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Book of Abstracts

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The bottom of each page contains links to the Table of Contents and Index.

Contents

Plenaries

Data to decisions via adaptive reduced models and multifidelity uncertainty quantification

Karen Willcox[1†] & Benjamin Peherstorfer[2]

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11:30 on Monday in Management School Lecture Theatre

This talk describes our recent work in adaptive model reduction and multifidelity uncertainty quantification methods for large-scale problems in engineering design. We combine traditional projection-based model reduction methods with machine learning methods, to create data-driven adaptive reduced models that exploit the synergies of physics-based computational modeling and physical data. We develop multifidelity formulations to combine these reduced models with higher-fidelity models. The multifidelity formulation means that we obtain the benefits of the reduction in computational efficiency but maintain quality guarantees on the resulting answer, even in the absence of rigorous error bounds on the reduced models themselves. Examples highlight concepts for future aerospace vehicles that are endowed with sensors and on-board computing capabilities, thus enabling new design concepts and new modes of decision-making.

Some challenges, threats and opportunities for applied mathematics

Peter Grindrod[†]

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16:30 on Monday in Management School Lecture Theatre

The public funding landscape for UK research is now changing more rapidly than ever: this may well pose an existential threat to mathematics R&D, so how should the maths community respond?

The digital wave of data and analytics both shortens and accelerates the impact supply chain: why and how should we engage with corporate users? Our modus operandi may be our strongest card.

Crafting some new themes for the next ten years—some emerging examples and suggestions. Are we making the most of what we can do?

Maternal effects and environmental change

Rebecca Hoyle[†]

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11:30 on Tuesday in Management School Lecture Theatre

Maternal effects are influences of the maternal phenotype on offspring phenotypes by routes other than direct genetic transmission. Potentially they provide an additional means of adaptation to changing environmental conditions over and above that afforded by within-generation phenotypic plasticity. However, maternal effects have also been implicated in the risks of heart disease, diabetes and obesity. I will show how mathematical modelling can provide insight into the interaction of maternal effects and phenotypic plasticity under different patterns of environmental change and suggest when maternal effects might be expected to evolve and why.

Challenges for climate and weather prediction in the post-Moore's-Law era of computing: oscillatory stiffness, time-parallelism, and the role of long-time dynamics

Beth Wingate†

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16:30 on Tuesday in Management School Lecture Theatre

For weather and climate models to achieve exascale performance on next-generation heterogeneous computer architectures they will be required to exploit on the order of hundred-million-way parallelism. This degree of parallelism far exceeds anything possible in today's models even though they are highly optimized. In this talk I will discuss one of the mathematical issues that leads to the limitations in space- and time-parallelism for climate and weather prediction models – oscillatory stiffness in the partial differential equations that leads to time scale separation. I will introduce time-parallelism and discuss the cases when the time scale separation is infinite and finite. I will highlight research with 1) a fast-converging HMM-parareal-type method and 2) a time-parallel matrix exponential.

Surfing with wavelets

Ingrid Daubechies†

Duke University

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11:30 on Wednesday in Management School Lecture Theatre

This talk gives an overview of wavelets: what they are, how they work, why they are useful for image analysis and image compression. Then it will go on to discuss how they have been used recently for the study of paintings by e.g. Van Gogh, Goossen van der Weyden, Gauguin and Giotto.

Monday PM: Mini-Symposia

Symmetries and conservation laws

Organised by: Peter Hydon and Elizabeth Mansfield

A beginner's guide to symmetries and conservation laws

Peter Hydon†

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14:00 on Monday in LTB

Symmetry-based methods can give a huge amount of information about a given differential equation, including exact solutions, conservation laws and structure-preserving numerical approximations. Their power has been under-exploited in applied mathematics, perhaps because symmetry methods are not generally taught to undergraduates.

This talk introduces the main ideas and some nice applications. It aims to give newcomers a sufficient background for the rest of the mini-symposium Symmetries and Conservation Laws.

Multiphase wavetrains, conservation laws and the emergence of nonlinear PDEs

Daniel Ratliff[1]† & Thomas J. Bridges[1]

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14:40 on Monday in LTB

The study of the Whitham equations for multiperiodic wavetrains (or in general, multiparameter relative equilibria) has been studied in the past, however there has been little discussion up to now as to how they morph at the elliptic-hyperbolic transition point. This talk aims to first formulate the Whitham system for Lagrangian PDEs in this setting, where the degeneracy can be formulated using conservation law criticality. From this, a phase dynamical approach is used to show how these conditions give rise to nonlinear PDEs at the transition point; in particular we discuss the cases that lead to the KdV and two-way Boussinesq equations. Their coefficients benefit from being directly related to the projections of the conservation laws and a linear operator analysis that can be undertaken a-priori. The theory is illustrated using two examples, one being stratified shallow water flows and the other a coupled NLS system, to demonstrate how the coefficients may be obtained and criticality assessed.

Discrete conservation laws and moving frames

Elizabeth Mansfield[1†]
Ana Rojo-Echeburua[1], Peter Hydon[1] & Linyu Peng[2]
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15:00 on Monday in LTB

The talk will begin with a gentle introduction to Lie group based moving frames. I will indicate how Noether's conservation laws for a Lie group invariant difference Lagrangian, can be written explicitly in terms of discrete invariants and a discrete moving frame. I will then show how the use of a moving frame can ease the solution of the Euler Lagrange equations. Examples will be Lagrangians invariant under rotations, translations and scalings.

Structure preserving numerical schemes and exact solutions for the mKdV system

Tristan Pryer[†]
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15:20 on Monday in LTB

In this talk we introduce the mkdv system, give an overview on the construction of exact solutions and present a methodology for the design of a class of conservative numerical methods.

Network Science

Organised by: Marianno Beguerisse, Jonathan Ward and James Rankin

Trees and networks in understanding pathogen transmission

Caroline Colijn†

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14:00 on Monday in LTD

The network of contacts between human hosts shapes how infections can spread in human communities, with some networks facilitating rapid spread and others hindering infections. Indeed, social contact networks are key in determining pathogens' opportunities to reproduce. Accordingly, when we analyse data on pathogen populations' ancestry and reproduction, we can hope to see some traces of the underlying host contact network. The question is complicated by the fact that networks are complex and change through time, and also because pathogen ancestry and reproduction are visible only indirectly, through phylogenetic trees. Here, we show how to use genetic data from a pathogen to estimate features of the host contact network, by studying the phylogenetic trees of a pathogens transmitted over networks. We introduce a suite of metrics on the space of unlabelled rooted binary trees, first by developing a metric (in the sense of a true distance function), and then by extending it to incorporate several informative, but non-metric, tree features. We illustrate how this metric gives us access to the underlying contact network, using a dynamic model of complex contact networks that exhibits clustering, a skewed degree distribution, stable size and turnover and rewiring dynamics.

Trophic Coherence: what it is and why it matters

Samuel Johnson†

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14:20 on Monday in LTD

We have recently found that many dynamical and topological properties of directed networks can be related to their "trophic coherence" [1]. This is a measure of how neatly nodes can be classified into distinct layers, or "trophic levels", which has its origin in ecology but can be applied to generic directed graphs. We have shown that trophic coherence plays a key role in determining the stability of ecosystems, percolation in spreading processes, graph eigenspectra, and the scarcity of feedbabck loops in networks of species or genes [1-4]. Our work also poses many new questions, such as which mechanisms lead to trophic coherence, and what trophic structure can tell us about node function.

[1] S Johnson, V Domínguez-García, L Donetti, and MA Muñoz, "Trophic coherence determines food-web stability", PNAS 111, 17923–17928 (2014).

[2] J Klaise and S Johnson, "From neurons to epidemics: How trophic coherence affects spreading processes", Chaos 26, 065310 (2016).

[3] V Domínguez-García, S Johnson, and MA Muñoz, "Intervality and coherence in complex networks", Chaos 26, 065308 (2016).

[4] S Johnson and NS Jones, "Looplessness", to appear in PNAS, arXiv:1505.07332 (2017).

Geometric dynamics of the endoplasmic reticulum

Peter Ashwin[†]

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14:40 on Monday in LTD

The endoplasmic reticulum (ER) is a large organelle within the cytoplasm of plant cells may take the form of a complex and highly dynamic network that spans large parts of the cell. For cells with a large vacuole, the cytoplasm is constrained to a thin sheet making it possible to view the ER dynamics using confocal microscopy. In this talk I will discuss collaborative work on modelling the geometry and dynamics of the ER from live cell imaging data. The networks formed seem to be close to local, but not global, minimal length networks that have many features of perturbed Steiner trees. By modelling the dynamics of junctions in the network we are able to explain some geometric and statistical features of the network. (Joint work with C. Lin, I. Sparkes and R. White)

The dynamics of evaporation

Organised by: Dominic Vella and Stephen Wilson

The role of kinetic effects in droplet evaporation

Matthew Saxton[1†]
Dominic Vella[2], Jonathan Whiteley[2] & James Oliver[2]

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14:00 on Monday in LTF

We consider the evaporation of a thin, axisymmetric, partially wetting drop into an inert gas. The common modelling assumption that vapour transport is limited by diffusion leads to a singularity in the mass flux at the contact line. Instead, we take kinetic effects into account through a linear constitutive law, and perform a local analysis to show that this leads to a finite mass flux at the contact line. Moreover, we perform a matched asymptotic analysis in the physically relevant regime in which the kinetic timescale is much smaller than the diffusive one to show that kinetic effects regularize the mass-flux singularity in an inner region near the contact line. We then discuss how the kinetics-based evaporation rate influences the evolution of the drop.

Low-order modelling of evaporation in asymmetric drops

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Omar Matar[2], Pedro Saenz[3] & Stephen Wilson[1]

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14:20 on Monday in LTF

The diffusion-limited model of evaporation has received extensive attention in recent years thanks to its utility in a variety of situations, especially its use in the explanation of the so called "coffee-stain effect". However, analytical solutions for the evaporation rate have relied on computation in a co-ordinate system where not only does the interface lie exactly in a co-ordinate plane, but also in which solutions to Laplace's equation are (R-)separable. While this does apply to the spherical-cap solution (or circular-arc in 2D), these constraints are otherwise rather restrictive, being inapplicable to even a situation as simple as a drop depressed slightly by the effect of gravity. We use a combination of exact analytical, asymptotic, and direct numerical approaches to investigate relaxing these constraints to better understand how non-spherical drops evaporate.

The lifetime of evaporating droplets with related initial and receding contact angles

Stephen Wilson[1†]
Jutta Stauber[1], Brian Duffy[1] & Khellil Sefiane[2]

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14:40 on Monday in LTF

The lifetime of a droplet depends on the manner in which it evaporates. There are many possible modes of evaporation, but in practice a stick-slide mode (in which the droplet first evaporates in a constant contact radius mode until the contact angle reaches the receding contact angle, after which it evaporates in a constant contact angle mode) often occurs. In this talk a physically plausible relationship between the initial and receding contact angles is proposed and used to give a complete description of the lifetime of a droplet evaporating in a stick-slide mode. In particular, it will be shown that the dependence of the lifetime on the initial contact angle is qualitatively different from that when the relationship between the initial and receding contact angles is not taken into account. This talk is an account of work with Jutta Stauber and Brian Duffy (University of Strathclyde) and Khellil Sefiane (University of Edinburgh), and further details can be found in our joint paper published in Phys. Fluids 27, 122101 (2015).

Watching sessile droplets evaporate...is beautiful and interesting!

Prashant Valluri[1†]
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15:00 on Monday in LTF

The evaporation of a liquid drop on a solid substrate is a remarkably common phenomenon. Yet, the complexity of the underlying mechanisms has constrained previous studies to spherically-symmetric configurations. We recently demonstrated (Sáenz et al, JFM, 772, 705) detailed evolution of thermoca-pillary instabilities during evaporation of hemispherical and non-hemispherical sessile droplets. Rigor-ous DNS (using our in house TPLS2 solver) showed for the first time, breakage of symmetry and the consequent development of a preferential direction for thermocapillary convection. This results in counter-rotating whirling currents in the drop playing a critical role in regulating the interface thermal and fluid dynamics. This work received the First ARCHER International Image Prize in 2014.

I will also present our recent investigations of well-defined, non-spherical evaporating drops of pure liquids and binary mixtures. We deduce a new universal scaling law for the evaporation rate valid for any shape and demonstrate that more curved regions lead to preferential localized depositions in par-ticle-laden drops. Furthermore, geometry induces well-defined flow structures within the drop that change according to the driving mechanism and spatially-dependent thresholds for thermocapillary in-stabilities. In the case of binary mixtures, geometry dictates the spatial segregation of the more volatile component as it is depleted. In the light of our results, we believe that the drop geometry can be ex-ploited to facilitate precise local control over the particle deposition and evaporative dynamics of pure drops and the mixing characteristics of multicomponent drops.

Lattice-Boltzmann simulations of evaporation dynamics

Rodrigo Ledesma-Aguilar[1†]
Julia Yeomans[2], Dominic Vella[2], Ginaluca Laghezza[2] & Elfego Ruiz-Gutierrez[1]

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15:20 on Monday in LTF

We present a computational algorithm to model the dynamics of evaporation based on a Lattice-Boltzmann method.

We illustrate the ability of our method to model evaporation dynamics by reporting simulation results for three different systems: a single droplet evaporating on a smooth solid surface, a collection of sessile droplets evaporating in the presence of a gas phase of higher density than the vapour, and a liquid meniscus evaporating in contact with a solid beam.

We discuss the interplay between the typical evaporation rate of the liquid, the fluid dynamics and the solid geometry, and how this can trigger convective and capillary flows during the evaporation process.

Drying of sessile thixotropic droplets

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15:40 on Monday in LTF

The well studied coffee drop phenomenon has been observed in the evaporation of simple nanoparticles-laden droplets. For a range of biological fluids, however, the drop can undergo a sol-gel transition during the drying process, leading to a considerable increase in the complexity of the phenomena. Here the rheology becomes non-uniform during the complex transition. In this study, we propose the use of thixotropy to model the evolving rheology during this transitional process. Thixotropy has become of increasing interest for a variety of applications in recent years. It is often used to describe fluids which exhibit a decrease in viscosity due to flow and subsequent slow recovery of viscosity after the cessation of the flow. Ideal thixotropy, proposed by Larson, describes fluids in which the rheological response is purely viscous. In this work, we will introduce an additional parameter to describe the internal structure of the droplet. Furthermore, we will use the lubrication approximation to simplify the governing equations. The study aims to systematically explore the effect of ideal thixotropic models upon the internal composition of the droplet. It is seen that due to the dependence of the shear rate on structure development, a range of behaviours can be observed; this highlights the importance of selecting appropriate models for the fluid rheology.

Asymptotic methods in applied sciences

Organised by: Phillippe Trinh

Organisation of vascular pattern in plant roots

John King[1†] & Nathan Mellor[1]

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14:00 on Monday in LTG

Asymptotic and (cell-based) numerical approaches to vascular patterning in plant roots will be described.

Invisible catastrophes: when to turn an asymptotic blind eye

Chris Howls[1†] & Jon Stone[1]

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14:40 on Monday in LTG

The talk deals with a class of functions that are present across asymptotic modelling that possess subtle asymptotic properties resulting from the coalescence of critical points with singularities. Far from leading to caustic divergences in the integral at these coalescences, the integral is actually asymptotically well-behaved and usual techniques lead to a good numerical approximation. However, delving deeper into the asymptotics uncovers additional caustic complications that, both analytically and numerically would have been better left alone. Examples from acoustics, optics and fluid dynamics will be presented.

Exponential asymptotics, self-similarity and thin-film rupture

Philippe Trinh[1†]
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15:20 on Monday in LTG

In many physical models, the formation of a singularity is described by a self-similar reduction of a nonlinear partial differential equation. In certain cases, numerical simulations may suggest the presence of coexisting solutions that form part of a discrete countably infinite family. However, a standard asymptotic analysis reveals a continuum of possible solutions, with no apparent selection mechanism serving to distinguish the discrete nature of the families. These types of problems are characterized by exponentially small effects.

This talk will introduce a range of such problems where this type of mechanism occurs. We shall then demonstrate how exponential asymptotics is applied to determine the selection process. We will focus on the case of finite-time rupture of a thin-film of viscous fluid, which presents some subtle challenges to the asymptotic procedure.

LMS Scheme 3 minisymposium Inverse problems and imaging

Organised by: Carola-Bibiane Schönlieb, Matthias Ehrhardt and Martin Benning

Fast compressed quantitative MRI

Mohammad Golbabaee[1†] & Mike Davies[1]

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14:00 on Monday in LTH

We will present a compressed sensing perspective of a novel form of MR imaging called Magnetic Resonance Fingerprinting (MRF). This enables direct estimation of the T1, T2 and proton density parameter maps for a patient through undersampled k-space sampling and BLIP, a gradient projection algorithm that enforces the MR Bloch dynamics. One of the key bottlenecks in MRF is the projection onto the constraint set. We will present both theoretical and numerical results showing that significant computational savings are possible through the use of inexact projections and a fast approximate nearest meighbor search.

Large noise in variational regularisation

Hanne Kekkonen[1†]
Martin Burger[2] & Tapio Helin[3]

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14:40 on Monday in LTH

We consider variational regularisation methods for inverse problems with large noise, which is in general unbounded in the image space of the forward operator. We introduce a Banach space setting that allows to define a reasonable notion of solutions for more general noise in a larger space provided one has sufficient mapping properties of the forward operator. As an example we study the particularly important cases of one- and p-homogeneous regularisation functionals. As a natural further step we study stochastic noise models and especially white noise, for which we derive error estimates in terms of the expectation of the Bregman distance. As an example we study total variation prior.

The parameter estimation problem under radial lines sampling

Clarice Poon[1†]
Charles Dossal[2] & Vincent Duval[3]

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[2]Université de Bordeaux
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15:20 on Monday in LTH

The problem of parameter estimation for a superposition of point sources is rooted in applications such as astronomy, NMR (nuclear magnetic resonance) spectroscopy and microscopy where the signal of interest can often be modelled as point sources. In recent years, the infinite dimensional formulation of this problem has been a topic of intense research within the mathematical community. However, these works have focussed on the case where one samples the Fourier transform at Cartesian grid points. On the other hand, physical constraints can sometimes restrict observations to certain angular directions, and in the case of NMR spectroscopy, one is required to sample along continuous trajectories such as radial lines. Our work considers the parameter estimation problem when given samples of its Fourier transform along radial lines. In particular, we present a numerical algorithm for the computation of solutions to this infinite dimensional problem. Our theoretical results make precise the link between the sampling operator and the recoverability of the point sources.

Recent progress in the mathematical theory of fluid dynamics

Organised by: Paolo Secchi and Marco Sammartino

Finite-time singularity formation for Euler interface problems

Daniel Coutand†
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14:00 on Monday in LTJ

We will present a new general methodology establishing the finite-time singularity formation for Euler vortex sheet problems when gravity effects are considered.

On the controllability of the Navier-Stokes equations

Franck Sueur[1]†
Jean-Michel Coron[2] & Frédéric Marbach[2]
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[2]Laboratoire Jacques-Louis Lions
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14:40 on Monday in LTJ

In this work we consider the incompressible Navier-Stokes equations in a smooth bounded domain, either in 2D or in 3D, with a Navier slip-with-friction boundary condition except on a part of the boundary. This under-determination encodes that one has control over the remaining part of the boundary. We prove that for any initial data, for any positive time, there exists a weak Leray solution which vanishes at this given time.

Approximate current-vortex sheets near the onset of instability

Alessandro Morando[1]†
Paolo Secchi[1] & Paola Trebeschi[1]
[1]University of Brescia
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15:20 on Monday in LTJ

This talk is concerned with the free boundary problem for 2D current-vortex sheets in ideal incompressible magneto-hydrodynamics near the transition point between the linearized stability and instability. In order to study the dynamics of the discontinuity near the onset of the instability, Hunter and Thoo have introduced an asymptotic quadratically nonlinear integro-differential equation for the amplitude of small perturbations of the planar discontinuity. We study such amplitude equation and prove its nonlinear well-posedness under a stability condition given in terms of a longitudinal strain of the fluid along the discontinuity. We first present the problem and discuss some known results about the stability of current-vortex sheets; then we give some new results on the well-posedness of the Cauchy problem associated to the amplitude equation.

Dynamical systems and applications

Organised by: Jan Sieber

Dynamics of human prostate cancer cells in response to low androgen levels

Marianna Cerasuolo[1†]
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14:00 on Monday in LTL

Prostate cancer (PCa) is one of the most common cancers among males worldwide. Androgens (type of male hormones) play a critical role in stimulating PCa growth. Historically, treatment for metastatic prostate cancer has been limited to androgen deprivation therapy, but often such endocrine therapy fails in blocking tumour cell growth and PCa progresses to a hormone independent status. In vitro experiments, designed to explore the progression of PCa to androgen independence status, found that deprived PCa cells trans-differentiate into non-malignant neuroendocrine phenotype (NE cells). The experiments could not explain the role that NE cells play in androgen independence acquisition though.

