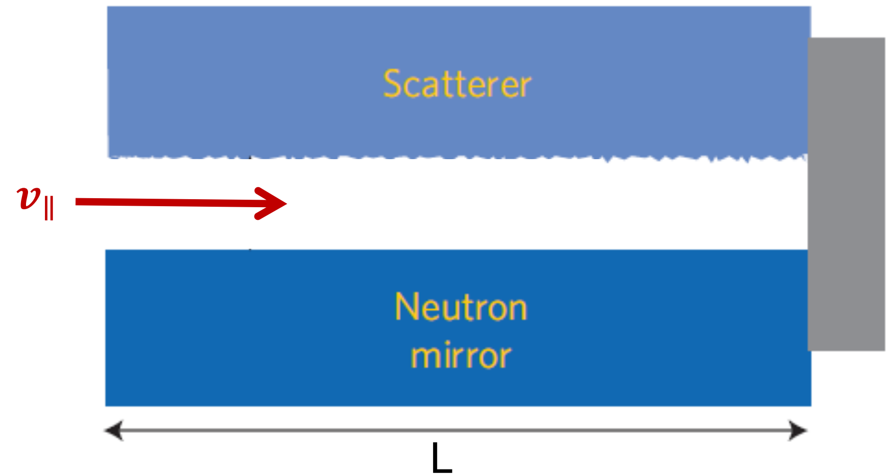


Connection to experiments

Simulation parameters

- **Excitation amplitude**: $L_0 = 1.34\ \mu m$
- **Initial frequency**: $f(0) = 350\ Hz$
- **Frequency at first transition**: $f_{1\to2} = 254\ Hz$
- **Frequency of 8 → 9 transition**: $f_{8\to9} \simeq 135\ Hz$, reached after $t_\star \approx 85\ ms$
- **Typical longitudinal velocity**: $v_\parallel = 6\ m/s$ ($\sim$1 mK)
- **Required mirror length**: $L = v_\parallel\, t_\star \simeq 50\ cm$





Conclusions and Perspectives

- We used **autoresonance** to address a high-N quantum gravitational state
- Works by climbing all levels from n=1 (ground state) to n=N
- Perspective: Use **quantum control** technique to jump directly from ground state to state N

G. Manfredi, O. Morandi, L. Friedland, T. Jenke, H. Abele, *Chirped-Frequency Excitation of Gravitationally Bound Ultracold Neutrons*, Phys. Rev. D **95**, 025016 (2017)