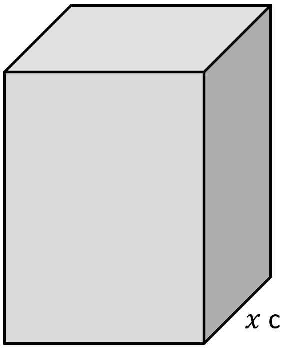
Maths for Chemical Engineering – A-level revision questions

The following questions are taken from AS and A2 past paper examinations for Pure Maths that will have been available to you during your A-level study. They have been selected as the topics they cover are crucial for you to understand in preparation for studying your Chemical Engineering course at the University of Surrey.

If you would like to practice further questions, you can access these through the brilliant website https://alevelmathsrevision.com/maths-categorised-exam-questions/ which is where these questions were sourced.

1) Figure 1 shows a solid cuboid with square base of side x cm and height h cm. Its volume is 120 cm^3.



   a. Find h in terms of x. Hence show that the surface area, A cm^2, of the cuboid is given by $A = 2x^2 + \frac{480}{x}$.
   b. Find $\frac{dA}{dx}$ and $\frac{d^2A}{ax^2}$.
   c. Hence find the value of x which gives the minimum surface area. Find also the value of the surface area in this case.

2) The polynomial $f(x)$ is defined by $f(x) = x^3 - 9x^2 + 7x + 33$.
   a. Find the remainder when $f(x)$ is divided by $(x + 2)$.
   b. Show that $(x - 3)$ is a factor of $f(x)$.
   c. Solve the equation $f(x) = 0$, giving each root in an exact form as simply as possible.

3) Find the binomial expansion of $(2 + x)^5$, simplifying the terms. Hence find the coefficient of y^3 in the expansion of $(2 + 3y + y^2)^5$.

4) a. Find and simplify the first three terms in the binomial expansion of $(2 + ax)^6$ in ascending powers of x.

   b. In the expansion of $(3 - 5x)(2 + ax)^6$, the coefficient of x is 64. Find the value of a.

5) Given that β is the obtuse angle such that $\sin\beta = \frac{3}{7}$, find the exact value of $\cos\beta$.

6) The cubic polynomial $ax^3 - 4x^2 - 7ax + 12$ is denoted by $f(x)$.
   a. Given that $(x - 3)$ is a factor of $f(x)$, find the value of the constant a.
   b. Using this value of a, find the remainder when $f(x)$ is divided by $(x + 2)$.

7) The equation of a cubic curve is $y = 2x^3 - 9x^2 + 12x - 2$.
   a. Find $\frac{dy}{dx}$ and show that the tangent to the curve when $x = 3$ passes through the point (-1 , -41).
   b. Use calculus to find the coordinates of the turning points of the curve. You need not distinguish between the maximum and minimum.
   c. Sketch the curve, given that the only real root of $2x^3 - 9x^2 + 12x - 2 = 0$ is $x = 0.2$ correct to one decimal place.

8) Evaluate the following definite and indefinite integrals.
   a. $\int (2x + 1)(x + 3)dx$
   b. $\int_0^9 \frac{1}{\sqrt{x}} dx$
   c. $\int \frac{6}{x^3} dx$
   d. $\int (x^3 + 8x - 5)dx$
   e. $\int_{-5}^{3} (x^3 - 19x + 30)\, dx$
   f. $\int_1^4 (3\sqrt{x} + 5)\, dx$
   g. $\int \frac{6x^4+4}{x^2} dx$

9) Solve the following sets of simultaneous equations.
   a. $x + 2y - 6 = 0$ and $2x^2 + y^2 = 57$
   b. $y = 2(x - 2)^2$ and $3x + y = 26$
   c. $2x + y - 5 = 0$ and $x^2 - y^2 = 3$
   d. $x^2 + y^2 = 34$ and $3x - y + 4 = 0$

10) a. Solve the simultaneous equations

$$y = 2x^2 - 3x - 5 \text{ and } 10x + 2y + 11 = 0$$

b. What can you deduce from the answer to part a. about the curve $y = 2x^2 - 3x - 5$ and the line $10x + 2y + 11 = 0$?

11) a. Show that the equation $2\sin^2 x = 5\cos x - 1$ can be expressed in the form

$$2\cos^2 x + 5\cos x - 3 = 0.$$

b. Hence solve the equation $2\sin^2 x = 5\cos x - 1$ giving all values of x between 0° and 360°.

12) A curve has equation $y = \sin(ax)$, where a is a positive constant and x is in radians.
   a. State the period of $y = \sin(ax)$, giving your answer in an exact form in terms of a.
   b. Given that $x = \frac{1}{5}\pi$ and $x = \frac{2}{5}\pi$ are the two smallest positive solutions of $\sin(ax) = k$, where k is a positive constant, find the values of a and k.

13) Given that a is the acute angle such that $\tan a = \frac{2}{5}$, find the exact value of $\cos a$.

14) The function f is defined for all real values of x by

$$f(x) = e^{2x} - 3e^{-2x}$$

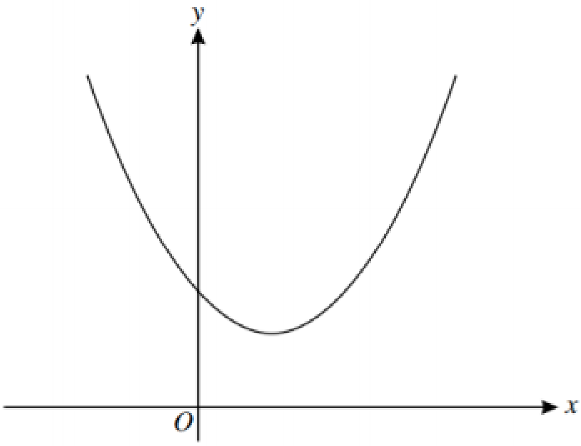
a. Show that $f'(x) > 0$ for all x.
b. Show that the set of values of x for which $f''(x) > 0$ is the same as the set of values for x for which $f(x) > 0$, and state what this set of values is.

15) Differentiate, with respect to x:
a. $x^3(x+1)^5$
b. $\sqrt{3x^4+1}$
c. $x^2(x+1)^6$

16) The function g is defined for all real values of x by

$$g(x) = e^{2x} + ke^{-2x}$$

Where k is a constant greater than 1. The graph of $y = g(x)$ is shown below. Find the range of g, giving your answer in simplified form.



17) The mass, m grams, of a substance is increasing exponentially so that the mass at time t hours is given by:

$$m = 250e^{0.021t}$$

a. Find the time taken for the mass to increase to twice its initial value, and deduce the time taken for the mass to increase to 8 times its initial value.
b. Find the rate at which the mass is increasing at the instant when the mass is 400 grams.

18) It is given that $f(x) = 1 - \frac{7}{x^2}$

a. Use the Newton-Raphson method, with a first approximation of $x_1 = 2.5$, to find the next approximations x_2 and x_3 to a root of $f(x) = 0$. Give the answers correct to 4 decimal places.
b. The root of $f(x) = 0$ for which x_1, x_2 and x_3 are approximations, is denoted by α. Write down the exact value of α.

19) A curve has parametric equations

$$x = t^2 - 6t + 4 \text{ and } y = t - 3$$

Find:

a. The coordinates of the point where the curve meets the x-axis

b. The equation of the curve in cartesian form, giving your answer in a simple form without brackets

c. The equation of the tangent to the curve at the point where $t = 2$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

20) Use the trapezium rule, with 3 strips each of width 2, to estimate the value of

$$\int_1^7 \sqrt{x^2 + 3}\, dx$$