A non-linear delayed mathematical model was developed within the study to fill in the knowledge gap, and was parameterised using independent experimental data. The model showed that after an initial transient phase, when human PCa (LNCaP) cell population appear almost extinguished and the non-malignant NE population dominate, a sudden trend reversal occurs, NE-like cells start sustaining the androgen-dependent malignant cells. The influence of the delay on the stability switches of the internal equilibria and on the long-term behaviour of the solutions was analysed. A numerical bifurcation analysis tool was used to explore the stability of the equilibria in dependence of unrestricted parameters.

Predicting financial stock crashes using ghost singularities

Damian Smug[1†]
Peter Ashwin[1] & Didier Sornette[2]
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14:20 on Monday in LTL

The analysis of a financial dynamical model which exhibits bubbles and crashes is presented. In addition to brief bifurcation analysis of equilibria and periodic orbits, the way of approximating the trajectories by extended centre manifold will be explained. Moreover, a notion of 'ghosts of finite-time singularities' is introduced. This concept will be used to predict time of a crash by estimating the periodicity of bubbles with periodicity of solutions for truncated normal form close to a bifurcation point. Finally, that idea is tested on the deterministic and stochastic system.

Meta-food-chains as a many-layer epidemic process on networks

Edmund Barter[1†] & Thilo Gross[1]
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14:40 on Monday in LTL

In ecological systems several different types of networks appear. One of these types is food webs, the networks of who eats whom. Another is dispersal networks, the networks on which animals travel between different locations, called patches. A simple modelling framework, proposed by Pillai et al., combines the dispersal of a meta-community of species over a network of patches, with feeding interactions between the species.

In this framework feeding interactions determine the availability of each patch to each species. Focussing on predator-prey interactions in a linear food chain, patches are only available to a predator if they are inhabited by its prey. So the patches available to a species are dependent on the dispersal process of its prey. The system can be represented by a multilayer network, where is layer is the network of patches available to a species, with interlayer interactions representing the feeding relationships. Each layer of the system evolves dynamically due to the dispersal of the other species in the meta-community.

We design operations connecting the rates of change of the generating functions for the degree distributions of the network of patches in consecutive layers. We find the solution for the steady state of the resulting equations, and hence for the degree distributions of the networks available to each species in the endemic state. We present the consequences of this analysis for the viability of species at all levels of the food chain. In particular we find that the variation in viability and abundance for species in the chain is dependent on the degree distribution of the underlying patch network.

Deterministic and stochastic modelling of pathogen-induced autoimmunity

Farzad Fatehi[1†]
Konstantin Blyuss[1] & Yulia Kyrychko[1]
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15:00 on Monday in LTL

In this talk I will discuss a new mathematical model for autoimmune diseases caused by viral infections. This detailed model takes particular account for different types of T cells, as well as interleukin 2 (IL 2). Stability analysis of various steady states is performed to identify parameter regions where the model exhibits different types of behaviour, including normal clearance of infection, chronic infection, as well as periodic oscillations associated with autoimmune response. I will also present a stochastic version of the model that allowed us to investigate the role of stochastic effects in facilitating possible oscillatory dynamics in the model. We have used van Kampen's system size to derive an approximation of the master equation and to obtain estimates for the variance of stochastic oscillations around deterministically stable steady states.

Sexual conflict accelerates species invasion

Kevin Minors[1†]
Tim Rogers[1], Richard James[1], Safi Darden[2] & Darren Croft[2]

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15:20 on Monday in LTL

Coupled interactions can significantly increase the invasion speed of a population. This talk will show that this is true in the case of sexual conflict in fish modelled by the FKPP equation. We consider one fish and show how 'run and tumble' movement can lead to diffusion with some rate at large times. Then, sexual conflict is introduced between two fish and we find that the effective diffusion rate of the two fish can be much larger than the diffusion rate of the one fish. Finally, the model is extended to consider larger populations of fish.

A numerical bifurcation analysis of spiral waves

Shreya Sehgal[1†] & Andrew J. Foulkes[1]

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15:40 on Monday in LTL

Spiral waves are spatio-temporal solutions to reaction-diffusion systems of equations. There are several types of motion of spiral waves. They can rotate rigidly in which the arm of the spiral is fixed in shape. They can also meander, which is a type of quasi-periodic motion. It has been shown numerically by other authors that a supercritical Hopf bifurcation is responsible for the transition from rigid rotation to meander. However, these studies were limited to small core spirals.

In this project, we study solutions to reaction-diffusion equations in a comoving frame of reference. This leads to a system of reaction-diffusion-advection equations in which the tip of the spiral wave is fixed in position and orientation. We study numerically spiral waves in a comoving frame of reference using the FitzHugh-Nagumo system of equations. We complete a numerical Hopf bifurcation analysis by studying the underlying limit cycles of meandering spiral waves in a comoving frame of reference. In this frame of reference, we can afford smaller domain sizes and therefore we can extend the analysis to large core spirals. Results show that indeed a Hopf bifurcation is responsible for the transition from rigid rotation to meander.

Monday PM: Contributed Talks

Miscellaneous fluids

Diffuse-interface modelling of flows in porous media

Douglas Addy[1†]
Marc Pradas[2], Markus Schmuck[3] & Serafim Kalliadasis[1]

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14:00 on Monday in LTA

Interfacial flows through porous media present a difficult challenge for numerical study, where the usual stress singularities are compounded by a complex and intricate substrate. By applying a diffuse-interface model it is possible to resolve the contact line singularity while homogenization techniques yield a coarse-grained description of the flow. We study a model we have recently developed which consists of a modified phase field equation valid at the macroscale with tensor coefficients given by a Poisson equation, and a Stokes equation in a representative microscopic geometry. The resulting equations are solved in series using a finite-element formulation with the open-source software FEniCS. We investigate the effects of different microscopic geometries in the bulk fluid flow and compare it with existing results, finding that this model can resolve the macroscale effects of the microstructure without simulating the microscale directly.

A Cole-Hopf-Feynman-Kac formula and quasi-invariance for the Navier-Stokes equations

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14:20 on Monday in LTA

We consider the fundamental issues of the Navier-Stokes equations using probabilistic methods to derive a condition for possible blowup. The basic idea is as follows. Given two similar equations; the closer they are, the harder it is for their solutions to behave in markedly different manners. Starting from the Navier-Stokes equations in the vector potentials A, we show that they can be written in a Wiener path-integral of the form $\theta =< F[\theta](W_t) >$ by generalising the Cole-Hopf transform and applying the Feynman-Kac formula. Here $A_j = k \log \theta_j, \quad j = 1, 2, 3$ with a constant $k (\neq 0)$, F represents essentially the nonlinear term, W_t Brownian motion and $< >$ an average.

Assuming that a solution blows up at $t_* = 1/(2a)$, by dynamic scaling we derive the Leray equations $\widetilde{\theta} =$ with a drift $h(t) = \int_0^t b(W_s) ds$, which have a smooth solution for all time. (The same notation t is used to denote time.) By transforming the probability measure, we can rewrite the Navier-Stokes equations as $\theta =< F[\theta](W_t + ah(t)) G_a >$ up to $t = \sqrt{2}/a > t_*$. Hence quasi-invariance (or, near-invariance) under dynamic scaling holds modulo the Maruyama-Girsanov density $G_a = \exp(-a \int_0^t (W_s) \cdot W_s ds - \frac{a^2}{2} \int_0^t |W_s|^2 ds)$. We therefore deduce that if a singularity shows up, it is necessary for G_a to shrink the lifetime of the solution below $1/(2a)$ to avoid a contradiction.

cf. K. Ohkitani, J. Phys. A: Math. Theor., 50 (2017) 045501.

Moving boundary problem of shape dependent oxidation of nanosized metal particle

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14:40 on Monday in LTA

I will describe analytic and numerical results of modeling Mott oxidation of nanosized aluminum particle, in Cabrera-Mott model. This is a moving boundary problem, a nonlinear version of Hele Shaw problem. Our results provide an estimate of oxidation time, and have important applications in modeling nano-energetic materials, composed of mixtures of nano-sized aluminum and oxidizer particles, with superior performance in highly energetic applications.

Non-Newtonian effects and Taylor dispersion in rivulet flow

Fatemah H. H. Al Mukahal[1†]
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15:00 on Monday in LTA

The previous work by Al Mukahal et al. [1,2] concerning rivulet flow of a power-law fluid provided a rare analytical benchmark for the study of rivulet flow of non-Newtonian fluids; however, it is of limited applicability to the flow of more realistic non-Newtonian fluids. In this talk we consider steady gravity-driven flow of a thin uniform rivulet of a generalised Newtonian fluid down a vertical planar substrate. We derive the parametric solution for any generalized Newtonian fluid whose viscosity is specified as a function of the shear rate (including, in particular, the solution for a Carreau fluid), and the explicit solution for any generalised Newtonian fluid whose viscosity is specified as a function of the shear stress (including, in particular, the solution for an Ellis fluid). These solutions are used to describe rivulet flow of a Carreau fluid and of an Ellis fluid, highlighting the similarities and differences between the behaviour of these two fluids. In particular, a rivulet of a strongly shear-thinning Ellis fluid can comprise two regions with different viscosities, with the velocity having a plug-like profile with large magnitude in a narrow central region of the rivulet. While Taylor dispersion [3] has been studied extensively for pipe and channels flows of various cross-sections, much less attention has been paid to transport in rivulet flow. Thus in this talk we also investigate both the short-time advection and the long-time dispersion of a passive solute in steady flow of a rivulet of a Newtonian fluid and subject to a uniform shear stress on its free surface down a vertical planar substrate. In particular, we derive the dispersion coefficient of the solute as a function of the shear stress.

[1] F. H. H. Al Mukahal, B. R. Duffy, S. K. Wilson, A rivulet of a power-law fluid with constant contact angle draining down a slowly varying substrate, Phys. Fluids 27 (2015) 052101.

[2] F. H. H. Al Mukahal, S. K. Wilson, B. R. Duffy, A rivulet of a power-law fluid with constant width draining down a slowly varying substrate, J. Non-Newt. Fluid Mech. 224 (2015) 30–39.

[3] G. I. Taylor, Dispersion of soluble matter in solvent flowing slowly through a tube, Proc. Roy. Soc. Lond. A 219 (1953) 186–203.

Explaining shear-thickening with friction

Adam Townsend[1†] & Helen Wilson[1]

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15:20 on Monday in LTA

Mixing cornflour and water into a thick paste which shear thickens is a classic kitchen science experiment. But the mechanism behind which cornflour shear thickens, when so many other mixtures shear thin, is not entirely clear. Over the last fifty years or so, there have been a few different theories to explain shear thickening, but recently, a consensus has developed that shear thickening is mostly caused by friction as particles rub past each other.

In this talk, we'll have a look at these theories and then see, through the use of Stokesian Dynamics simulations, how the addition of friction affects the behaviour of particles, both 'zoomed in' on the particles, and 'zoomed out' on the suspension as a whole. We'll also have a quick look (for Stokesian Dynamics fans) at how we can implement this in an efficient way.

Modeling self-organized shear flows in 0D, 1D and 2D

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15:40 on Monday in LTA

Self-organisation is a novel property of complex systems whereby a (macroscopic) order emerges out of complexity. In this talk, self-organization is invoked as a paradigm to explore the processes governing the evolution of shear flows. By examining the probability density function (PDF) of the local flow gradient (shear), we show that shear flows reach a quasi-equilibrium state as its growth of shear is balanced by shear relaxation. Specifically, the PDFs of the local shear are calculated numerically and analytically in reduced 1D and 0D models, where the PDFs are shown to converge to a bimodal distribution. This bimodal PDF is then shown to be reproduced in the nonlinear simulation of 2D hydrodynamic turbulence.

Acoustic Waves

Scattering from a row of aligned cylinders of arbitrary cross-section; tail-end asymptotics for the periodic Green's function

Georgia Lynott[1†]
William Parnell[1], David Abrahams[2] & Mike Simon[1]

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14:00 on Monday in LTE

The problem of wave scattering from a periodic row of parallel cylinders, of arbitrary cross-section, is studied via the boundary element method (BEM). The standard procedure of introducing the periodic Green's function is followed to give rise to a simplified integral equation. However, in general this requires the computation of an infinite sum that is slow to converge. Here a novel method is presented in order to approximate this infinite sum via a finite sum and asymptotic corrections; the scheme is rapidly convergent and straightforward to implement for cylinders of arbitrary cross-section. Numerical results for the transmission and reflection coefficients from arrays with different cross-sections are obtained.

Complex aeroacoustic catastrophes: fangs and the memory of singularities past

Jonathan Stone[1†]
Christopher J. Howls[1] & Rodney H. Self[1]

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14:20 on Monday in LTE

The downstream propagation of high frequency noise from a point source in a jet obeying Lilley's equation is well known to be organised around the so called "cone of silence", a fold catastrophe across which the amplitude may be modelled uniformly using Airy functions. In this talk we show that not only does significant jet noise unexpectedly propagate upstream, but is also organised at constant source distance around the cusp catastrophe with amplitude modelled locally by the Pearcey function. Furthermore the cone of silence is revealed to be a cross-section of a swallowtail catastrophe singularity structure with local form containing a Riemann sheet structure that appears to terminate the caustics but in fact transmutes them from order one coalescences to exponentially small types. We analyse the mathematical structure of the local form, assess the impact of both new singularities in the context of symmetric and asymmetric source distributions and discuss the existence of a global catastrophe field description.

Effect of relaxation on sonic booms

Eman Aljabali[1†] & Paul Hammerton[1]
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14:40 on Monday in LTE

A sonic boom is the sound associated with the pressure shock wave generated by disturbances in the atmosphere that lead to rapid increases in pressure over a short time. This paper studies the shock waves as it propagates through the atmosphere . When the shock is controlled solely by thermoviscous diffusion the disturbance is governed by Burgers' equation. Our interest is focused on investigating the shock location, width, and amplitude by applying different schemes such as Matched Asymptotic Expansions, Cole-Hopf transformation and numerical methods then combining the results. Results are then presented when relaxation processes associated with polyatomic molecules are included. Each relaxation mode is characterized by two parameters: a relaxation time and an effective concentration. A parametric study of the case of a single relaxation mode is presented.

Ultrasound propagation through contrast agent suspensions

Nicholas Ovenden[1†]
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15:00 on Monday in LTE

Gas microbubbles stabilised by a surfactant or polymer coating have been in clinical use as contrast agents for ultrasound imaging now for several decades. They are widely used in echocardiography and, increasingly, for quantitative studies of tissue perfusion. It is the highly nonlinear response of microbubbles to ultrasound excitation that makes them such effective contrast agents. It is this property also, however, that leads to significantly nonlinear propagation through regions of tissue containing microbubbles and both to image artefacts and difficulties in obtaining accurate quantitative information. There are very few theoretical models describing the response of a contrast agent population that take into account both the nonlinear behaviour and multiple bubble interactions. Most quantitative imaging algorithms rely on empirically derived correction factors and assume backscatter and attenuation respond linearly with concentration. Computational complexity arises when the medium contains a polydisperse bubble population because, to close the model, a nonlinear ordinary differential equation (ODE) governing the bubble response must be computed for the current radius of each bubble size present at each spatial location at every time step. Sonovue, a commercial contrast agent, has a range of bubble sizes from less than 0.5 μm to 5 μm, and, at a ultrasound frequencies of clinical interest, these bubble types exhibit a variety of differing behaviours. This can mean that many ODEs need to be computed at every space and time step to determine the integral on the right hand side of the wave equation, which unfortunately makes the numerical model impractical for real-time clinical use. In this work, we investigate an approximation that can potentially reduce significantly the computational complexity and demonstrate that, under certain parameter regimes, the approximation simulates the nonlinear propagation of an ultrasound pulse through a polydisperse population of bubbles to reasonably high accuracy.

Acoustic-gravity waves generated by impulsive sources at the ocean

Wade Parsons[1†] & Usama Kadri[2]

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15:20 on Monday in LTE

Impulsive sources at the ocean surface generate propagating compression-type modes known as acoustic–gravity waves (AGWs) that travel in the water column at speeds near the speed of sound in water, i.e. c=1500 m/s leaving a measurable pressure signature. Possible sources include solid objects impacting the water surface, e.g. falling meteorites, landslides, sudden formation of rogue waves, or storm surges. A lot of promising work has been reported on AGWs in the last few years due to sea floor sources, with an emphasise on remote sensing as an early detection of tsunami [1, 2]; and only very recently have surface sources started to attract more attention. Here, we extend some of these studies to the remote sensing of general events generated at the ocean surface.

To this end, we developed an analytical model for AGWs generated by an impulsive source at the free surface (the Green's function) from which the solution for various sources can be extracted. The results are compared with various solutions in literature whereby the source is located at the sea-floor [3, 4, 5, 6]. For the validation of the model, we carried out experiments in a water tank where neutrally buoyant spheres impacted the water surface, and the generated acoustic modes were recorded from a distance using a hydrophone. The shape of the pressure signature revealed three main regions that are associated with the impact, cavitation, and secondary wave formation. Employing these findings and solving the inverse problem allows remote sensing and prediction of the main source properties.

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[4] Parsons W. and Kadri U., (2016), Acoustic remote sensing of rogue waves, EGU General Assembly 2016.

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Two-dimensional wave propagation in a layered medium

Hilary Ockendon[1†]
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15:40 on Monday in LTE

This talk will consider linear waves propagating in a half-space in which the sound speed varies periodically in the y-direction and the waves are generated by an impulsive forcing at the boundary. It is known that the dominant large-time signal propagates with a homogenised wave speed irrespective of the layer structure and the direction of propagation. The detailed structure of the signal with decaying precursors and trailing wave trains will be described asymptotically and compared with numerical results.

Population models

The fundamental equation of life

Rowena Ball[1†] & John Brindley[2]

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14:00 on Monday in 22AA04

We model the putative hydrothermal rock pore setting for the origin of life on Earth as a train of continuous flow units coupled in series. Perfusing through the train are reactants that give rise to thermochemical and pH oscillations,and an activated nucleotide, which produces monomer and dimer monophosphates. The dynamical equations that model this system are fully thermally self-consistent. Crucially, we build stochasticity of the inputs into the model. Interrogating the computational results, we find that they infer various constraints and conditions on, and insights into, the origin of life and its physical setting: long, interconnected porous structures would have been essential, longitudinal nonuniformity of pores favourable, and the ubiquitous pH-dependences of all biology may well have been established in the prebiotic era. We demonstrate the important role of input fluctuations in driving the growth, evolution and diversification of the prebiotic world to a living world. In particular, we show explicitly that the resulting outputs must have a left-skewed, right-weighted probability distribution for a prebiotic system to evolve towards a living system. These results also vindicate the general approach of constructing and running a simple toy model to learn important new information about a complex system.

"Sense and Sensitivity": using mathematical modelling to understand how a plant remains responsive to bacteria

Lydia Rickett[1†]
Silke Robatzek[1], Richard Morris[2], Georg Felix[3], Justin Lee[4], Fabian Trempel[4], Joseph McKenna[5], Julia Vorholt[6] & Frank Menke[1]

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[2]John Innes Centre
[3]University of Tübingen
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14:20 on Monday in 22AA04

Plants are constantly surrounded by millions of microbes, often outnumbering the plant's own cells. These microbes may be beneficial, providing nutrition and maintaining growth [1], or they may be pathogenic, causing disease [2] and potentially leading to major crop losses. Since such losses present a worldwide threat to food security [3], a pressing but yet unanswered question is what is the exact nature of the relationship between the plant microbial community and plants? Much work in this area has focussed on the dedicated receptors through which plants sense the molecular patterns that are associated with bacteria via receptor-ligand binding, and the rapid, transient defence responses that are triggered as a result [4]. However, laboratory experiments hint that such responses are reduced in level, or even absent all together, when treating a second time with the same bacterial stimulus [5,6]. One hypothesis is that a continuous ability to respond to bacterial signals relies on a continuous supply of plant receptors. However, to test this hypothesis directly would require an unattainable quantitative experimental knowledge of receptor dynamics. We provide a means to investigate this hypothesis through the development of a probabilistic model which uses the Hill equation to tie together genomic studies of bacterial sequences and steady-state receptor binding data with experimental measurements of plant defence responses.

Our model shows that an ongoing supply of fresh plant receptors is key for the continued perception of bacterial stimuli, and at higher bacterial concentrations, predicts that receptor demand is not matched to supply, leading to a lag-time during which the plant becomes blind to the bacteria. We have found our predictions to be validated by ongoing experimental data, and the results of both raise some further interesting questions. In future work we will consider how different bacterial molecular patterns, or different bacterial populations in general, may compete, and whether the maintenance of receptor numbers comes at the expense of plant growth.

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Re-evaluating the vaccination against Haemophilus influenzae type b in England: insights from a mathematical model

Jasmina Panovska-Griffiths[1,2†]
Jodie McVernon[3], Lindsey Sillifant[1], Stefan Flasche[2]

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14:40 on Monday in 22AA04

Routine vaccination against Haemophilus influenzae type b (Hib) in UK started in late 1992 and has been effective in reducing Hib burden from a reported 838 cases in 1990 to only 33 cases in 1998. Outbreaks of invasive Hib over the period 1999-2003 motivated additional vaccination against Hib to be administered to all individuals under the age of 48 months in 2003, and, following its success, an additional vaccine booster dose was included in the UK immunisation schedule from 2006 and administered at 12 months of age.

In this work we use mathematical modelling, statistical data analysis and extensive numerical simulations to investigate the impact of including these vaccine boosters on the number of invasive Hib cases in England and thus assess the impact of changing the UK vaccination schedule against Hib on the burden of the disease. We develop an age-structured dynamic model that describes Hib transmission in the UK. The model is parametrised to available data for England and calibrated to data on invasive Hib cases in England 1990-2014. The calibrated model is used to explore the impact on changing the timing of the vaccine booster and the presence of a vaccine scare.

Our results suggest that the current vaccine schedule with the booster given at 12 months is most effective in reducing Hib burden, both in the short-term and the long-term. Therefore, our results support the current immunisation schedule in the UK. In presence of a vaccine scare, the number of Hib cases starts to increase and, at a low enough coverage level, the model projections suggest that the number of disease cases could return to pre-vaccine levels. This indicates the necessity for the continuation of the current vaccine programme against Hib in order to maintain low burden of this invasive disease in the UK.

Modelling and analysis of TGF-beta pathway on tumour progression and cell adhesion

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15:00 on Monday in 22AA04

Transforming growth factor beta (TGF-beta) signalling has been reported to play a crucial dual role in cancer progression and metastasis. Considering TGF-beta effect on cancer cell proliferation and adhesion we present a model of nonlocal PDEs describing the dynamics of two cancer cell populations and TGF-beta concentration, coupled with ODEs describing the dynamics of extracellular matrix and integrins. We discuss results regarding the existence and boundedness of solution, as well as the patterns obtained under different assumptions for cell-cell/cell-matrix adhesion.

Convexity of invariant manifolds in competitive population models

Stephen Baigent†

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15:20 on Monday in 22AA04

In the late 80's Morris Hirsch showed that many competitive population models have attracting invariant manifolds of codimension-one. These manifolds are Lipschitz and they are unordered in the sense that no two points on the manifold can be ordered component-wise. I have found that in some cases it is also possible to show that the manifolds are convex or concave. I will give simple graph-transform proofs of the existence of an attracting invariant manifold in both planar continuous-time and discrete-time competitive population models. Moreover, I will show how to determine when these manifolds are either convex or concave by proving the convexity/concavity of each iterate in the graph transform process. Finally, I will discuss how the convexity/concavity of the manifold relates to the stability of coexistence equilibria.

Elasticity

Numerical modelling of layered composite pipes under bending and pressure

Oleksandr Menshykov[1†]
Marina Menshykova[1] & Igor Guz[1]
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14:00 on Monday in 24AA04

The current study is focused on the optimal design of multi-layered thick-walled fibre reinforced composite pipes subjected to bending loading and pressure. The problem is considered for perfectly bonded layers and stress and displacement continuity conditions imposed on interfaces. The system of integral equations obtained from the moment equilibrium relation is solved numerically. Stress-strain state is analysed for pipes with different lay-ups and reinforcement structure.

Resolution of the threshold fracture energy paradox for solid particle erosion

Daniel Peck[1†]
Grigory Volkov[2], Gennady Mishuris[1] & Yuri Petrov[2]
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14:20 on Monday in 24AA04

Erosion impact is a long-studied phenomenon, with the previously developed models being widely accepted. Unfortunately, although surface fractures resulting from solid particle erosion are encountered in both nature and industry, the current models of brittle fracture mechanisms remained underdeveloped. This is particularly true for small scale 'dynamic' impacts, where effects ignored by the classical Griffith (or Griffith-Irwin) fracture criterion begin to play a decisive role.

In response to this situation, models have recently been developed for small scale single erosion impacts. They utilize the Schtaerman-Kilchevsky theory of quasi-static blunt impact to model an infinite elastic half-plane being impacted by a rigid axisymmetric indenter defined by the shape function $z = B^{(1-\lambda)}r^{\lambda}$. Dynamic effects where accounted for using specific derived relations for the impact loading pulse. Surface fractures resulting from the impact where predicted using an incubation time based fracture criterion.

This model was able to obtain a bound on the threshold fracture energy of an impact, defined as being the minimum energy required for fracture, however produced an unexpected result. Namely, that there existed a critical shape parameter $\lambda^* = 5.5$, with impacts involving indenters with shape function $\lambda < \lambda^*$ having a unique minimum threshold fracture energy, while no such minimum existed when $\lambda > \lambda^*$. No physical reason for this parameter could be provided by the initial authors, while further investigation was able to replace this bound with an exact relation but yielded no explanation for this phenomenon.

In this talk it will be demonstrated that this paradoxical value was a result of previous approaches failure to account for the dynamic properties of the elastic medium. These properties will be incorporated by introducing supersonic modelling during the initial stages of the impact. An algorithm capable of determining the threshold fracture energy from the new model will be demonstrated. Results showing that the paradox has been resolved will be provided.

Steady-state fracture of a double chain under a moving load

Nikolai Gorbushin[1†] & Gennady Mishuris[1]

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14:40 on Monday in 24AA04

Development of cellular materials led to a wide range of their mechanical properties which sometimes are superior to those of conventional materials. Analysis of dynamic fracture propagation in such materials is crucial to understand their performance. Successful approach to study such problems was proposed by Slepyan (1984) and further developed by him and his colleagues. We revising the problem of the crack propagating between the double chain considering more general load imposed by mowing forces.

Solution of the problem is obtained by using Wiener-Hopf technique. We derive the dependence between crack speed and loading parameters as well as the respective balance of energy. Analysis of the displacement field revealed existence of admissible and forbidden regimes of crack propagation at certain velocities. Moreover, anisotropy introduced in the structure may qualitatively influence on these regimes. It was shown that the energy-crack tip speed diagram alone is not enough to establish stability of the crack propagation and the force-speed relationship is instrumental for this purpose. The latter possesses monotonic character while the former does not.

Acknowledgements: This work was supported by FP7 Marie Curie ITN transfer of knowledge programme under project PITN-GA-2013-606878-CERMAT2.

Energy release rate in hydraulic fracture: can we neglect an impact of the hydraulically induced shear stress?

Gennady Mishuris[1†]
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15:00 on Monday in 24AA04

We discuss a novel model of hydraulic fracture (HF) formulation which accounts for the hydraulically induced shear stress at the crack faces. It utilizes a general form of the elasticity operator alongside a revised fracture propagation condition based on the critical value of the energy release rate.

We show that the revised formulation is always in agreement with the linear elastic fracture mechanics. Furthermore, our numerical simulations have highlighted advantages of the revised HF model in comparison with those which fail to account for the shear stress. In particular, we found that the small toughness regime no longer presents a significant computational challenge. The modified formulation opens new ways to analyse the physical phenomenon of HF, as well as improving the reliability and efficiency of its numerical simulation.

Averaging the strain in elastic equilibria

Mitchell Berger[†]

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15:20 on Monday in 24AA04

The elastic properties of structured materials are often averaged over sub-volumes of various scales inside the material. For sub-volumes smaller than a representative volume element, simple volume averaging of the stress and strain may not preserve the elastic energy. We introduce an averaging process which preserves the energy for all boundary conditions. This averaging process emphasises the parts of the material which carry the most stress. Here the effective strain is weighted by the local stress, and can be interpreted as an average strain over all paths taken by loads and forces through the volume. This alternative effective strain may be especially appropriate for materials with voids such as foams and granular matter, as the averaging only involves the material itself.

Tuesday AM: Mini-Symposia

Why I compute - in honour of Robert Rosner's 70th birthday

Organised by: Eun-Jin Kim and Mitchell Berger

Why do I compute?

Robert Rosner†

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09:00 on Tuesday in LTB

The non-dimensional numbers characterizing most astrophysical phenomena are sufficiently large that direct numerical simulations describing these phenomena are not just difficult – in most cases, such calculations are even in principle impossible. Despite this inconvenient fact, modern astrophysics clearly revels in modeling complex astrophysical phenomena – and the skeptical observer might well ask, 'to what effect?' I will discuss my answer to this question.

Quasi-cyclic behaviour in non-linear simulations of the shear dynamo

Michael R.E. Proctor[1†] & Robert J. Teed[1]

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09:40 on Tuesday in LTB

The solar magnetic field displays features on a wide range of lengthscales including spatial and temporal coherence on scales considerably larger than the chaotic con- vection which generates the field. Explaining how the Sun generates and sustains such large-scale magnetic field has been a major challenge of dynamo theory for many decades. Traditionally the 'mean-field' approach, utilising the well-known α-effect, has been used to explain the generation of large-scale field from small-scale turbulence. However, with the advent of increasingly high-resolution computer simulations there is doubt as to whether the mean-field method is applicable under solar conditions. Models such as the 'shear dynamo' provide an alternative mechanism for the genera- tion of large-scale field. In recent work we showed that while coherent magnetic field was possible under kinematic conditions (where the kinetic energy is far greater than magnetic energy), the saturated state typically displayed a destruction of large-scale field and a transition to a small-scale state. In this paper we report that the quenching of large-scale field in this way is not the only regime possible in the saturated state of this model. Across a range of simulations we find quasi-cyclic behaviour where large- scale field is preserved and oscillates between two preferred lengthscales. In this regime the kinetic and magnetic energies can be of a similar order of magnitude.

Strong field dynamo action in rotating convection with no inertia

David Hughes[1†] & Fausto Cattaneo[2]

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10:20 on Tuesday in LTB

The earth's magnetic field is generated by dynamo action driven by convection in the outer core. For numerical reasons, inertial and viscous forces play an important role in geodynamo models; however, the primary dynamical balance in the earth's core is believed to be between buoyancy, Coriolis, and magnetic forces. Here we adopt an approach complementary to that of most geodynamo models by considering a model of dynamo action driven by rapidly rotating convection in which inertial forces are neglected from the outset. Within this framework we are able to construct a branch of solutions in which the dynamo generates a strong magnetic field that satisfies the expected force balance. The resulting strongly magnetised convection is dramatically different from the corresponding solutions in which the field is weak.

Instability and transition shear flows

Organised by: Adam Butler

The modulation and excitation of stationary crossflow vortices by surface roughness

Adam Butler[1†] & Xuesong Wu[1]

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09:00 on Tuesday in LTD

A key factor in determining the characteristics of the laminar-turbulent transition process in shear layers is the receptivity of the flow to external disturbances. For swept-wing boundary layers in flight conditions, the development of the flow is typically dominated by stationary crossflow vortices, which are highly sensitive to surface roughness.

We consider the weakly-nonlinear interactions of stationary crossflow vortices with surface roughness. For certain modes, these interactions can be thought of as modulations of the modal amplitudes. We find that for typical roughness heights, this modulation can be significant compared to the leading order growth rate, or even larger than it, and that the amplitudes of pairs of interacting modes are fixed relative to one-another.

The largest response occurs for modes near the upper branch. If the critical layer of a nearly-neutral crossflow mode coincides with that of the mean flow perturbation induced by the roughness, a further interaction can occur within this layer. This interaction is quadratic in the amplitude of the crossflow mode.

These interactions occur soon after the stationary modes are first generated, and so can play an important role in determining the subsequent development of the perturbed flow.

On generation of instability modes in the wake of a trailing edge

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09:20 on Tuesday in LTD

This paper focuses on the generation of the symmetric-type (varicose) instability modes in the wake of a finite flat plate by a sound wave propagating in the external subsonic stream. This is an essential scattering problem acting in the vicinity of the trailing edge, which can be described by triple-deck formalism. In the near wake, where the viscous sublayer (lower deck) is thinner than the main shear layer, two instability modes, respectively of viscous and inviscid nature and corresponding to low and high frequencies, emerge. The asymptotic behaviours of both modes are analyzed, and the receptivity coefficients, defined as the ratio of the amplitude of the generated instability modes to that of the Stokes-shear layer induced by the freestream, are obtained by solving the local scattering system. Interestingly, the high-frequency modes, regardless of the viscous or the inviscid ones, always with small receptivity coefficients, are found to be more unstable as propagating to the far wake, due to their relatively high growth rates. Additionally, it is found that if the triple-deck frequency $\omega << R^{1/28}$, where R is the Reynolds number based on the length of the flat plate, the near-wake viscous mode will evolve to the far wake, and become the viscous-inviscid-interactive (VII) and the long-wavelength Rayleigh(LWR) modes eventually. On the other hand, if $\omega >> R^{1/28}$, the LWR modes will directly appear in the near wake, bypassing the emergence of the viscous and the VII modes. It is also confirmed that the LWR is essentially the solution of the Rayleigh equation in lower branch limit.

Nonlinear evolution and secondary instability of Gortler vortices induced by free-stream vortices

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09:40 on Tuesday in LTD

We investigate the nonlinear development and secondary instability of Görtler vortices driven by free-stream vortical disturbances (FSVD) in a boundary layer over a concave wall. For the general case where the Görtler number G_Λ (based on the spanwise wavelength Λ of the disturbance) is of order one, the formation and evolution of Görtler vortices are governed by the initial-boundary-value problem consisting of the nonlinear unsteady boundary-region equations (NUBR) and the appropriate upstream and far-field boundary conditions which characterise the impact of FSVD on the boundary layer. The initial-boundary-value problem is solved numerically. It is found that the nonlinear interactions inhabit the response and cause the Görtler vortices to saturate. The predicted modified mean-flow profiles and the structure of Görtler vortices are compared with experimental measurements. With the frequency increasing, the saturated amplitude of harmonic component (0,2) becomes larger, for which the excited nonlinear Görlter vortices are quasi-steady. A secondary instability analysis indicates that Görtler vortices become inviscidly unstable in the presence of FSVD with a high enough intensity. The characteristics of these unstable modes are also compared with recent experiments. When FSVD is unsteady, the secondary instability is intermittent. However, the intermittence diminishes with the frequency increasing. The present theoretical framework, which allows for a detailed description of entire transition process, from generation, through linear and nonlinear evolution, to secondary instability, represents a useful step towards predicting the pre-transitional flow and transition itself of the boundary layer over a blade in turbo-machinery.

Nonlinear evolution of unsteady streaks in a compressible boundary layer subject to free-stream vorticity

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10:00 on Tuesday in LTD

The nonlinear response of a compressible boundary layer to unsteady free-stream vortical fluctuations of the convected-gust type is investigated theoretically and numerically. The free-stream Mach number is assumed to be of $O(1)$ and the effects of compressibility, including aerodynamic heating and heat transfer at the wall, are taken into account. Attention is focused on low-frequency perturbations, which induce strong streamwise-elongated components of the boundary-layer disturbances, known as streaks or Klebanoff modes. The generation and nonlinear evolution of the streaks, whose amplitudes become comparable with the mean flow, are described on a self-consistent and first-principle basis using the mathematical framework of the nonlinear unsteady compressible boundary-region equations. The free-stream flow is studied by including the boundary-layer displacement effect and the solution is matched asymptotically with the boundary-layer flow. The nonlinear interactions inside the boundary layer drive an unsteady two-dimensional flow of acoustic nature in the outer inviscid region through the displacement effect. A close analogy with the flow over a thin oscillating airfoil is exploited to find analytical solutions. In the subsonic regime the disturbances propagate in all directions from the plate, while at supersonic speeds the fluid ahead of the body remains undisturbed and the perturbations are confined within the Mach dihedron. Numerical computations are performed for carefully chosen parameters that characterize three practical applications: turbomachinery systems, supersonic flight conditions and wind tunnel experiments. The results show that nonlinearity plays a marked stabilizing role on the velocity and temperature streaks, and this is found to be the case for low-disturbance environment such as flight conditions. Increasing the free-stream Mach number inhibits the kinematic fluctuations but enhances the thermal streaks, relative to the free-stream velocity and temperature respectively, and the overall effect of nonlinearity becomes weaker. An abrupt deviation of the nonlinear solution from the linear one is observed in the case pertaining to a supersonic wind tunnel. Large-amplitude thermal streaks and the strong abrupt stabilizing effect of nonlinearity are two new features of supersonic flows. The present study provides an accurate signature of nonlinear streaks in compressible boundary layers, which is indispensable for the secondary instability analysis of unsteady streaky boundary-layer flows.

Nonlinear spatio-temporal patterns in inclined layer convection

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10:20 on Tuesday in LTD

Patterns driven by buoyancy (e.g., the Rayleigh-Bénard system) can be associated with a sequence of bifurcations of the uniform base state. Consequently, methods that exploit the linear instability such as weakly nonlinear analysis are employed to analyse them. In contrast, shear driven patterns (e.g. plane Couette flow) occur even when the associated basic state is linearly stable. Analysis of such subcritical patterns requires a fully nonlinear analysis and thus remains challenging. To investigate the formation of spatio-temporal patterns due to the interaction of buoyancy and a mean shear, we focus on the inclined layer convection (ILC) system. In the ILC cell, the fluid layer is inclined to the horizontal plane and subject to a temperature gradient and generates different patterns due to the interaction of buoyancy and shear. Three relevant parameters characterize this system: the ratio of momentum to thermal diffusivity (Prandtl number, Pr), the ratio of buoyancy to viscous forces (Rayleigh number, R) and the angle of inclination. At small angles of incline, the uniform base state becomes unstable to secondary instabilities in the form of buoyancy dominated longitudinal rolls. There exists a critical angle of incline, a co-dimension 2 point, above which shear driven transverse roll instabilities take over as the secondary instabilities. By varying the thermal driving and the inclination angle for a chosen Prandtl number fluid and we compute the location of co-dimension 2 point, all secondary bifurcations and the resulting tertiary states using weakly nonlinear analysis. Calculated secondary and tertiary states are employed as initial solutions and the full nonlinear bifurcation behaviour of the ILC system at different angles of incline is obtained through numerical continuation methods.

Stabilisation of finite-amplitude solutions in shear flow

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10:40 on Tuesday in LTD

Pipe flow, channel flow and Couette flow are linearly stable at flow rates for which turbulence frequently occurs. In other words, there exist two attractors for the same flow rate - one corresponding to the simple 'laminar' solution to the governing equations, and a complicated chaotic attractor corresponding to the turbulent state. Probing the boundary between the two attractors has led to the discovery of many new solutions to the governing Navier-Stokes equations over the last 10-15 years for these geometries. Probing the boundary requires bisection between two initial conditions that are attracted to the two different states. This can be very expensive for full 3-dimensional simulations. In this work we investigate the possibility of accessing these solutions, and potentially undiscovered ones, by stabilising them via a dynamic flow rate, thus requiring only a single simulation.

Lighthill-Thwaites prize talks

Organised by: IMA

From one-dimensional fields to Vlasov equilibria: theory and application of Hermite polynomials

Oliver Allanson[1†]
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09:00 on Tuesday in LTF

It is estimated that more than 99% of the visible matter in the universe is in the plasma state. Most astrophysical plasmas - and indeed many other plasmas - are extremely diffuse and/or exhibit sufficiently dynamic phenomena such that particle collisions need not be considered in the mathematical models. On the level of the distribution of particles in phase-space (position and velocity), such collisionless plasmas admit an infinite number of statistically steady states. This is in contrast to collisional gases and plasmas, for which there is a unique statistical equilibrium: the Maxwellian.

Many plasma phenomena critically depend on the kinetic physics of particle distributions. Hence, it is not only interesting, but important to understand the equilibrium distribution functions that are self-consistent with a given macroscopic configuration, as defined by the electromagnetic fields, electric currents and thermal pressures: the 'inverse problem in collisionless equilibria'.

I will discuss recent progress on this inverse problem, and focus on the method of representing the particle distribution functions using expansions in Hermite polynomials: a complete set of orthogonal functions. It is often possible to solve the inverse problem by this method in a formal sense, but there are important questions regarding the convergence, boundedness, and non-negativity of the resultant expression.

Asymptotic analysis of a drying model motivated by coffee bean roasting

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09:20 on Tuesday in LTF

/BMCabstract Understanding heat, moisture, and mass transport during the roasting of a coffee bean is essential to identifying the colour and flavours of the final product. Recent modelling of coffee bean roasting suggests that in the early stages of roasting, there are two emergent regions: a dried outer region and a saturated interior region. These two regions are separated by a moving transition layer (or drying front). We will consider the asymptotic analysis of this drying process in order to gain a better understanding of its salient features. The model consists of a PDE system governing the moisture and gas pressure profiles throughout the interior of the bean. By obtaining asymptotic expansions for these quantities in relevant limits of the physical parameters, we are able to determine the qualitative behaviour of the outer and interior regions, as well as the dynamics of the drying front. Indeed, we find that for all of the asymptotic limits considered, our approximate solutions faithfully reproduce the qualitative features evident from numerical simulations.

An integral equation method for the homogenization of unidirectional fibre reinforced media; antiplane elasticity

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09:40 on Tuesday in LTF

In Parnell and Abrahams (2008 Proc. Roy. Soc. A 464, 1461-1482) (doi:10.1098/rspa.2007.0254) a homogenization scheme was developed that gave rise to explicit, analytical forms for the effective antiplane shear moduli of a periodic unidirectional fibre-reinforced medium. The expressions are rational functions in the volume fraction. In that scheme a (non-dilute) approximation was invoked in order to determine leading order expressions. It was shown that agreement with existing methods is good except at very high volume fractions. Here the theory is extended in order to determine higher order terms in the rational function expansions. The methodology is attractive in that the expressions can be derived for a large class of fibres with noncircular cross section. Terms are identified as being associated with the lattice geometry of the periodic structure, fibre cross-sectional shape, and host/fibre material properties. The expressions are derived in the context of antiplane elasticity but the analogy with the potential problem illustrates the broad applicability of the method to, e.g., thermal, electrostatic and magnetostatic problems. The efficacy of the scheme is illustrated by comparison with the well-established method of asymptotic homogenization where for fibres of general cross section, the associated cell problem is usually solved by some computational scheme.

Edge behaviour in the glass sheet redraw process

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10:00 on Tuesday in LTF

Thin glass sheets are used in smartphone, battery and semiconductor technology, and may be manufactured by first producing a relatively thick glass slab and subsequently redrawing it to a required thickness. In this latter redraw process, the sheet is fed into a furnace where it is heated and stretched so that its thickness decreases. However, undesirable features such as thick edges commonly form, and the resulting sheets are not suitable for purpose. Experimental observations indicate that thickness variations can be confined to a small region near the sheet edge, if the heater zone through which the sheet is drawn is short compared to its width. In this talk I will present a mathematical model for a viscous sheet undergoing redraw, and use asymptotic analysis in the thin-sheet, short-heater-zone limit to investigate the formation of thick edges and the dependence on process parameters.

The effect of ions on the motion of an oil slug in a charged capillary

Zachary Wilmott[1†]
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10:20 on Tuesday in LTF

Experimental studies have documented that injecting low salinity water into an oil reservoir can increase the amount of oil recovered. However, due to the complexity of the chemical interactions involved in this process, there has been much debate over the dominant mechanism causing this effect. One proposal suggests that ion exchange reactions on the oil and mineral surfaces are responsible for the increased recovery.

We discuss the implications of these reactions by studying the motion of an oil slug through a clay capillary in the presence of salt water. We use an ion dissociation model for the exchange reactions, and discuss how this affects the surface charges of the oil and the clay. By using lubrication theory to describe the thin-film flow in the water layer separating the oil from the clay surface, we study how the surface charges affect the thickness of the water film.

From this we determine the macroscopic flow rate of the oil through the capillary, assuming that the thin film of water reduces to an effective boundary condition allowing partial slip on the clay surface. By modelling the ion exchange mechanism from the molecular scale to the pore scale, we are able to determine whether ion exchange reactions are responsible for the low salinity effect.

Quantum theory information

Organised by: Davide Girolami

Quantum coherence, time-translation symmetry, and thermodynamics

Matteo Lostaglio[1†]
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09:00 on Tuesday in LTH

The first law of thermodynamics imposes not just a constraint on the energy content of systems in extreme quantum regimes but also symmetry constraints related to the thermodynamic processing of quantum coherence. We show that this thermodynamic symmetry decomposes any quantum state into mode operators that quantify the coherence present in the state. We then establish general upper and lower bounds for the evolution of quantum coherence under arbitrary thermal operations, valid for any temperature. We identify primitive coherence manipulations and show that the transfer of coherence between energy levels manifests irreversibility not captured by free energy. Moreover, the recently developed thermomajorization relations on block-diagonal quantum states are observed to be special cases of this symmetry analysis.

Extracting a quantum distance from relative Renyi entropy measures

Reevu Maity[1†] & Davide Girolami[1]
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09:40 on Tuesday in LTH

The non-commutativity between two observables is interpreted via the uncertainty principle whereas between state and observable via asymmetry/superselection rules. It is an interesting question to ask about an interpretation regarding the non-commutativity between two quantum states. We define a class of measures of quantum distances $D_q(\rho,\sigma)$ based on the non-commutativity between two states and which are always lesser than or equal to the total distance $\mathrm{D}(\rho,\sigma)$ given by the relative entropy, relative-Renyi entropy and the sandwiched-Renyi entropy. They depend on a parameter α which is an important quantity for the family of relative entropies. We find an operational interpretation of the non-commutativity between two quantum states in the problem of binary quantum Hypothesis testing.

Quantum processes which do not use coherence

Benjamin Yadin[1†]
Jiajun Ma[2], Davide Girolami[1], Mile Gu[3] & Vlatko Vedral[1,2]

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10:20 on Tuesday in LTH

A major distinction between classical and quantum physics is coherence, the existence of superposition states. This has been identified as a fundamental element powering quantum technologies, and also plays a key role in situations as diverse as quantum thermodynamics, many-body physics and transport processes. Inspired by techniques learned from quantum information, a resource theory approach to quantifying coherence has been developed recently. This provides a mathematical formalism for describing the manipulation of coherence and for its rigorous quantification. At the heart of the resource theory of coherence are the so-called incoherent operations – those operations which cannot create coherence. However, it is not straightforward to identify exactly which operations are the relevant ones; a number of alternatives have been proposed.

Here, we show that the "strictly incoherent" (SI) operations define a physically-motivated resource theory of coherence. Firstly, we characterise these operations as being unable to either create coherence or to use it to go beyond classical dynamics. Next, we prove that the SI set captures all operations which can be constructed from the basic building-blocks of incoherent unitary dynamics and measurements. We give a physical picture for this result in terms of interferometry.

Finally, we study the implications for coherence in multipartite settings. General quantum correlations (in the form of discord) can be described in terms of coherence shared between subsystems. Using the set of SI operations performed locally, we are able to discuss the interplay between coherence and correlations in situations where both are considered resources.

Recent progress in the mathematical theory of fluid dynamics

Organised by: Paolo Secchi and Marco Sammartino

Stability of boundary layer flows

David Gerard-Varet[1†]
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09:00 on Tuesday in LTJ

We shall present a recent work on the short time justification of boundary layer expansions in 2D Navier-Stokes. This justification is only possible under strong monotonicity and regularity assumptions on the data, due to underlying hydrodynamic instabilities. We shall discuss these instabilites, and state an optimal stability result in Gevrey regularity.

Long time behaviour for a dissipative shallow water model

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09:40 on Tuesday in LTJ

We consider a two-dimensional dissipative shallow water model, describing the motion of an incompressible viscous fluid, confined in a shallow basin, with varying bottom topography. We construct the approximate inertial manifolds for the associated dynamical system and estimate its order. Finally, working in the whole space $\mathbb{R}^2$, under suitable conditions on the time dependent forcing term, we prove the L^2 asymptotic decay of the weak solutions.

Local existence of MHD contact discontinuities

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Alessandro Morando[1] & Yuri Trakhinin[2]
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10:20 on Tuesday in LTJ

We prove the local-in-time existence of solutions with a contact discontinuity of the equations of ideal compressible magnetohydrodynamics (MHD) for 2D planar flows provided that the Rayleigh-Taylor sign condition $[\partial p/\partial N] < 0$ on the jump of the normal derivative of the pressure is satisfied at each point of the initial discontinuity. MHD contact discontinuities are characteristic discontinuities with no flow across the discontinuity for which the pressure, the magnetic field and the velocity are continuous whereas the density and the entropy may have a jump. We first prove the well-posedness in Sobolev spaces of the linearized problem under the Rayleigh-Taylor sign condition satisfied at each point of the unperturbed discontinuity. Then, the proof of the resolution of the nonlinear problem follows from a suitable tame a priori estimate in Sobolev spaces for the linearized equations and a Nash-Moser iteration. This is a joint work with A. Morando and Y. Trakhinin.

Spatial localisation in fluids

Organised by: Cédric Beaume

Localized binary convection in a slightly inclined rectangular cavity

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09:00 on Tuesday in LTL

In a binary fluid layer heated from below, stationary localized structures, the so-called convectons, have been identified. We studied their origin, properties and the snaking branches of solutions where convectons were located. In the present work, convective flows are disturbed by a small inclination of the cell. This effect is particularly interesting as it induces a large scale flow at any value of the heating and, thus, prevents the existence of the no-flow conductive state found in the horizontal case. We investigate the new localized structures that arise in this situation, where the vertical buoyancy competes with the lateral one, mainly induced by a strong concentration gradient characteristic of the large scale flow.

From convectors to complexity in doubly diffusive convection

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09:20 on Tuesday in LTL

Doubly diffusive convection arises frequently in natural phenomena and industrial processes and occurs in systems characterised by competing fields that diffuse at different rates. Well-known examples are provided by thermohaline convection and the salt finger instability. In this talk, we consider three-dimensional thermohaline convection where a binary mixture is confined between vertical walls maintained at different temperatures and salinities. In this configuration, we found stationary spatially localised solutions consisting of spots of convection embedded in a background conduction state. These convectons are formed through a subcritical bifurcation from the conductive state (motionless fluid) and display a variety of patterns while simulations above onset reveal chaotic dynamics.

Dynamics of spatially localized patterns in binary fluids

Yasumasa Nishiura[1 †]
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09:40 on Tuesday in LTL

Binary fluid is a mixture of two miscible fluids such as water and ethanol. When a temperature gradient is applied to a binary fluid, "Soret effect", an effect where the temperature gradient induces a concentration gradient, appears. There are both positive and negative Soret effects. In the positive case, the heavier component is condensed in cooler parts and vice versa. Our work focused on the negative effect, in which case, the temperature field activates the convection, which is in turn inhibited by the concentration field. In contrast to mono fluid, there appears spatially localized convection patterns such as steady pulse (SP, also known as "convecton") and time-periodic traveling pulse (PTP). We discuss about the collision dynamics between two PTPs. A variety of patterns appear after the complex transition pro- cess when counter-propagating PTPs collide with each other. In these transition processes, even-symmetric SPs play a role as "separators" and thus the network of saddles consisting of even-symmetric SPs is very important to predict the asymptotic state of collisions. We present this network structure as well as a detailed analysis of the even-SP branch.

Creation and annihilation bifurcation mechanisms of localized travelling pulses in shear flows

Alvaro Meseguer[1 †]
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10:00 on Tuesday in LTL

In this talk we will study different localization and delocalizations mechanisms of travelling waves in plane channel flows. One of the mechanisms consist of subharmonic Hopf bifurcations of replicated Tollmien-Schlichting Waves (TSW) in arbitrary long domains. We will show how these bifurcations sometimes exhibit Benjamin-Feir instabilities leading to localized modulated travelling pulses. We will show how these instabilities may also lead to modulated travelling waves that do not necessarily exhibit localization but serve as connection between upper and lower branches of TSW. Finally we will see that localized pulses may exhibit snaking scenarios that are not always robust, being structurally very sensitive to changes of channel length.

Oblique stripes of turbulence in plane Poiseuille flow

Yohann Duguet[1†]
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10:20 on Tuesday in LTL

The onset of turbulence in extended three-dimensional plane Poiseuille flow is characterised by oblique stripe patterns of turbulent motion co-existing spatially with laminar flow. We first show experimental evidence that these stripes relaminarise by themselves for low enough flow rates. Using direct numerical simulation in a reduced oblique geometry, we unfold for the first time a sequence of bifurcations leading from a simple localised (unstable) travelling wave to a strange repeller, via relative periodic orbits and unstable relative tori. This study, parametrised by the angle of the stripe, suggests a minimal value of the Reynolds number for the appearance of localised turbulent motion.

LMS Scheme 3 minisymposium Quantitative evolutionary biology

Organised by: Rebecca Hoyle, Gianne Derks and Daniele Avitabile

Fidelity of parent-offspring transmission and the evolution of social behaviour in structured populations

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09:00 on Tuesday in 22AA04

The theoretical investigation of how spatial structure affects the evolution of social behavior has mostly been done under the assumption that parent-offspring strategy transmission is perfect, i.e., for genetically transmitted traits, that mutation is very weak or absent. In this talk, we investigate the evolution of social behavior in structured populations under arbitrary mutation probabilities. We consider spatially structured populations of fixed size N, in which two types of individuals,A and B, corresponding to two types of social behavior, are competing. Under the assumption of small phenotypic differences (weak selection), we provide a formula for the expected frequency of type A individuals in the population, and deduce conditions for the long-term success of one strategy against another. We then illustrate this result with three common life-cycles (Wright-Fisher,Moran Birth-Death and Moran Death-Birth),and specific population structures. Qualitatively, we find that some life-cycles (Moran Birth-Death,Wright-Fisher, when social interactions affect fecundities) prevent the evolution of altruistic behavior, confirming previous results obtained with perfect strategy transmission. Imperfect strategy transmission also alters the balance between the benefits and costs of staying next to one's kin, leading to surprising results in subdivided populations, in that higher emigration probabilities can be favourable to the evolution of altruistic strategies.

Is the world full of species?

Thomas Ezard†

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09:40 on Tuesday in 22AA04

The intuition of a finite limit to the number of species that can co-exist on Earth appeals because Earth itself is a finite object. Population biologists call such a limit the "carrying capacity". The chief regulating factor is purported to be a density-dependent skeleton: as density goes up then population growth goes down because of resource limitation. Around the deterministic pattern, environmental noise generates short-term fluctuations. The co-option of such ecological theory into palaeobiology has a long history but simple re-interpretation of elementary ecology into "diversity dependence" is compromised by environmental and ecological changes. Despite the preposterous suggestion of an infinite number of things within a finite area, the evidence for species limits remains the subject of much recent debate. I adapted mathematical models of dynamic diversity dependence and fitted them statistically to plankton data from the Cenozoic Era. The best supported models suggest an environmentally dependent macroevolutionary form of contest competition that yields finite upper bounds on species richness. Models of biotic competition assuming unchanging environmental conditions were overwhelmingly rejected. In the best-supported model, temperature affects the per-lineage diversification rate, while both temperature and an environmental driver of sediment accumulation defines the upper limit. These models show how a sufficiently complete and fine-grained fossil record strongly supports a more dynamic diversity-dependence than is usually considered.

The mathematics of cultural accumulation

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10:20 on Tuesday in 22AA04

The ways in which human culture is created, spreads, and accumulates profoundly impacts human evolution as well as important nuances of human behaviour, from political decisions to responses to epidemics. Cultural evolution is a highly interdisciplinary field that seeks to understand human culture as a Darwinian system of variation, transmission, and competition. One focus of cultural evolution theory has been the dynamics of cultural accumulation and loss: how to we gain culture and how do we lose it?

Recent comparisons of modern hunter-gatherer populations have shown considerable variation in the number of tools used to get and process food – from fewer than ten tools to over one hundred. Mathematical models and laboratory experiments have suggested that this variation is driven by population size – that smaller populations can maintain less culture and larger populations can maintain more. However, regional and worldwide statistical analyses of toolkit sizes in hunter-gatherer populations provide, at best, weak support for this conclusion. Instead, the data suggest that the environmental variation to which a population is subject determines cultural complexity. Mathematical models rarely focus on environmental fluctuation explicitly.

Here I present a model drawing on theoretical population genetics that accounts for the effects of environmental fluctuation as well as population size in determining cultural complexity. The relative importance of environmental fluctuations, demographic factors, and innovation are directly compared in a novel theoretical framework.

Multi-scale systems: from analysis to numerics to applications

Organised by: Ben Goddard, Serafim Kalliadasis and Greg Pavliotis

Multi-scale modelling of nanoparticle suspensions

Pietro Asinari[1†]
Annalisa Cardellini[1], Matteo Alberghini[1], Matteo Fasano[1] & Eliodoro Chiavazzo[1]
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09:00 on Tuesday in 24AA04

Self-assembly of nanoparticles (NPs) into mesoscopic ordered structures plays a crucial role in a large variety of applications including pharmaceutical, food, drug delivery, immunology and technological. On the one hand, trying to prevent and avoid the self-organization of nanoparticles has traditionally been the main issue for stabilizing nano-suspensions, foams and emulsions. On the other hand, the aggregation of building-blocks into mesoscopic structures has allowed to explore new materials with desired functionalities and properties. For example, many experiments and some theoretical studies have shown that the chain-forming morphologies in nano-suspensions allow an enhancement of thermal properties [1]. However, due to the challenges of controlling the multiscale phenomena occurring in nano-suspensions, clear guidelines for their rational design are still missing. Despite a wide range of experimental observations, there is an increasing need to establish rigorous modelling techniques, able to explore and describe the multiscale nature of nano-suspensions [2].

In the present work a multiscale modelling approach is implemented to relate the nanoscale phenomena to the macroscopic bulk properties of nano-suspensions. Specifically, Molecular Dynamics (MD) simulations and Brownian Dynamics (BD) are synergistically integrated to understand the mechanisms driving the building-block interactions and hence to predict the shapes of assembled clusters. First, the pair Potential of Mean Forces (pPMF) is computed between atomistic modelled NPs dispersed in aqueous solutions. A sensitivity analysis is carried out by altering the hydrophilicity of the nanoparticles, their surface charge and the salt concentration of the bulk solutions. The role of anionic (Sodium Dodecyl Sulfate -SDS-) and cationic (Dodecyl Trimethyl Ammomium -DTAB-) surfactants is also investigated. Second, Brownian Dynamics simulations are carried out to understand how nanoscale phenomena, like the hydration layer or steric interactions, affect the mesoscale dynamics.

The Coarse Grained procedure here suggested offers a practical multiscale approach for guiding a robust and optimal design of nanoparticle suspensions.

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DFT study of a system with competing interactions

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09:40 on Tuesday in 24AA04

Classical density functional theory (DFT) is a powerful tool to study the equilibrium structures and corresponding thermodynamics of a system with competing interactions. Competing interactions are, beside a hard core that excludes particle overlap, attractive at short and repulsive at longer distances, and are modeled here by two Yukawa potentials. In such systems it is known that particle clusters can form and that these clusters can build inhomogeneous periodic structures on a length-scale significantly larger than the particle size. We study these inhomogeneous structures by freely minimizing the DFT in three spatial dimensions within the framework of the so-called fundamental measure theory (FMT), which is the state of the art theory for hard-sphere repulsion. We briefly discuss some methods to speed up the numerical computation in three dimensions by fast Fourier transforms (FFT) and the use of modern graphics cards (GPU computing).

Multiscale modelling of burning on neutron stars

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10:20 on Tuesday in 24AA04

Type I X-ray bursts are nuclear explosions which occur on the surfaces of neutron stars. The physics of these explosions spans a wide range of scales: the burning itself is confined to a thin flame front, several centimetres thick, while the propagation of this burning front is driven by the Coriolis force which acts on the order of kilometres. This makes modelling the bursts quite challenging. In our work, a multiscale model is used, with the large scale physics described using the relativistic shallow water equations, the intermediate scale described by the compressible relativistic fluid equations and the smallest scale described by the relativistic low Mach number equations. The different scales are coupled together using mesh refinement methods, allowing us to capture the relevant physics at the different scales whilst keeping the problem feasible numerically and produce the first whole neutron star ocean model.

Transport properties at fluids interfaces: macroscopic relations for microscopic phenomena

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10:40 on Tuesday in 24AA04

Recent technological progress across a wide spectrum of applications involving fluids and soft matter in general, from the rapidly growing field of micro- and nanofluidics, the design of surfaces with controllable wetting properties to colloidal science and bioengineering, require a rational and systematic understanding of fundamental physical properties of matter at the nanoscopic-molecular level. Integration and scaling out of the associated products, processes and devices requires appropriate macroscopic relations able to integrate a microscopic description of fluid and soft matter properties at liquid-vapour and multi-fluids interfaces is missing. Nevertheless, despite numerous works in the literature (e.g. [1, 2, 3]), such macroscopic relations are not available as of today one more often than not studies on interfacial phenomena rely on oversimplified assumption, e.g. that the shear or dynamic viscosity can be considered the same constant across interfaces separating different phases and throughout the phases (e.g. [4]). For these reasons but also for the crucial role played by fluid-fluid interfaces in a wide variety of fundamental multiphase problems and applications, detailed theoretical-computations investigations are necessary. At the same time, experiments at the nanoscale are extremely difficult and are hampered by errors, while continuum models based on Navier-Stokes equations are not suitable for molecular-scale phenomena where the discrete behaviour of matter comes to the fore. Molecular dynamics (MD) offers an attractive alternative in view of these difficulties and indeed it is the computational tool of choice in the nanoscale regime. This numerical technique, together with elements from statistical mechanics, allows the computation of the physical properties of interest, with a molecular resolution. In our work, non-equilibrium MD simulations of a Lennard-Jones fluid are used to scrutinise efficiently and systematically, through the tools of statistical mechanics, the main properties of fluids, namely density variations, stress tensor,and shear viscosity, at the interface between liquid and vapour and between two partially miscible fluids. Our analysis has led to the formulation of a general relation between shear viscosity and density variations validated for various interfacial fluid problems. In addition, it provides a rational description of other interfacial quantities of interest, including surface tension and its origins.

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[3] P. Geysermans, N. Elyeznasni, V. Russier, Layered interfaces between immiscible liquids studied by density- functional theory and molecular-dynamics simulations, The Journal of Chemical Physics 123 (20).

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LMS Scheme 3 minisymposium Inverse problems

Organised by: Carola-Bibiane Schönlieb, Matthias Ehrhardt and Martin Benning

Model identification, prediction, and validation in inverse problems

Franz J Király[†]
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9:00 on Tuesday in 39AA04

Over-fitting and model complexity mismatch are severe practical problems whenever the underlying inversion model (physical/mechanistic, analytic, statistical, or otherwise) is not fully known. Furthermore, even when regularization techniques are employed to alleviate the issue, it is often difficult to get an exact quantitative certificate for the type or complexity of the model appropriate for the data at hand.

While modern supervised machine learning offers a wealth of meta-modelling techniques for model selection and model assessment of predictive models, their application to the inversion setting is not immediate - simply, since the primary goal in inversion is not prediction.

In the talk, we will discuss in some stylized examples connections between inversion-type parameter estimation and predictive modelling, with implications for selecting and assessing inversion models.

Gradient descent in a generalised Bregman distance framework

Martin Benning[1†]
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9:40 on Tuesday in 39AA04

We discuss a special form of gradient descent that in the literature has become known as the so-called linearised Bregman iteration. The idea is to replace the classical (squared) two norm metric in the gradient descent setting with a generalised Bregman distance, based on a more general proper, convex and lower semi-continuous functional. Gradient descent as well as the entropic mirror descent by Nemirovsky and Yudin are special cases, as is a specific form of non-linear Landweber iteration introduced by Bachmayr and Burger. We are going to analyse the linearised Bregman iteration in a setting where the functional we want to minimise is neither necessarily Lipschitz-continuous (in the classical sense) nor necessarily convex, and establish a global convergence result under the additional assumption that the functional we wish to minimise satisfies the so-called Kurdyka-Łojasiewicz property.

Level set and sparsity regularization for an inverse problem of the 2D time-dependent transport equation in optical tomography

Oliver Dorn[1†] & Kernel Prieto[2]

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10:20 on Tuesday in 39AA04

The time-dependent linear transport equation is a model used in medical imaging for describing the propagation of near-infrared light through human tissue. It is of importance in emerging fields such as Diffuse Optical Tomography (DOT), Fluorescence Tomography or Photoacoustic Tomography. In the inverse problem two parameters are needed to be estimated, the absorption cross section and the scattering cross section, from boundary measurement of outgoing flux. Practically a severe cross-talk is usually observed when trying to recover both parameters from the same data set simultaneously. Moreover, the inverse problem is severely ill-posed which means that details of embedded structures and their precise boundaries are usually blurred in the reconstructions .We will investigate in this talk two novel reconstruction techniques which are designed to address these issues by special regularization techniques. The first one is a level set based shape reconstruction method, and the second one a sparsity regularized Tikhonov style approach. We will compare those with a more traditional inhouse approach using numerical experiments in 2D. The results show that each of these three approaches has merits and pitfalls, and works well in certain situations but might show weaknesses in others.

Tuesday AM: Contributed Talks

Electromagnetic waves

Wigner function approach to analyse near-to-far field propagation

Deepthee Madenoor Ramapriya[1†]
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09:00 on Tuesday in LTA

In nature and technology there are many examples of complex sources, such as printed circuit boards (PCBs) that emit electromagnetic radiation that is stochastic. The modelling of such radiation from complex sources is challenging and a long standing research issue. In our approach we use Wigner Distribution Function (WDF), which is a quasi-probability distribution that helps analyse radiation from complex sources statistically in phase space. We treat problems in which the complex sources are random and which therefore call for a statistical approach instead of a deterministic one. Our model helps characterize wave propagation both in the near field and far field. This is done by exploiting the relationship between the WDF and Correlation function (CF). We also see that the theory extends well from 2D to the 3D case which is again validated using experimental measurements. Though the model presented deals with free space propagation, this can be extended to complex environments such as electronic devices in enclosures etc.

Propagation of electromagnetic waves in large scale environments—an operator approach

Valon Blakaj[1†]
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09:20 on Tuesday in LTA

New wireless communication systems will operate at much higher frequencies than used so far for 3G and 4G ranging from microwave (6 GHz) to mmWave (above 30 GHz). Wave propagation prediction is a challenging task at these frequencies, and will need new modelling tools compared to those used for current wireless systems and need to include the complexitiy of the environment at much finer scale. In this talk, a linear operator formulation of ray tracing will be introduced, the so-called Dynamical Energy Analysis (DEA), which is a phase-space method for predicting the distribution of waves intensities in complex environments. We focus here in particular on integrating the effects of scattering from rough surfaces in DEA type calculations. We choose top-hat functions (or 'pixels') as the DEA basis functions in phase- space. In addition, we characterize the rough surfaces as a Gaussian random process, that is, the surface fluctuations is Gaussian and the correlation function is Gaussian as well. Simulations have been carried out for different statistics of rough surfaces. We show that even for small fluctuations of the rough surfaces, the significant local variations in the ray-field are observed in the scattered field. We will furthermore present propagation of the field-field correlation from rough surfaces based on a Wigner function approach. The latter calculation provides the diffusion coefficient of the scattered field, which can be used as input in large-scale DEA calculation.

Electromagnetic wave diffraction of perfect electric conducting wedges with arbitrary linear polarization

Matthew Nethercote[1†]
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09:40 on Tuesday in LTA

In this talk, we will present a method to find the electromagnetic field and the Poynting vector resulting from a time harmonic electromagnetic plane wave of an arbitrary linear polarization incident on an infinite perfect electric conducting wedge. The aim is to find out how the polarization of this incident electromagnetic wave impacts the solution to diffraction by perfectly conducting wedges. We will use the third dimensional invariance of the scatterer and the perfect conducting boundary conditions to rewrite the electromagnetic field, governed by the source free Maxwell's equations, in terms of two uncoupled scalar potentials or Debye potentials. These potentials depend on the polarization angle and can be shown to respectively solve the Dirichlet and Neumann scalar (acoustic) wedge problems. They can therefore be found using the Sommerfeld-Malyuzhinets technique. After this, we approximate the electromagnetic field and the time averaged Poynting vector for high frequency and plot the components of both quantities for different values of the polarization angle to determine its impact.

Waves in slowly varying band-gap media

Ory Schnitzer†

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10:00 on Tuesday in LTA

This talk is concerned with the asymptotic description of high-frequency waves in locally periodic media (period wavelength). A key issue is that the Bloch-dispersion curves vary with the local microstructure, giving rise to hidden singularities associated with band-gap edges and branch crossings. I will describe an asymptotic approach for overcoming this difficulty, and take a first step by studying in detail the simplest case of 1D Helmholtz waves. The method entails matching adiabatically propagating Bloch waves, captured by a multiple-scale Wentzel-Kramers-Brillouin (WKB) approximation, with complementary multiple-scale solutions spatially localised about spectral singularities. Within the latter regions, the Bloch wavenumber is nearly critical; this allows their homogenisation, following the method of high-frequency homogenisation (HFH), over a scale intermediate between the period (& wavelength) and the macro-scale. Analogously to a classical turning-point analysis, close to a spatial band-gap edge the solution is an Airy function, only that it is modulated on the short scale by a standing-wave Bloch eigenfunction; asymptotic matching between the WKB and HFH solutions furnishes a detailed description of Bloch-wave reflection from a band-gap edge. Excellent agreement between the asymptotic theory and exact numerical calculations is demonstrated for layered media.

Active cloaking for flexural waves in a pinned Kirchhoff plate

Stewart Haslinger[1†]
Özgür Selsil[1], Natasha Movchan[1], Jane O'Neill[1] & Richard Craster[2]

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10:20 on Tuesday in LTA

We present new results for the active cloaking of flexural waves in a structured Kirchhoff plate. A cluster of periodically spaced pins is surrounded by active sources, represented by the non-singular Green's function for the two-dimensional biharmonic operator. The sources possess complex amplitudes that are determined by solving an algebraic system of equations enforcing the cancellation of selected multipole orders of the scattered wave field. For frequencies in the zero-frequency stop band, a characteristic feature of a pinned plate, we find that only a small number of active sources are sufficient for efficient cloaking, and may be placed at nearest-neighbour lattice sites exterior to the cluster. For higher frequencies, active cloaking is achieved by positioning the sources on a circle surrounding the cluster. We demonstrate the cloaking efficiency with several numerical illustrations, considering key frequencies from band diagrams and dispersion surfaces for a Kirchhoff plate pinned in a doubly periodic fashion.

Cells and biomechanics

A vertex-based model relating cell shape and mechanical stress in an epithelium

Oliver Jensen[1†]
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09:00 on Tuesday in LTE

Using a popular vertex-based model to describe a spatially disordered planar epithelial monolayer. we examine the relationship between cell shape and mechanical stress at the cell and tissue level. Deriving expressions for stress tensors starting from an energetic formulation of the model, we show that the principal axes of stress for an individual cell align with the principal axes of shape, and we determine the bulk effective tissue pressure when the monolayer is isotropic at the tissue level. Using simulations for a monolayer that is not under peripheral stress, we fit parameters of the model to experimental data for Xenopus embryonic tissue. The model predicts that mechanical interactions can generate mesoscopic patterns within the monolayer that exhibit long-range correlations in cell shape. The model also suggests that the orientation of mechanical and geometric cues for processes such as cell division are likely to be strongly correlated in real epithelia.

Discrete and continuum modelling of cell-ECM adhesion

Linda Irons[1†]
Markus Owen[1], Reuben O'Dea[1] & Bindi Brook[1]
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09:20 on Tuesday in LTE

Integrin-mediated adhesions between airway smooth muscle (ASM) cells and the extracellular matrix (ECM) regulate how contractile forces generated within the cell are transmitted to the cell's external environment. Experimental evidence indicates that this could have important consequences for asthmatics. In this talk I will introduce two models of cell-ECM adhesion: firstly, a stochastic-elastic computational model, and then a continuum approximation. These will be presented by means of a 1D case study in which we consider the effect of an oscillatory loading (representing environmental fluctuations associated with tidal breathing) on the formation and stability of adhesions. Dependent on the amplitude of the oscillations, either bond rupture or bond formation dominate, and we expect this to affect the attainable levels of force transmission. For intermediate magnitudes of forcing we observe a region of bistability and hysteresis. Changes in the region of bistability for different parameters are observed and could have implications for contractile force transmission from ASM cells to the airway tissue during tidal breathing or deep inspirations.

Microtubule curling

Simon P Pearce[1†]
Matthias Heil[1], Oliver Jensen[1], Gareth Wyn Jones[1] & Andreas Prokop[1]
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09:40 on Tuesday in LTE

The brain is wired by delicate cable-like projections (axons) extruding from neurons, which must be maintained throughout life. A main component of axons are well-organised, parallel bundles of microtubules, which serve as structural backbones and highways for intracellular transport. This microtubule order must be highly regulated over the organism's lifetime, but in neurodegenerative diseases and ageing microtubules often become disorganised, leading to cell death. In vitro, microtubules have been shown to have a large persistence length (2-4mm), meaning that on cellular length-scales they should act like rigid rods. However, in areas of neurodisorganisation highly curved microtubules are seen, with radii of curvature of less than 1μm. Here we model the microtubules as inextensible elastic rods with a preferred curvature, and explore how this preferred curvature can evolve due to internal or external factors, enabling the generation of metastable curved states without high applied forces.

Micro-scale undulatory locomotion in heterogeneous viscoelastic environments

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10:00 on Tuesday in LTE

While many microorganisms swim in viscoelastic fluids, there are notable examples where the suspended microstructure that makes the fluid viscoelastic is at the same length scale as the swimmers. Here, the swimming cells experience the surrounding medium as a set of obstacles suspended in a viscous fluid, rather than a viscoelastic continuum. Using simulations based on the force-coupling method, we explore this situation for a simple undulatory swimmer as it moves through an environment of obstacles that are tethered to random points in space via linear springs. We examine how swimming behaviour is altered by mechanical interactions with the obstacles by varying obstacle density and tether stiffness. We find that the mechanical interactions can either enhance or hinder locomotion, and often for fixed tether stiffness, there is an obstacle density for which the average speed is maximized. We also find cases where the swimmer is completely trapped by the environment. In addition, we find that the velocity fluctuations, and consequently the effective swimmer diffusion, are also highly dependent on environment composition and a non-monotonic dependence on the relevant parameters can be found here as well.

Growth dynamics of tubular structures

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10:20 on Tuesday in LTE

Many living biological tissues are known to grow in response to their mechanical environment, such as changes in the surrounding pressure. This growth response can be seen, for instance, in the adaptation of heart chamber size and arterial wall thickness to changes in blood pressure. We discuss the growth dynamics of tubular structures, which are very common in biology (e.g. arteries, plant stems, airways). Many living elastic tissues actively maintain a preferred level of mechanical internal (residual) stress, called the mechanical homeostasis. The tissue-level feedback mechanism by which changes of the local mechanical stresses affect growth is called a growth law within the theory of morphoelasticity, a theory for understanding the coupling between mechanics and geometry in growing and evolving biological material. In this talk we analyse a homeostasis-driven growth law and explore issues of heterogeneity and growth stability in tubes. We show that the stability of the homeostatic state non-trivially depends on the anisotropy of the growth response. The key role of anisotropy may provide a foundation for experimental testing of homeostasis-driven growth laws.

Differential contractility as a new mechanism for mechanotransduction in epithelial cells

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10:40 on Tuesday in LTE

It is well known that cells can sense and respond to the mechanical properties of their environments. In particular, a range of studies have shown that changes in substrate stiffness can be linked to changes in behaviours as diverse as growth, motility and type specification. The mechanisms by which cells effect such stiffness sensing is, however, still unclear. There has been significant experimental activity focusing on the role of Focal Adhesions (FAs), adhesive regions connecting the cell and the extracellular matrix, in mechanotransduction. Here we present an alternative mechanism based on the implications of differential contractility within epithelial cells. We use a continuum elasticity description of a contractile cell coupled to an elastic foundation with a single non-dimensional control parameter quantifying substrate stiffness. We predict how and where regions of localised stretch are generated within the cell as substrate stiffness increases and thus elucidate a novel potential mechanism for linking conformational change with downstream molecular mechanotransduction. The areas of mechanotransduction predicted by the model are significantly displaced from areas of maximal FA density, with important implications for experimental design.

Thin films

Stability of thin liquid curtains

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09:00 on Tuesday in LTG

We investigate the stability of thin liquid curtains with respect to two-dimensional perturbations. The dynamics of perturbations with wavelengths exceeding (or comparable to) the curtain's thickness are examined using the lubrication approximation (or a kind of geometric optics). It is shown that, contrary to the previous theoretical results, but in agreement with the experimental ones, all curtains are stable with respect to small perturbations. Large perturbations can still be unstable, however, but only if they propagate upstream and, thus, disrupt the curtain at its outlet. This circumstance enables us to obtain an effective stability criterion by deriving an existence condition for upstream propagating perturbations.

Destabilisation of self-similar rupture solutions in a thin film equation

Michael Dallaston[1]†
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09:20 on Tuesday in LTG

In this talk we consider the lubrication model of a thin film destabilised by an effect such as van der Waals force, thermocapillarity or the Rayleigh-Taylor instability (buoyancy), while being stabilised by surface tension. The resulting thin film equations differ only in the exponent of the coefficient function of the destabilising term (depending on the physical effect being modelled), but have markedly different dynamics, particularly concerning finite time rupture (where the thickness goes to zero at a point).

Using numerical continuation, we examine the existence and stability of self-similar rupture solutions of the thin film equation as the exponent of the destabilising term is varied. Our main finding is the existence of a Hopf bifurcation by which the main branch of similarity solutions loses stability and, as confirmed by computation of the thin film equation itself, leads to the formation of successively smaller 'iterated structures' in the film as rupture is approached.

Long-wave analysis and control of the viscous Rayleigh-Taylor instability using electric fields

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Thomas G. Anderson[2], Peter G. Petropoulos[3] & Demetrios T. Papageorgiou[1]

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09:40 on Tuesday in LTG

We investigate the electrostatic stabilisation of a viscous thin film wetting the underside of a horizontal surface in the presence of an electric field applied parallel to the surface. The model includes the effect of bounding solid dielectric regions above and below the liquid-air system that are typically found in experiments. The competition between gravitational forces, surface tension and the nonlocal effect of the applied electric field is captured analytically in the form of a nonlinear evolution equation. A semi-spectral solution strategy is employed to resolve the dynamics of the resulting partial differential equation. Furthermore, we conduct direct numerical simulations (DNS) of the Navier-Stokes equations using the volume-of-fluid methodology and assess the accuracy of the obtained solutions in the long-wave (thin film) regime when varying the electric field strength from zero up to the point when complete stabilisation occurs. We employ DNS to examine the limitations of the asymptotically derived behaviour as the liquid layer thickness increases, and find excellent agreement even beyond the regime of strict applicability of the asymptotic solution. Finally, the asymptotic and computational approaches are utilised to identify robust and efficient active control mechanisms allowing the manipulation of the fluid interface in light of engineering applications at small scales, such as mixing.

Existence and stability of stationary solutions of a thin film equation with Derjaguin disjoining pressure

Abdulwahed Alshaikhi[1†]
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10:00 on Tuesday in LTG

Using the Liapunov-Schmidt reduction, we describe the structure of stationary solutions of a thin film equation with Derjaguin disjoining pressure of the form $\Pi(h) = h^{-n-m} - h^{-m}$, where $h(x, y, t)$ is the thickness of the film and the exponents n, m are positive, as the mean height of the film is varied. Using AUTO, we perform a continuation analysis of these stationary solutions and establish the existence of both "spinodal" and of "nucleation and growth" regimes, and discuss the structure of the global attractor of the equation in both cases.

Capturing fluid structure in thin-film models for the motion of terraced drops

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10:20 on Tuesday in LTG

In recent two dimensional computations of contact lines, the location where two fluid phases and a solid surface meet, it is seen that the macroscopic assumption of homogeneous densities in the fluid phases breaks down dramatically [1]. Instead, highly structured fluid layers occur and give rise to an effective wetting energy, i.e. a binding potential, which accounts for an additional contribution to the free energy from the interaction between the fluid-fluid and fluid-solid interfaces. Efforts have been made to extract this binding potential from nanoscopically accurate density functional theory (DFT) computations [1,2] at equilibrium, and in this presentation we discuss the rich behaviour these additional contributions can make to the dynamics of droplet motion. Of particular interest [3] are in situations where droplets can become terraced due to oscillatory binding potentials, and additionally, in situations at very small scales where the thin-film equation must be modified to account for single-particle hopping, leading to a diffusive dynamics, rather than the collective hydrodynamic motion usually modelled.

[1] A. Nold, D.N. Sibley, B.D. Goddard, and S. Kalliadasis, Phys. Fluids (2014) http://dx.doi.org/10.1063/1.488612

[2] A.P. Hughes, U. Thiele and A.J. Archer, (2017) preprint at http://arxiv.org/abs/1611.06957

[3] H. Yin, D.N. Sibley, U. Thiele and A.J. Archer, (2017) preprint at http://arxiv.org/abs/1611.00390

Ice formation within a thin film flowing over a flat plate

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10:40 on Tuesday in LTG

We present a model for ice formation in a thin, viscous liquid film driven along a flat plate by a Blasius boundary layer after heating is switched off along part of the plate. The plate cooling is applied downstream of a point, Lx_0, located an O(L)-distance from the tip of the plate, where L is much larger than the film thickness. The cooling is assumed to be slow enough that the flow is quasi-steady. We present a thorough asymptotic derivation of the governing equations from the incompressible Navier-Stokes equations in each fluid and the corresponding Stefan problem for ice growth. The problem breaks down into two temporal regimes, corresponding to the relative size of the temperature difference across the ice, which we analyse asymptotically and numerically. In each regime, there are two distinct spatial regions, an outer region on the lengthscale of the plate, and an inner region close to x_0 in which the film and air flows are driven over the growing ice layer. Moreover, in the early-time regime, there is an additional intermediate region in which the the air-water interface propagates a slope discontinuity downstream due to the sudden onset of the ice at the switch-off point. For each regime, we present ice profiles and growth rates, and show that for large times, the film is predicted to rupture in the outer region when the slope discontinuity becomes sufficiently enhanced. We conclude with a discussion on the applicability of the model.

Tuesday PM: Mini-Symposia

Why I compute - in honour of Robert Rosner's 70th birthday

Organised by: Eun-Jin Kim and Mitchell Berger

The fun of simulating MHD turbulence

Joanne Mason†

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14:00 on Tuesday in LTB

Computations are key for understanding astrophysical turbulence, for which Reynolds numbers are huge. Numerical simulations can provide a way of differentiating between competing theoretical models. However, if numerical effects are misinterpreted as being physical, numerical results can lead us astray. Here I will describe 'why I compute' incompressible, field-guided, MHD turbulence.

Why I compute...statistics of astrophysical flows and dynamos

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14:40 on Tuesday in LTB

Astrophysical flows and dynamos are characterised by the huge range of spatial and temporal scales that need to be resolved. Here I shall describe a programme based on computing the statistics of such flows directly - termed Direct Statistical Simulation (DSS), by comparing the results from DSS with DNS. I shall further discuss how DSS can be made computationally efficient using model reduction and how it may form the basis of a conservative and efficient subgrid model for schemes that wish to describe the dynamics of resolved scales.

Simulating convectively-driven astrophysical dynamos

Paul Bushby†

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15:20 on Tuesday in LTB

BMCabstract In many astrophysical systems, it is believed that the observed magnetic fields are generated and maintained by a hydromagnetic dynamo mechanism. Astrophysical dynamos typically operate in rather extreme conditions that cannot be modelled directly using numerical simulations. Most simulations therefore occupy regions of parameter space that are far removed from astrophysical reality. I will discuss some recent numerical work relating to the problem of dynamo action driven by convection in a (non-rotating) layer of fluid that is heated from below. I will also describe some of the limitations of these existing numerical models. For example, it is very difficult to simulate dynamos in the astrophysically-relevant parameter regime of low magnetic Prandtl number, in which the kinematic viscosity of the fluid is very much smaller than the magnetic diffusivity. Even modest reductions in this parameter from a value of order unity lead to a significant increase in the level of driving needed to produce a dynamo. What does this imply for dynamos in nature? I will conclude by describing alternative approaches to direct numerical simulations of this type, focusing upon their possible strengths and weaknesses.

Instability and transition shear flows

Organised by: Adam Butler

Streak instability in near-wall turbulence revisited

Andrea Cassinelli[1†]
Matteo de Giovanetti[1] & Yongyun Hwang[1]
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14:00 on Tuesday in LTD

The regeneration cycle of near-wall streaks and streamwise vortices is of fundamental importance in sustaining turbulence. The streak breakdown phase is the process responsible for the formation of streamwise vortices, but the understanding of the underlying physical mechanism is limited to the case of laminar flows. In this study, a series of direct numerical simulations in turbulent channel flow are employed to drive amplified streaks, by applying an optimal forcing profile computed using linear theory. Subsequently, dynamic mode decomposition allows to extract the dominant flow features at specific wavelengths. Upon increase of the forcing amplitude, streaks generate intense fluctuations in the cross-stream plane, with peaks in the range $\lambda_x^+ \approx 200 - 300$, corresponding to the characteristic wavelengths of near-wall vortical structures. No amplification is found to occur for structures longer than $\lambda_x^+ \approx 2000 - 3000$. The flow is also shown to be composed of sinuous meandering streaks and alternating cross-streamwise velocity structures, associated with sinuous-mode streak instability in previous studies. Finally, the structures found are aligned along the critical layer of the secondary instability, suggesting that the surrounding turbulence does not change the inviscid inflectional breakdown mechanism of streaks.

Globally unstable behaviour in the rotating-disc boundary layer

Christian Thomas[1†] & Christopher Davies[2]
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14:20 on Tuesday in LTD

Linear disturbance development in the von Karman boundary layer over a rotating-disc is investigated for an extensive range of azimuthal mode numbers n. The study expands upon earlier investigations that were limited to modes $n < 75$ that established globally stable characteristics (Davies & Carpenter 2003). Numerical simulations of larger valued n suggest that a form of global instability can be engineered that is similar to that found on a rotating-disc with mass suction (Thomas & Davies 2010). The numerical solutions indicate that a change in the response of the global behaviour arises about $n = 85$ that is marginally greater than those disturbances studied previously. Hence, why global instability was not found earlier. Furthermore, the Reynolds number corresponding to the large valued n-calculations coincides with the upper bound of experimental predictions for transition. The new findings are directly related to the Huerre & Monkewitz (1990) local-global stability criterion that states that in order to generate global instability, the region of local absolute instability (for a fixed n) must exceed a certain threshold size. Thus, for large enough n, the extent of absolute instability in the von Karman flow (Lingwood 1995) has become sufficient to generate globally unstable behaviour. Comparisons are drawn with the solutions of the linearized complex Ginzburg-Landau equation for a spatially varying stability parameter. Using solutions of disturbances in the rotating-disc boundary layer based on the parallel flow assumption, conditions are determined that can be used to predict the global stability characteristics. The global response is governed by a detuning effect, based on the radially varying temporal frequency and growth rate. Radial variations in the temporal growth rate are found to increase with larger n, eventually attaining values sufficient to establish global instability.

Global instability of mixing layers over sloping sea beds

Jonathan Healey†

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14:40 on Tuesday in LTD

Wind blowing over the sea surface can generate a mixed layer of approximately uniform flow lying over essentially stationary fluid. The mixing layer between the lower stationary fluid and the flow in the upper mixed layer is subject to the Kelvin-Helmholtz instability. Healey (2009, JFM) showed that such flows can become absolutely unstable for some ranges of the distance between the mixing layer and the sea surface and the distance between the mixing layer and the sea bed. In the present study we investigate the effects of gently sloping sea beds on the global stability properties of the flow. WKB theory is used and amplitude equations derived near turning points that act as wavemaker regions driving the entire flow. Globally unstable sea-bed topographies are identified, and also topographies leading to novel forms of amplitude equation. Global instability is expected to enhance mixing in coastal regions.

Ice-fluid coupling

Organised by: Frank Smith and Alexander Korobkin

Water entry problem in the presence of floating ice flows

Alexander Korobkin[1†] & Tatyana Khabakhpasheva[2]

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14:00 on Tuesday in LTE

The two-dimensional model of water impact in between several floating flat ice floes is presented. Initially the liquid and the floes are at rest. A solid body touches the free water surface at a single point. Then the body starts to penetrate liquid at a given speed. Motions of the ice floes, wetted region of the entering body and forces acting on the body are studied for different numbers of floes, their dimensions and characteristics. Vertical displacement and inclination of each floe are determined as functions of time. The hydrodynamic part of the problem is linearised. The boundary conditions are imposed on the initial equilibrium water level. Gravity, surface tension and the liquid viscosity are not included in the model. The resulting problem is nonlinear due to nonlinear equations for the size and position of the contact region between the surface of the entering body and the liquid. The solution of the problem is obtained for arbitrary number of floes.

Freezing droplets on solid surfaces with arbitrarily complex geometry

Davide Bella[†]

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14:20 on Tuesday in LTE

Ranging over different phenomena, the matter of nucleation is for sure one of the oldest and richest in terms of human empirical experience and yet it is still a subject with many grey areas (not just white). Understanding physically what it is happening is one aspect; but also giving a firmer mathematical framework and having a reliable set of experiments with which to compare are two big challenges to address. Our project tackles these two challenges in the case of ice nucleation in water droplets. In particular we are interested in understanding how such behaviour is influenced by the surface on which the droplet is sitting. The method used is basically a generalization of a Stefan problem with arbitrary boundary conditions, applying variational calculus of a suited functional to obtain different solutions in various environments with and without gravity. From that it is also possible is in principle to extend the study in a "dynamical sense", so that the growing ice nuclei motions are on a slope or they bounce on a substrate. The aim is to build a framework strong enough to be applied in general cases and reliable enough to be gauged in a simple way. The implications of a deep understanding of an apparently mundane phenomenon such as nucleation can be quite deep and the applications are numerous and interesting. Designing coatings for aircraft that may even avoid the icing problem altogether or at least reduce it, is but one example.

Impact ice growth and properties

David Hammond†

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14:40 on Tuesday in LTE

Ice growing on aerial structures such as wind turbines and aircraft in cloud has a range of properties and appearance. Using shape, micro-structural and properties data, we can probe the ice growth process and reflect on some of the remaining challenges to modelling ice growth.

Numerical study of solitary wave propagation in a fragmented ice plate

Emilian Parau[1]† & Philippe Guyenne[2]

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15:00 on Tuesday in LTE

A numerical model for simulation of nonlinear ocean waves propagating through fragmented sea ice is proposed. This model solves the full equations for nonlinear potential flow coupled with a nonlinear thin-plate formulation for the ice cover. The coefficient of flexural rigidity is allowed to vary spatially so that distributions of ice floes can be directly specified in the physical domain. Two-dimensional simulations are performed to examine the attenuation of solitary waves by scattering through an irregular array of ice floes.

The response of the sea ice edge to atmospheric and oceanic jet formation

Harold Heorton[1]†
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15:20 on Tuesday in LTE

/BMCabstract The sea ice edge presents a region of many feedback processes between the atmosphere, ocean and sea ice. Here we focus on the impact of on-ice atmospheric and oceanic flows at the sea ice edge. Mesoscale jet formation due to the Coriolis effect is well understood over sharp changes in surface roughness such as coastlines. This sharp change in surface roughness is experienced by the atmosphere and ocean encountering a compacted sea ice edge. This paper presents a study of a dynamic sea ice edge responding to prescribed atmospheric and oceanic jet formation. An idealised analytical model of sea ice drift is developed and compared to a sea ice climate model (the CICE model) run on an idealised domain. The response of the CICE model to jet formation is tested at various resolutions.

We find that the formation of atmospheric jets at the sea ice edge increases the wind speed parallel to the sea ice edge and results in the formation of a sea ice drift jet in agreement with an observed sea ice drift jet. The increase in ice drift speed is dependent upon the angle between the ice edge and wind and results in up to a 40% increase in ice transport along the sea ice edge. The possibility of oceanic jet formation and the resultant effect upon the sea ice edge is less conclusive. Observations and climate model data of the polar oceans have been analysed to show areas of likely atmospheric jet formation, with the Fram Strait being of particular interest.

Wave interaction with floating ice sheets

Richard Porter†

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15:40 on Tuesday in LTE

The subject of this talk is the interaction of water waves with thin elastic ice floes floating freely on the surface of the fluid. A variety of problems will be discussed in which the use of Fourier transforms allow problems to be reduced to integral equations whose numerical solutions are efficiently computed. A selection of results will illustrate the method and connections will be made to problems in renewable ocean wave energy.

Applying maths to public health

Organised by: Jasmina Panovska-Griffiths

Modelling influenza transmission and vaccination for public health decision-making

Katherine Atkins[1†]
Marc Baguelin[2], David Hodgson[3], Jasmina Panovska-Griffiths[3], Richard Pebody[2] & Edwin van Leeuwen[2]

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14:00 on Tuesday in LTF

In 2013 England and Wales began to fund a live attenuated influenza vaccine programme for individuals aged 2–16 years. Mathematical modelling predicts substantial beneficial herd effects for the entire population as a result of reduced influenza transmission. With a decreased influenza-associated disease burden, existing immunisation programmes might be less cost-effective. We used a mathematical model coupled with an economic evaluation to assess the impact of the existing elderly and risk group vaccination programme under the new policy of mass paediatric vaccination in England.

For this cost-effectiveness analysis, we used a transmission model of seasonal influenza calibrated to 14 seasons of weekly consultation and virology data in England and Wales. We combined this model with an economic evaluation to calculate the incremental cost-effectiveness ratios, measured in cost per quality-adjusted life-years (QALY) gained.

Our results suggest that well timed administration of paediatric vaccination would reduce the number of low-risk elderly influenza cases to a greater extent than would vaccination of the low-risk elderly themselves if the elderly uptake is achieved more slowly. Although high-risk vaccination remains cost-effective, substantial uncertainty exists as to whether low-risk elderly vaccination remains cost-effective, driven by the choice of cost-effectiveness threshold.

A novel approach for whole cell tracking based on geometric partial differential equations

Anotida Madzvamuse[†]

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14:40 on Tuesday in LTF

In this talk, I will present a novel approach for whole cell tracking based on geometric partial differential equations for the cell surface motion where the physics of the migrating cell is easily encoded. In order to fit to experimental data an optimal control framework using phase-field theory is presented. A highly efficient, adaptive and fast multigrid solver is then employed to allow for realistic 2D and 3D simulations. Numerical examples will be exhibited that show the applicability of the mathematical framework for whole cell migration. Cell migration is a multistep process essential for mammalian organisms and is closely linked to processes such as development, immune response, wound healing, tissue differentiation and regeneration, inflammation, tumour invasion and metastasis formation.

Vaccine induced herd immunity for control of RSV

Timothy Kinyanjui[1†]
Thomas House[1,4], Moses Kiti[2], Patricia Cane[3], James Nokes[2,4] & Graham Medley[5]

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15:20 on Tuesday in LTF

Epidemic models have proved useful in a number of applications in epidemiology. In this work, I will present an age structured mathematical model used to describe the transmission of Respiratory Syncytial Virus in a developing country setting. The model captures the key epidemiological characteristics of RSV and fitted to age-specific hospitalisation data using statistical maximum likelihood. Routine vaccination at different ages and coverage is then implemented at the stable limit cycle and was assumed to elicit an immune response equivalent to primary infection. We found that delayed infant vaccination has significant potential in reducing the number of hospitalisations in the most vulnerable group and that most of the reduction is due to indirect protection. It also suggests that marked public health benefit could be achieved through RSV vaccine delivered to age groups not seen as most at risk of severe disease. A full sensitivity and uncertainty analysis suggests that the results are robust to the model structure and parameters.

Asymptotic methods in the applied sciences

Organised by: Phillippe Trinh

Computation of the coefficients appearing in the uniform asymptotic expansions of integrals

Adri Olde Daalhuis[1†] & Sarah Farid Khwaja[2]

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14:00 on Tuesday in LTG

The coefficients that appear in uniform asymptotic expansions for integrals are typically very complicated. In the existing literature the majority of the work only give the first two coefficients. In a limited number of papers where more coefficients are given the evaluation of the coefficients near the coalescence points is normally highly numerically unstable. In this paper, we illustrate how well-known Cauchy type integral representations can be used to compute the coefficients in a very stable and efficient manner. We discuss the cases: (i) two coalescing saddles, (ii) two saddles coalesce with two branch points, (iii) a saddle point near an endpoint of the interval of integration. As a special case of (ii) we give a new uniform asymptotic expansion for Jacobi polynomials $P_n^{(\alpha,\beta)}(z)$ in terms of Laguerre polynomials $L_n^{(\alpha)}(x)$ as $n \to \infty$ that holds uniformly for z near 1. Several numerical illustrations are included.

Analysis of Carrier's problem

Jon Chapman[1†] & Patrick Farrell[1]

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14:40 on Tuesday in LTG

A computational and asymptotic analysis of the solutions of Carrier's problem is presented. The computations reveal a striking and beautiful bifurcation diagram, with an infinite sequence of alternating pitchfork and fold bifurcations as the bifurcation parameter tends to zero. The method of Kuzmak is then applied to construct asymptotic solutions to the problem. This asymptotic approach explains the bifurcation structure identified numerically, and its predictions of the bifurcation points are in excellent agreement with the numerical results. The analysis yields a novel and complete taxonomy of the solutions to the problem, and demonstrates that a claim of Bender & Orszag is incorrect.

An asymptotic method for long waves in a curved layer

C. John Chapman†

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15:20 on Tuesday in LTG

This talk presents a technique, based on a deferred approach to a limit, for analysing the dispersion relation for propagation of long waves in a curved waveguide. The technique leads to the introduction of a new concept, that of an analytically satisfactory pair of Bessel functions, which is strikingly different from the familiar concept of a numerically satisfactory pair, and is shown to be of use in simplifying the dispersion relations for curved waveguide problems. Details are presented for a non-trivial example, that of long elastic waves in a curved layer, for which symmetric and anti-symmetric waves are strongly coupled. The technique gives higher-order correction terms to dispersion relations based on kinematic hypotheses, and provides, for the first time, a rigorous determination of which of their coefficients are exact by comparison with the full equations.

Quantum theory information

Organised by: Davide Girolami

Generalized geometric quantum speed limits

Marco Cianciaruso[1†]
Diego Paiva Pires[2], Lucas C. Celeri[3], Gerardo Adesso[1] & Diogo O. Soares-Pinto[2]

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14:00 on Tuesday in LTH

In recent years there has been an intense theoretical and experimental research activity to understand, on one hand, a fundamental concept in quantum mechanics such as time, and to devise, on the other hand, efficient schemes for the implementation of quantum technologies. A basic question that combines and underpins both areas of research is: "How fast can a quantum system evolve in time?" Progress towards answering such a question has led to the establishment of quantum speed limits, intended as lower bounds setting the minimum time that a quantum system takes to undergo a given dynamics between an initial and a target quantum state. Establishing general and tight quantum speed limits is indeed crucial to assess how fast quantum technologies can ultimately be, and can accordingly guide in the design of more efficient protocols operating at or close to the ultimate bounds.

In this work we adopt a unifying and general information geometric framework to construct an infinite family of quantum speed limits valid for any dynamical evolution. We take advantage of the fact that in quantum theory there is not a unique bona fide measure of distinguishability on the state space, but rather an infinite family of so-called contractive Riemannian metrics that are all equally appropriate for this purpose. A different quantum speed limit arises from each of these metrics, in such a way that the tightest bound for a given dynamics is specified by the metric whose geodesic is best tailored to the given dynamical path. By resorting to this intuitive geometric criterion, we derive bounds that are tighter than any previously established one in some relevant instances (e.g., for open system evolutions), and demonstrate the optimality of previously proposed bounds in some other instances (e.g., for closed system evolutions).

Information theory and observable thermalisation

Fabio Anza[1†] & Vlatko Vedral[1]

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14:40 on Tuesday in LTH

In statistical mechanics, a point of crucial importance is the definition of thermal equilibrium, which is usually given exploiting Jaynes principle, via maximisation of von Neumann entropy. Arguing that such notion can never be experimentally probed in a many body quantum system, we propose a new way of describing thermal equilibrium, focused on observables rather than on the full state of the system and we study the properties that it heralds. Such notion is given via maximisation of the Shannon entropy of the observable. The relation with Gibbs ensembles is brought to light. Furthermore, we apply the results to study the emergence of thermal equilibrium in a closed quantum systems and prove that there is always a class of observables which exhibits thermal equilibrium properties and we give a recipe to explicitly construct them.

Characterizing the structure of multipartite correlations

Davide Girolami†
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15:20 on Tuesday in LTH

We provide an information-theoretic analysis of the correlation hierarchy in multipartite classical and quantum systems. We identify a class of measures satisfying desirable criteria for quantifiers of genuine multipartite correlations. We then introduce the concept of Weaving to discriminate equally correlated but differently built multipartite states. The Weaving of a state is given by the weighted sum of correlations at any degree, where the weights are monotones of the correlation degree. Measures of Weaving are reliable descriptors of the scaling pattern of multipartite correlations.

Recent progress in the mathematical theory of fluid dynamics

Organised by: Paolo Secchi and Marco Sammartino

The vanishing viscosity limit in porous media

Anna Mazzucato[1†] & Christopher LaCave[2]

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14:00 on Tuesday in LTJ

We consider the flow of a viscous, incompressible, Newtonian fluid in a perforated domain in the plane. We study the simultaneous limit of vanishing particle size and inter-particle distance, and vanishing viscosity. Under suitable conditions on the particle size, inter-particle distance, and viscosity, we prove that solutions of the Navier-Stokes system in the perforated domain converges to solution of the Euler system in the full plane. That is, the flow is not disturbed by the porous medium and becomes inviscid in the limit.

Error estimates and 2nd order corrections to approximate fluid models

Bin Cheng[†]

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14:40 on Tuesday in LTJ

In weather and climate studies/predictions, geophysical fluid dynamics (GFD) plays a central role across a wide range of temporal and spatial scales. Various constraints in multiscale simulation and observation make it necessary to enlist approximate fluid models which are typically "easier" to study and simulate and thus have long attracted the attention of theoretical and applied scientists alike. Notable examples include the incompressible approximation and quasi-geostrophic approximation. Part of this talk is proof-based analysis in getting sharp error estimates of some approximate models which essentially filters out the majority of fast waves. In this analysis, an important and difficult aspect is the physically relevant solid-wall boundary. Another part of this talk tries to establish connections to numerical analysis and geophysical studies. Approximate fluid models and their error estimates can make fundamental contribution to the development and refinement of next-generation weather/climate codes. These codes are essentially multiscale and ultimately aim at capturing GFD at regional scales, but such attempt is only meaningful if their performance at larger and longer scales are sufficiently close to the prediction of "easier" approximate models.

Singular behavior for regularized vortex sheet motion

Francesco Gargano[1†]
Russel E.Caflisch[2], Marco Sammartino[1] & Vincenzo Sciacca[1]

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15:20 on Tuesday in LTJ

We present in this talk a numerical analysis concerning the regularization of a vortex sheet motion governed by Birkhoff-Rott (BR) equation for a flow induced by an infinite array of planar vortex-sheets . The Euler-a regularization applied to the BR equation is considered, and its solutions is compared with the dynamics of the non regularized vortex sheet by means of the analysis of the complex singularities of the solutions through the singularity tracking method ([1]). We show that the regularized solution has several complex singularities that approach the real axis, and we relate their presence to the formation of high-curvature points in the vortex sheet during the roll-up phenomenon [2] . The motion of the sheet in the Euler-a regularization is shown to be compatible also with the the motion of a viscous layer of non-uniform vorticity governed by the Navier-Stokes equation in the zero viscosity limit. A numerical analysis of the complex singularities of the curve supporting the vortex layer is carried out and results are compared with those obtained from the same analysis for the Euler-a vortex sheet case.

[1] C. Sulem, P.L. Sulem, and H. Frisch, Tracing complex singularities with spectral methods. J. Comput. Phys. 50 (1), 138–161 (1980)

[2] Caflisch, R.E., Gargano, F., Sammartino, M. & Sciacca, V. 2016 Regularized Euler-a motion of an infinite array of vortex sheets. Boll. Unione Mat. Ital. pp. 1–29

Applied delay differential equations

Organised by: Yulia Kyrychko and Konstantin Blyuss

Time-delayed model of RNA interference

Konstantin Blyuss†

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14:00 on Tuesday in LTL

RNA interference is a fundamental cellular process responsible for a number of important functions, from immune defence to regulation of development and morphogenesis. In this talk I will discuss a mathematical model of RNA interference with a particular account for time delays associated with primed amplification. I will present analytical and numerical results of stability analysis for various states of the model and discuss their biological significance. I will demonstrate how time delays can affect bi-stability associated with the hysteresis loop in the system. Numerical results will illustrate various types of behaviour, including simultaneous co-existence of a stable steady state and a stable periodic orbit.

Dynamic motifs in networks of delay-coupled delay equations

Thilo Gross†

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14:20 on Tuesday in LTL

The dynamics of networks of interacting systems depends intricately on the interaction topology. When the dynamics is explored, generally the whole topology has to be considered. However, here we show that there are certain mesoscale subgraphs that have precise and distinct consequences for the system-level dynamics. In particular, if mesoscale symmetries are present then eigenvectors of the Jacobian localise on the symmetric subgraph and the corresponding eigenvalues become insensitive to the topology outside the subgraph. Hence, dynamical instabilities associated with these eigenvalues can be analysed without considering the topology of the embedding network. While such instabilities are thus generated entirely in small subgraphs, they generally do not remain confined to the subgraph once the instability sets in and thus have system-level consequences. Here we illustrate the analytical investigation of such instabilities in an ecological meta-population model consisting of a network of delay-coupled delay oscillators.

Synchronization in multiagent systems with reaction delays and multiplicative noise

Jan Haskovec†

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14:40 on Tuesday in LTL

We investigate a generalization of the Cucker–Smale model for collective behavior, formulated as a system of delayed stochastic differential equations. It incorporates two processes that are realistic but often neglected in modeling: (i) stochasticity (imperfections) of individual behavior and (ii) delayed responses of individuals to signals in their environment. By designing a suitable Lyapunov functional, we derive sufficient conditions for asymptotic velocity synchronization of the agents. As a by-product we obtain a new result regarding the asymptotic behavior of delayed geometric Brownian motion. The analytical results are complemented with systematic numerical simulations. They hint at a somehow surprising behavior of the system, which has yet to be understood - namely, that the introduction of an intermediate time delay may facilitate synchronization.

Dynamics of neural systems with discrete and distributed delays

Yulia Kyrychko†

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15:00 on Tuesday in LTL

In this talk I will present a neural system of coupled elements, where one sub-system receives a delayed input from another sub-system. This model includes a combination of both discrete and distributed delays in connections between its elements. I will discuss the stability properties for different commonly used distribution kernels, and will compare results to the corresponding findings on stability for such systems with no distributed delays.

Normal forms in delay-differential equations with state-dependent delays

Jan Sieber†

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15:20 on Tuesday in LTL

Recently, B. Wage, M. Bosschaert, S. Janssens and Y. Kusnetsov have proposed and implemented an efficient method to determine normal form coefficients for local codimension-one and -two bifurcations of equilibria in delay-differential equations (DDEs) with constant delays. Their approach generalised earlier efforts for ODEs (as implemented in MatCont) to DDEs. I will show that with minor modifications this approach can be generalized to DDEs with state-dependent delays (sd-DDEs). The method assumes the existence of a smooth local centre manifold for sd-DDEs, which is still an open problem. All proposed methods are implemented and publically available as part of DDE-Biftool.

Mathematical modelling of lineage commitment of human hematopoietic progenitors in early erythroid culture

Daniel Ward[1†]
Deborah Carter[1], Martin Homer[1], Lucia Marucci[1], Alexandra Gampel[1]
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15:40 on Tuesday in LTL

Erythropoietin is a hormone that is essential for the production of mature erythroid cells, promoting both proliferation and survival. Whether erythropoietin (and other cytokines) can influence lineage commitment of blood stem and differentiated cells is of significant interest. We have developed a mathematical model that accurately predicts population dynamics of erythroid culture, by applying delay differential equations to represent the transition of cells through their various states (i.e common myeloid progenitor and megakaryocyte/erythroid progenitors of peripheral blood cells). This method allowed us to capture the apparent delays between cell emergence in a specific state and the continuation of proliferative dynamics. Fitting the model on experimental data that we generated, we are able to show that the exclusion of this hormone does not affect rate of lineage restriction. By contrast, more differentiated cell proliferation appears to be sensitive to its inclusion. These results shed new light on the role of erythropoietin in erythropoiesis and provide a powerful tool for further study of hematopoietic progenitor lineage restriction and erythropoiesis.

LMS Scheme 3 minisymposium Quantitative evolutionary biology

Organised by: Rebecca Hoyle, Gianne Derks and Daniele Avitabile

Modelling evolution in structured populations using multiplayer games

Mark Broom[1†]
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14:00 on Tuesday in 22AA04

Within the last ten years, models of evolution have begun to incorporate structured populations, including spatial structure, through the modelling of evolutionary processes on graphs (evolutionary graph theory). One limitation of this otherwise quite general framework is that interactions are restricted to pairwise ones, through the edges connecting pairs of individuals. Yet many animal interactions can involve many individuals, and theoretical models also describe such multi-player interactions. We shall discuss a more general modelling framework of interactions of structured populations, including the example of competition between territorial animals. Depending upon the behaviour concerned, we can embed the results of different evolutionary games within our structure, as occurs for pairwise games such as the Prisoner's Dilemma or the Hawk-Dove game on graphs. For a population to evolve we also need an evolutionary dynamics, and we demonstrate a birth-death dynamics for our framework. Finally we discuss some examples together with some important differences between this approach and evolutionary graph theory.

Social influence increases cooperation in the public goods game on networks

James Allen[1†]

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14:40 on Tuesday in 22AA04

Social dilemmas such as climate change and volunteering within communities are important issues for our societies. Inspired by measurements of real world cooperation, we use the public goods game on a multiplex network to address questions around these social dilemmas. Our model represents people and their relationships in a social system as a two-layer network. The first layer models the rational, economic relationships between people, whilst the second represents their social connections. We model cooperation in this system through the public goods game: players in the game choose how much to donate to the rest of the population through a combination of how cooperative they are, and how much the rest of their group donates. Players then alter how cooperative they are through imitation of better performing neighbours on the economic layer, or influence from the social layer. Using both computational simulations and mean-field approaches we find that a small amount of social influence results in large frequencies of cooperation, even when the economic incentive to cooperate is very small, and that the initial distribution of strategies is the key factor in the final amount of cooperation. These findings help our understanding of real world cooperation.

Invariant manifolds of a model from population genetics

Belgin Seymenoglu[1†] & Stephen Baigent[1]
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15:20 on Tuesday in 22AA04

In 1976, Nagylaki and Crow proposed a continuous-time model for the population frequencies, which focuses on one gene with two variants (or alleles). Much of my time has been spent plotting phase plane diagrams for this model, but whatever values I put in for the parameters, I always find a stubborn special curve in my diagram - an invariant manifold. I proved that the manifold does indeed exist in the model for a certain case, and more recently relaxed the conditions so there is no need to assume the system is competitive. You can also look forward to a gallery of my colourful phase plane plots showing that the invariant manifold need not be unique, smooth or convex.

Structure in time and space

Organised by: Sofia Olhede and Russell Rodrigues

FRESH - An algorithm for resolution enhancement of piecewise smooth signals and images

Pier Luigi Dragotti†

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14:00 on Tuesday in 39AA04

We study the problem of enhancing the resolution of signals and images by exploiting structure and dependency of the wavelet coefficients across scales. Specifically, we model piecewise smooth signals as the sum of a piecewise polynomial signal, which is fully specified by a finite number of parameters, and a globally smooth term. The wavelet coefficients of the piecewise polynomial function at fine scales are estimated by using methods developed in sparse sampling theory and the resulting high resolution signal is then fused with the globally smooth part. We then apply this method along vertical, horizontal, and diagonal directions in an image to obtain a single-image super-resolution algorithm. Simulation results show that our method outperforms state-of-the-art algorithms under different blurring kernels.

Surfaces, shapes and anatomy

Adrian Bowman†

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14:20 on Tuesday in 39AA04

Three-dimensional surface imaging, through laser-scanning or stereo-photogrammetry, provides high-resolution data defining the surface shape of objects. In an anatomical setting this can provide invaluable quantitative information, for example on the success of surgery. Two particular applications are in the success of facial surgery and in developmental issues with associated facial shapes. An initial challenge is to extract suitable information from these images, to characterise the surface shape in an informative manner. Landmarks are traditionally used to good effect but these clearly do not adequately represent the very much richer information present in each digitised image. Curves with clear anatomical meaning provide a good compromise between informative representations of shape and simplicity of structure, as well as providing guiding information for full surface representations. Some of the issues involved in analysing data of this type will be discussed and illustrated. Modelling issues include the measurement of asymmetry and longitudinal patterns of growth.

Joint motion estimation and image reconstruction

Carola-Bibiane Schönlieb[1†]
Martin Burger[2] & Hendrik Dirks[2]

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14:40 on Tuesday in 39AA04

In this talk we will derive and analyze a variational model for the joint estimation of motion and reconstruction of image sequences, which is based on a time-continuous Eulerian motion model. The model can be set up in terms of the continuity equation or the brightness constancy equation. We rigorously prove the existence of a minimizer in a suitable function space setting. Moreover, we discuss the numerical solution of the model based on primal-dual algorithms and investigate several examples. Finally, the benefits of our model compared to existing techniques, such as sequential image reconstruction and motion estimation, will be shown.

Mean-field control hierarchy

Giacomo Albi[1,2†]
Young-Pil Choi[3], M. Fornasier[1] & D. Kalise[4]

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15:00 on Tuesday in 39AA04

Mean-field optimal control can be viewed as an useful tool to approximate the optimal control problem of a high-dimensional system of interacting agents. In this talk, I will briefly review some theoretical results about mean-field optimal control problems, and second I will focus on their numerical solution. In particular, I will introduce a novel approximating hierarchy based on a Boltzmann approach, whose solution requires a moderate computational effort compared with standard direct approaches. Different examples will show the effectiveness of the proposed strategies.

Smooth curve fitting to 3D shape data

Huiling Le[1†]
Kwang-Rae Kim[1] & Ian L Dryden[1]

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15:20 on Tuesday in 39AA04

To carry out the smoothing spline fitting method for shapes of configurations in a 3-dimensional Euclidean space using the technique of unrolling and unwrapping, we link parallel transport along a geodesic on Kendall shape space to the solution of a homogeneous first-order differential equation, some of whose coefficients are implicitly defined functions. This enables us to approximate the procedure of unrolling and unwrapping by simultaneously solving such equations numerically, and so to find numerical solutions for smoothing splines fitted to 3-dimensional shape data. We apply this fitting method to the analysis of some moving peptide data, and a test for the best model among those considered is given.

Extracting rapidly-varying oscillations

Sofia Olhede[1†]
Arthur Guillaumin[1], Adam Sykulski[2], Jonathan Lilly[3] & Jeffrey Early[3]

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15:40 on Tuesday in 39AA04

Traditional time series models can only encapsulate slow variation in the underlying generative mechanism. However, in many scenarios, this is not a realistic assumption. There seems to be an unavoidable conflict between how rapidly the structured part of the model can change, versus how much we need to average in order to retrieve parameters stably. We here introduce a new class of nonstationary time series, and show how efficient and rapid inference is still possible in this scenario, despite the generating mechanism changing quickly. The methods are illustrated on drifter time series, from the global drifter programme. Depending on the latitude of the observations, the underlying generative mechanism of the observed phenomenon is either slowly or rapidly changing, and we show how the newly introduced methodology can resolve both scenarios.

Tuesday PM: Contributed Talks

Density function theory

Dynamical density functional theory with a short range hydrodynamic interaction

Rory Mills†

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14:00 on Tuesday in LTA

Ever since Chan & Finken [1] established a rigorous time-dependent density functional theory of classical fluids, complex fluids have received theoretical study by mathematicians [2], [3], [4], [5] motivated by applications in engineering, physics, biology and chemistry. A complex fluid consists of a discrete collection of particles immersed in an essentially continuous fluid. They may be used to model organic molecules in plant cells, tobacco smoke, pigments in paints, the formation of clouds and gravity currents, amongst many systems. This talk focuses on Dynamical Density Functional Theory (DDFT) as a technique to model complex fluid flow, in particular with short range inter-particle forces turned on. Such interactions are crucial in capturing phenomena associated with the dynamics of differently-sized and shaped species of particles, the effects of confinement and systems with enforced background flow. For computational efficiency the lubrication forces are included in the friction term of the coarse grained equations arising from the Langevin dynamics. We will see that existing definitions of lubrication forces, such as those used in discrete methods for colloidal flow, can be incompatible with the present continuum method. Better particle interaction models at low Reynolds number are required, particularly those with outer solutions for the velocity, and these are presented. We will also present some preliminary numerical results.

[1] G.K.-L. Chan & R. Finken, PRL 94, 183001 (2005).

[2] B.D. Goddard, A. Nold, N. Sava, G.A. Pavliotis & S. Kalliadasis, PRL 109, 120603 (2012).

[3] A. J. Archer, J. Chem. Phys. 130, 014509 (2009).

[4] U. M. B. Marconi and P. Tarazona, J. Chem. Phys. 110, 8032 (1999).

[5] M. Rex and H. L. Löwen, Eur. Phys. J. E 28, 139 (2009).

Dynamic density functional theory for phase transitions: non-classical nucleation pathways in colloidal fluids

Miguel A. Duran-Olivencia[1†]
Peter Yatsyshin[1], James F. Lutsko[2] & Serafim Kalliadasis[1]

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14:20 on Tuesday in LTA

Classical density functional theory (DFT) for fluids and its dynamic extension (DDFT) provide an appealing mean-field framework for describing equilibrium and dynamics of complex soft matter systems. For a long time, homogeneous nucleation was considered to be outside the limits of applicability of DDFT. However, our recently developed mesoscopic nucleation theory (MeNT) based on fluctuating hydrodynamics, reconciles the inherent randomness of the nucleation process with the deterministic nature of DDFT. As a matetr of fact, it turns out that in the weak-noise limit, the most likely path (MLP) for nucleation to occur is determined by the DDFT equations. We present computations of MLPs for homogeneous and heterogeneous nucleation in colloidal suspensions. For homogeneous nucleation, the MLP obtained is in excellent agreement with the reduced order-parameter description of MeNT, which predicts a multistage nucleation pathway. For heterogeneous nucleation, the presence of impurities in the fluid affects the MLP, but remarkably, the overall qualitative picture of homogeneous nucleation persists. Finally, we highlight the use of DDFT as a simulation tool, which is especially appealing as there are no known applications of MeNT to heterogeneous nucleation.

Bifurcations of equilibrium metastable states of inhomogeneous classical fluids in contact with patterned substrates

Peter Yatsyshin[1†]
Miguel A. Duran-Olivencia[1], Andrew O. Parry[1], Carlos Rascon[2] & Serafim Kalliadasis[1]

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14:40 on Tuesday in LTA

Classical density-functional theory (DFT) is a rigorous and self-consistent theoretical-computational framework originating from the statistical mechanics of classical fluids, allowing to systematically account for the spatial inhomogeneity of confined fluids, as well as for the non-local character of intermolecular fluid-fluid and fluid-substrate interactions. Although the end-user DFT governing equations often look like phase-field models, DFT is "ab initio" in the sense that at its cornerstone is the one-body distribution function of an N-body system. And as most macroscopic quantities of interest (e.g., pressure, surface tension) can be cast as averages over the one-body distribution, DFT has proved to be a rather powerful tool for the numerical exploration of such quantities. Applications of classical DFT range from phase transitions in macro-molecule solutions and interfacial phenomena to colloid sedimentation and homogeneous nucleation. In this talk we will attempt to give a flavour of what simple equilibrium DFT models look like, and what physical phenomena they can capture. We will discuss at length the approximations underlying the governing DFT equations and ways to solve them on a machine. As an example, we will apply DFT to the computation of substrate-fluid interfaces at the nano-scale, when the substrate is heterogeneous. Recent studies have shown the highly non-trivial behaviour of such interfaces, including existence of multiple metastable equilibria and exciting new phase transitions. We will show how these can be studied numerically within a unified approach based on the bifurcation analysis of the underlying model equations. Our results may have ramifications for the design of lab-on-a-chip devices and controlled nanofluidics.

Fluctuations in solute transport in a spatially disordered environment

Matthew J Russell[1†]
Oliver E Jensen[1] & Tobias Galla[1]
[1]University of Manchester
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15:00 on Tuesday in LTA

Solute transport in physiological systems involves advection, diffusion and uptake in environments with complex structures which may be difficult to characterise in a given realisation. In addition, the transport process itself may be stochastic. We investigate a class of transport models which arise from an individual-based process in which non-interacting particles perform a biased random walk past a series of point sinks. We consider sinks that are either randomly distributed throughout the domain, or distributed periodically with randomly varying strengths, reflecting the uncertainty in the environment. We analyse the models using a combination of discrete and continuum techniques, including a stochastic homogenization approximation that predicts the spatial fluctuations in the transport and their effect on the mean concentration profile, averaged over intrinsic and extrinsic noise. In an ensemble of realisations, the disorder in the uptake introduces long-range spatial correlations into the concentration field. Additionally, for sinks of variable strength, the fully-averaged mean solute concentration is elevated compared to that arising from a spatially uniform sink distribution.

Dynamical density functional theory and hydrodynamic interactions in confined systems

Ben Goddard[1†]
Serafim Kalliadasis[2] & Andreas Nold[3]
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15:20 on Tuesday in LTA

Colloidal systems consist of nano- to micrometer-sized particles suspended in a bath of many more, much smaller and much lighter particles. Motion of the colloidal particles through the bath, e.g. when driven by external forces such as gravity, induces flows in the bath. These flows in turn impart forces on the colloid particles. These bath-mediated forces, known as Hydrodynamic Interactions (HIs) strongly influence the dynamics of the colloid particles. This is particularly true in confined systems, in which the presence of walls substantially modifies the HIs compared to unbounded geometries. For many-particle systems, the number of degrees of freedom prohibits a direct solution of the underlying stochastic equations and a reduced model is necessary. We employ elements from the statistical mechanics of classical fluids, namely Dynamical Density Functional Theory (DDFT) [1,2], the computational complexity of which is independent of the number of particles, to include both inter-particle and particle-wall HI and demonstrate the physical importance of using the correct description of HIs in confined systems. In addition, DDFT allows us to isolate and investigate different components of HIs [3].

[1] Phys. Rev. Lett. 109, 120603 (2012) [2] J. Phys.: Condens. Matter 25, 035101 (2013) [3] J. Chem. Phys. 145, 214106 (2016);

Elasto-dynamics

Unidirectional localisation and control of waves in chiral lattices: the DASER effect

Giorgio Carta[1 †]
Ian S. Jones[1], Natasha V. Movchan[2], Alexander B. Movchan[2] & Michael J. Nieves[1]

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14:00 on Tuesday in 24AA04

We present a novel design of a chiral elastic system [1], where waves generated by an external source can be channelled along a single direction without a significant loss in the wave amplitude. The system is an elastic lattice, consisting of a triangular array of point masses linked by non-inertial springs, which is connected to a non-uniform system of gyroscopic spinners. The intrinsic preferential directionality of the spinners confers chirality properties to the medium. First, we determine the analytical expression of the dispersion relation for the chiral lattice, and we discuss the changes in the dispersion properties and the polarisation effect induced by the spinners. Then, we prove that at specific frequencies, which can be varied by modifying the spinning rates of the spinners, the slowness contours are parallel straight lines. Accordingly, waves propagate in one direction, which depends on the geometry of the medium. By means of numerical simulations in the frequency domain, we demonstrate that an external excitation applied to any point of the lattice creates a very localised wave pattern, characterised by a small reduction in the wave amplitude along its path. Furthermore, the unidirectional wave can be diverted by changing the arrangement of the spinners within the medium. Thus, a wave can be sent from one to any other point of the lattice plane, leaving the remaining part of the medium undisturbed. In addition, resonant effects leading to wave amplification can be obtained by creating a closed wave pattern inside the lattice. This phenomenon has been named DASER ("Dynamic Amplification by means of Spinners in an Elastic Reticulated system"). We envisage that the proposed chiral system may have a huge impact in physics and engineering applications, related to the directional control of elastic waves.

[1] Carta G., Jones I.S., Movchan N.V., Movchan A.B., Nieves M.J., 2017. "Deflecting elastic prism" and unidirectional localisation for waves in chiral elastic systems. Scientific Reports (doi: 10.1038/s41598-017-00054-6)

Asymptotic modelling of the Rayleigh wave field caused by a regular array of surface resonators

Peter Wootton[1 †]
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14:20 on Tuesday in 24AA04

A robust approach suppressing the effect of surface Rayleigh Waves by using a regular array of surface resonators was recently suggested. This consideration was based on exact formulation in 3D linear elasticity. In this talk we revisit the aforementioned problem but, unlike previous approaches, utilise a specialised asymptotic model for the Rayleigh wave which involves a scalar wave equation along the surface, with the wave speed coinciding with the Rayleigh wave. We derive a simple explicit dispersion relation for the surface wave affected by the resonators. Numerical comparisons with the exact solution for the problem justify the developed approximate procedure.

On the coupling of shear and pressure waves in a triangular lattice elastic lattice containing tilted resonators

Domenico Tallarico[1†]
Natalia V. Movchan[1], Alexander B. Movchan[1] & Daniel J. Colquitt[1]

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14:40 on Tuesday in 24AA04

Coupling of shear and pressure waves in a structured lattice is investigated. Specifically, we consider a vibrating triangular lattice whose unit cell contains a rigid resonator of triangular shape . The resonator is linked to the triangular lattice's nodal points via trusses and it is "tilted", i.e. rigidly rotated with respect to the triangular lattice's unit cell through a non-zero angle. We demonstrate that at zero tilting angle the structured lattices is statically undetermined. At non-zero tilting angle we study the dispersion surfaces for the elastic Bloch waves in the structured lattice, with special attention to the coupling of rotational and shear Bloch modes. The effective properties of the dispersion surfaces provide invaluable information for the design of structured interfaces capable to focus, localise and guide waves.

Over-damped elastic 'snap-through'

Michael Gomez[1†]
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15:00 on Tuesday in 24AA04

Elastic 'snap-through' occurs when a system is in an equilibrium state that either disappears or becomes unstable as a control parameter varies. The switch from one state to another is generally rapid and hence is used to generate fast motions in biology and engineering. While the conditions under which simple elastic objects undergo snap-through have been reasonably well studied, how fast snapping happens is much less well understood. Recently, it has been shown that snap-through can be subject to critical slowing down near the snapping transition, so that the dynamics may be slow even in the absence of viscous damping. Here, we study the interaction of snap-through with the flow of a viscous fluid. We begin by showing how snap-through may be used to create a channel whose hydraulic conductivity changes discontinuously in response to fluid flow. We then study the dynamics of snap-through for an elastic element embedded in a viscous fluid, showing that the fluid damping modifies the slowing down effect that is observed.

An efficient semi-analytical scheme for determining the reflection of Lamb waves in a semi-infinite waveguide

Robert Davey[1†]
Raphaël Assier[1], David L. Abrahams[2] & Rich Hewitt[1]

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15:20 on Tuesday in 24AA04

The reflection of Lamb waves from a free edge perpendicular to an elastodynamic plate is studied. It is known that extant methods for finding the reflected field have poor convergence due to irregular behaviour near corners. The form of the irregularity for an elastodynamic corner is derived asymptotically. A new method for incorporating this form of the corner behaviour is then implemented. Results are presented showing this new method improves convergence in the reflection problem.

Macroscopic dynamic models for periodically heterogeneous structures

Vladyslav Danishevskyy[1 †]
Julius Kaplunov[1] & Graham Rogerson[1]

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15:40 on Tuesday in 24AA04

Propagation of elastic waves in periodically heterogeneous structures is studied. As illustrative examples, a simple monatomic lattice and a piecewise continuous string are considered. Using the Floquet-Bloch approach, asymptotic expansions of the dispersion curves in the vicinity of band gaps are obtained. Matching these expansions by two-point Pade approximants we derive a macroscopic equation of motion, which is valid in the entire frequency range for the both short and long-wave cases. The developed model includes three dispersive terms and may be considered as a generalisation of double-dispersive equations that are well known in micropolar theories. It should be emphasised that unlike many phenomenological approaches, the proposed method allows one to determine coefficients of the macroscopic equation on a rigorous theoretical basis incorporating information about the internal structure of the medium and its properties.

Using the two-scale asymptotic procedure, we show that the obtained macroscopic model can describe asymptotically propagation of different types of modes, namely, locally periodic and locally anti-periodic ones.

The developed model provides not only a precise approximation of the dispersion curves, but is also applicable for the solution of dynamic boundary value problems. As an illustrative example, we study a dynamic response of a semi-infinite heterogeneous structure to a pulse load. The analytical solution is evaluated with the help of Laplace transform. The obtained results demonstrate a very good accuracy comparing with the data of a direct numerical simulation.

This work has received funding from the European Union's Horizon 2020 research and innovation programme under Marie Sklodowska-Curie grant agreement no. 655177.

Wednesday AM: Mini-Symposia

Symmetries and conservation laws

Organised by: Peter Hydon and Elizabeth Mansfield

Discretising the action: from variational integrators to singular solutions of nonlinear PDEs

Colin Cotter†

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09:00 on Wednesday in LTB

Variational integrators are a central tool in the construction of structure-preserving numerical methods. Instead of discretising the equations directly, we start from the principle of least action. By applying spatial and/or temporal discretisations to the action we obtain variational integrators. These integrators inherit many properties from the original system: conservation of (modified) energy, and conserved momenta associated with symmetries of the action. I will describe a particular type of least action principle, the "Clebsch optimal control principle" (closely related to the Hamilton-Pontryagin maximum principle), which can be used to derive the equations of a rotating rigid body, the incompressible Euler equations of fluid dynamics, geodesics on the diffeomorphism group (used in medical image registration), and many other systems. I will discuss how to obtain variational time discretisations from this framework, and their properties. Then I will show how Clebsch variational principles can be used to obtain finite dimensional subsets of solutions of infinite dimensional PDEs, such as the jetlet solutions of the regularised Euler equations. Finally I will discuss some recent work applying this idea to obtain weak piecewise-linear solutions of the p-Hunter-Saxton equation, which has applications in image registration.

Stochastic blow up

John King[1]† & Robert Jenkins[2]

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09:40 on Wednesday in LTB

Blow up in a simple stochastic model will be characterised, with a focus on self-similar behaviour.

Transformation groups and discrete structures in continuum description of defective crystals

Maxim Zyskin[1†] & Gareth Parry[1]

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10:00 on Wednesday in LTB

Davini description of elasticity and plasticity of defective crystal involves a frame of continuum 'lattice vector' fields (lvf), and dislocation density matrix, capturing the structure constants of the Lie bracket of those vector fields. Those fields together describe kinematics of a defective crystal, allowing for elastic and certain plastic deformations, preserving certain invariants A truncation assumption for the energy functional leads to consider finite dimensional Lie algebras of lvf and corresponding transformation groups. In low spatial dimensions, such groups may be classified. Discrete crystal structures emerge in such context as discrete subgroups of the corresponding Lie groups. This approach includes the usual crystal lattices as a particular case. I will describe explicit form of lvf and discrete structures for defective crystals with non-constant dislocation density in nilpotent (see our paper http://rdcu.be/nD07), solvable, and simple cases, corresponding to 2d crystals with 3d algebras of lattice vector fields.

Integrable nematic liquid crystals

Francesco Giglio[1†]
Giovanni De Matteis[2] & Antonio Moro[1]

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10:20 on Wednesday in LTB

Inspired by recent formulation of Thermodynamics via the theory of nonlinear conservation laws, we propose a novel approach to the Statistical Mechanics of Liquid Crystals (LCs) based on the analysis of differential identities for the partition function. In particular, we study the discrete Maier-Saupe model for nematic liquid crystals interacting with an external field. We show that the partition function, as a function of thermodynamic variables, solves the 2D Heat equation, where the heat conductivity is identified with the inverse of the number of molecules.

The thermodynamic limit, i.e. the limit for which the number of molecules tends to infinity, has been studied in the case of uniaxial molecules. We show that the Gibbs free energy satisfies a Hamilton-Jacobi type equation whose solution provides the equations of state of the model. We obtain the first exact equations of state for a model of nematic liquid crystal in presence of external fields.

Recent progress in the mathematical theory of fluid dynamics

Organised by: Paolo Secchi and Marco Sammartino

Explicit estimates on the torus for the sup-norm and the problem of the "crest factor" of solutions of the two-dimensional Navier-Stokes equations

Michele Bartuccelli†

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09:00 on Wednesday in LTJ

In this talk we have obtained explicit and accurate estimates of the sup-norm for solutions of the Navier-Stokes Equations (NSE) in two space dimensions. By using the best (so far) available estimates of the embedding constants which appear in the classical functional interpolation inequalities used in the study of solutions of dissipative partial differential equations, we have evaluated in an explicit manner the values of the sup-norm of the solutions of the NSE. In addition we have computed the so-called "crest factor" associated to the above solutions.

Nonlinear stability of relativistic vortex sheets in two spatial dimensions

Tao Wang[1]†
Gui-Qiang Chen[2] & Paolo Secchi[3]

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09:40 on Wednesday in LTJ

We study vortex sheets for the relativistic Euler equations in three-dimensional Minkowski spacetime. The problem is a nonlinear hyperbolic problem with a characteristic free boundary. The so-called Lopatinskiĭ condition holds only in a weak sense, which yields losses of derivatives. A necessary condition for the weak stability is obtained by analyzing roots of the Lopatinskiiĭ determinant associated to the linearized problem. Under such stability condition, we prove short-time existence and nonlinear stability of relativistic vortex sheets by the Nash-Moser iterative scheme.

Vortex layers of small thickness

Marco Sammartino[1†]
Russ Caflisch[2] & Maria Carmela Lombardo[1]

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10:20 on Wednesday in LTJ

In this talk we shall consider a 2D incompressible non viscous flow with an initial datum with vorticity concentrated close to a curve $y = \phi_0(x)$ and exponentially decaying away from it. We shall suppose the vorticity intensity to be $O(\epsilon^{-1})$ while the exponential decay occurs on a scale $O(\epsilon)$. We shall prove that, if the initial data are analytic, the solution of the above problem will preserve the vortex layer structure for a time that does not depend on ϵ. Moreover the dynamics of the layer is well approximated by the motion predicted by the Birkhoff-Rott equation for a vortex sheet of equivalent vorticity intensity.

Mathematical modelling of crime

Organised by: Andrew Lacey and Toby Davies

Self-exciting point processes and their applications to crime data

Craig Gilmour[1†] & Des Higham[1]
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09:00 on Wednesday in LTL

The question of how crime spreads is an important issue for police and society alike, with some crime types displaying highly clustered sequences in time and space. It has been proposed that certain types of crime, such as burglary, gang crime and gun violence, take place in highly clustered event sequences, and therefore can be modelled in much the same way that seismic events are, where earthquakes increase the risk of aftershocks occurring in close proximity to the original earthquake. Self-exciting point processes can be used in situations where the occurrence of an event increases the chances of a subsequent event taking place. We propose and analyse a new triggering function. We look at issues of calibration, inference and prediction, and give results on real crime data made publicly available by the Chicago Police Department.

Sequential data assimilation for urban crime models

Naratip Santitissadeekorn[1†]
David Lloyd[1] & Martin Short[2]
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[2]Georgia Institute of Technology
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09:20 on Wednesday in LTL

This talk will present an ensemble methodology to carry out sequential Bayesian inference for the parameters and crime rate of an urban crime model where the crime occurrences are modelled as a history-dependent Poisson process. This type of process can be used to approximate the Hawkes process where the crime rate is evolved based on event-triggered jumps in intensity.

Geographic profiling in biology

Sally Faulkner[†]
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09:40 on Wednesday in LTL

Geographic profiling is a statistical technique originally developed in criminology to prioritise large lists of suspects in cases of serial crime. It has been extremely successful in this field, and is routinely used by organisations including the Metropolitan Police, FBI and the United States Marine Corps. In this talk I will describe how geographic profiling has been applied to biological data and discuss how different mathematical models compare. I will explain in detail the Bayesian Dirichlet Process Mixture (DPM) model of GP recently introduced by our group and go on to discuss how the model can be extended to incorporate other information (for example, landscape data).

A data-driven approach to modelling serious and minor crime

Michael Tsardakas[1†] & Andrew Lacey[1]
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10:00 on Wednesday in LTL

In this talk we will examine data from serious and minor criminal activities in urban and rural areas of England. Results of exploratory data analyses will be presented, with a focus on extracting information that can be used in constructing models of crime involving ODEs and PDEs. We will also look at the data from time series point of view and compare time series forecasts with ODE models.

The concentration of victimisation and criminality

Rafael Prieto Curiel[1†]
Sofía Collignon Delmar[1] & Steven Bishop[1]
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10:20 on Wednesday in LTL

Generally speaking, crime is a rare event, which means that traditional measures of the concentration of crime in a population are not appropriate because they are not comparable across time and populations.

An estimate of the distribution of crime suffered (or committed) by a population based on a mixture model will be presented, which allows new and standardised measurement of the concentration of the rates of suffering a crime based on that distribution.

The new measure is applied to the case of robbery of a person in Mexico to show that the risk of suffering a crime is not uniformly distributed across the population and there are certain population groups which are statistically immune to suffering crime but there are also groups which suffer chronic victimisation.

The resulting new measurement is comparable between different regions and can be tracked over different time periods, allowing us to determine if a policy results in victim displacement rather than crime reduction.

Where next for the mathematical modelling of crime?

Toby Davies†
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10:40 on Wednesday in LTL

The mathematical modelling of crime has experienced a surge of interest in recent years, inspired by an increasing awareness that the mechanisms which give rise to crime can be modelled by analogy with a number of social and physical systems. Significant progress has been made in this period, particularly in developing an array of approaches for modelling the spatial and temporal properties of crime, and the potential value of such research is illustrated by its role in real-world ‘predictive policing’ systems. Furthermore, diverse areas of mathematics, including network theory and control theory, have been applied in both analysing and seeking to prevent crime. However, current technological developments, and the changing nature of crime more generally, present a number of opportunities and challenges for ongoing research in this area. In this talk, I will first review the state of the art with respect to the mathematical modelling of crime, and give an overview of the approaches which have been used to date. I will then identify a number of trends in real-world offending, and outline a number of future threats which will require new approaches but which may allow mathematics to take a more prominent role in crime prevention. The talk will aim to identify new areas for research and suggest topics of current interest for both police forces and policymakers.

Modelling epidemics in human and plant populations

Organised by: Marianna Cerasuolo

Generalization of pairwise models to non-Markovian epidemics on networks

Istvan Kiss[1†]
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09:00 on Wednesday in 22AA04

In this talk, a generalization of pairwise models to non-Markovian epidemics on networks is presented. For the case of infectious periods of fixed length, the resulting pairwise model is a system of delay differential equations, which shows excellent agreement with results based on stochastic simulations. Furthermore, we analytically compute a new R_0-like threshold quantity and an analytical relation between this and the final epidemic size. Additionally, we show that the pairwise model and the analytic results can be generalized to an arbitrary distribution of the infectious times, using integro-differential equations, and this leads to a general expression for the final epidemic size. By showing the rigorous link between non-Markovian dynamics and pairwise delay differential equations, we provide the framework for a more systematic understanding of non-Markovian dynamics.

Household modelling of Yaws data indicates that case finding and contact tracing may be unsuccessful at disease eradication

Louise Dyson[1†]
Michael Marks[2], Oliver M. Crook[1], Oliver Sokana[3], Anthony W. Solomon[2], Alex Bishop[1], David C. W. Mabey[2] & Deirdre Hollingsworth[1]
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09:20 on Wednesday in 22AA04

Yaws is a painful and disabling infectious disease that causes skin and bone lesions. It is most commonly seen in children in warm, humid tropical areas. Individuals with the disease can be effectively treated with antibiotics. Yaws is known to be clustered, even within endemic regions, but the spatial epidemiology remains poorly understood. Previous eradication campaigns used either mass drug treatment regardless of disease status (MDA) or, when the disease was less prevalent, more targeted treatment of cases and their contacts.

As many individuals are latently infected without clinical evidence of disease, this often resulted in failure to adequately treat latently infected individuals, resulting in subsequent rebound in disease incidence and ultimately failure of the eradication programme. The new WHO strategy mandates an initial round of MDA followed either by further MDA or active case finding and treatment of cases and their contacts.

We analyse data from the Solomon Islands using a household model to investigate whether targeting cases and their household contacts would be sufficient to interrupt transmission. Our model and data indicate that treating only active cases and their household contacts would leave between 70% and 100% of latent infections untreated.

Mathematical modelling of the transmission dynamics of opisthorchis viverrini

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09:40 on Wednesday in 22AA04

The trematode liver fluke, Opisthorchis viverrini, which causes the chronic disease, opisthorchiasis, is prevalent is southeast Asia. We develop ordinary (ODE) and paratial differential equation (PDE) models of the transmission dynamics of O. viverrini through its life cycle in snails, fish, humans, and reservoir hosts such as domestic cats and dogs. We calibrate these models to data collected from two communities in Khong Island in Southern Lao PDR. We determine equilibrium points and threshold conditions, including the basic reproductive number and the type-reproductive number for the different mammalial hosts, for the ODE models; and show the existence of steady state solutions for the PDE models. We conduct sensitivity analysis of and numerically simulate all models. Our results show that interventions such as treatment of humans and improved sanitation (reducing transmission from humans to snails) are most effective in reducing transmission potential and the mean burden of worms in humans. Crucially humans are likely to be the only maintenance host so interventions targeting humans can eliminate opisthorchiasis transmission.

Modelling latent tuberculosis infection

Lara Gosce[†]

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10:00 on Wednesday in 22AA04

Tuberculosis (TB) is an old disease, afflicting humankind for centuries and, to this day, is still one of the top 10 causes of death worldwide. Even if TB incidence has been steadily falling since 2001, it is not enough to reach the 2020 milestones of the "End TB Strategy" and different approaches needs to be taken. It is known that TB is caused by bacteria Mycobacterium tuberculosis and that, after infection, individuals enter a latency state that could last from a few months to a lifetime and never develop in the active state of the disease. One possible strategy in the control and, eventually, eradication of TB is treatment of what is called latent tuberculosis infection (LTBI), however this is usually offered only to targeted population (HIV co-infected patients, direct contacts of known active cases) and little is known on the biological effects that this treatment has on the patient regarding the possibility of future re-infections. After presenting a simple model describing the spread of TB in the general population, we are going to modify it and apply it to different scenarios analysing the connection between LTBI, treatment, vaccination and re-infection. By introducing and discussing several biological assumptions at the base of LTBI treatment we are going to evaluate different strategies in order to lower the disease incidence in low and high income countries.

Plant virus transmission pathways: the evolution of virulence and mutualism

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10:20 on Wednesday in 22AA04

The main transmission pathways for plant viruses are through vectors and pollen/seed. We use a semi-discrete model to investigate whether evolution leads to the coexistence of multiple virus transmission pathways. An ecological stability analysis is used to inform an evolutionary study of trade-offs between pollen/seed versus vector transmission. Mixing vector and pollen/seed transmission may be evolutionarily stable. Non-vector-borne and vector-borne variants may evolve from a single ancestral strain and coexist in the long run. Virus-plant interactions may also range from parasitism with reduced fecundity of infected plants (a measure of virulence) to mutualism, where increased fecundity of infected plants benefits both the plant and the virus. Two trade-offs are considered, vertical seed transmission vs infected plant fecundity, and horizontal vector transmission vs infected plant fecundity. We show that a trade-off between virulence and vertical transmission can lead to virus extinction, that evolutionary branching can occur with subsequent coexistence of mutualistic and parasitic virus strains, and that mutualism can out-compete parasitism in the long-run.

How a competing endemic pathogen strain affects the dynamics and control of an invading strain in a vegetatively propagated crop

Ryan T. Sharp[1†]
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10:40 on Wednesday in 22AA04

We evaluate methods of controlling both the spread of invading pathogen strains, and disease incidence, in pathosystems with and without an endemic pathogen strain present. The model is motivated by the cassava mosaic virus complex, which has caused serious problems in east Africa. When conventional forms of control are applied to a crop infected by both an invading and a less fit endemic pathogen strain, the incidence of both strains is reduced, because the control is nonspecific. This generates a larger proportion of healthy hosts. However, as the invasive strain is fitter than the endemic, the invasive strain can take advantage of the increase. The result is a decrease to total incidence, but an increase in the proportion of the invasive strain. This can consequently result in an increase in the speed with which the invasive pathogen spreads. Cassava is a vegetatively propagated crop and therefore when trade occurs, it is cassava cuttings that are traded. These cuttings may or may not already be infected. The only forms of control found in this study that effectively reduce the speed of spread when an endemic strain is present are those that reduce trade. We find that the spread of the invading strain is chiefly governed by the incidence at the wave front. Therefore stepping up conventional control once the wave front has passed proves to be the most effective way of both slowing down spread and controlling incidence.

Wednesday AM: Contributed Talks

Free-surface and interfacial flows

Coupled sloshing in a 2-vessel system

Ying Huang[1†]
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09:00 on Wednesday in LTA

A Tuned Liquid Damper (TLD) is a damping device used to mitigate wind and earthquake induced vibrations in tall buildings and other large structures. It consists of a vessel containing a fluid which is attached to a solid wall through a spring. There have been many studies on this problem both numerically and experimentally in both the linear and nonlinear regimes.

In this presentation we present natural frequency analysis for a model of two TLDs joined by an additional spring. The middle spring plays an important role on the dynamics of the 2-vessel system essentially coupling the modes of the two systems together. Results are presented for the natural frequencies of the system when the TLD setup is symmetric, leading to in-phase and out-of-phase modal solutions, and when the system symmetry is broken. In the latter case the modal configuration becomes more complicated.

Decay of solitary waves, a comparison between asymptotic and numerical results

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09:20 on Wednesday in LTA

The KdV equation is known for having solitary wave solutions. This presentation will consider a single solitary wave affected by small dissipation. Such a scenario sees terms accounting for dissipation added to the KdV equation, with a solitary wave solution as an initial condition. Our model will focus on two cases. The first considers a perturbed solitary wave solution to the KdV equation with a small second order derivative accounting for dissipation; this is consistent with the KdV-Burgers equation. The second case investigates a single solitary wave affected by the presence of a viscous boundary layer at the bottom boundary of a body of fluid. In both instances we give a detailed examination of how the amplitude and structure of the solitary waves are affected, as well as discussing some key numerical results. An overview of the pseudo-spectral scheme used in obtaining the results including an integrating factor manipulation will also be discussed, with comparisons to asymptotic approximations.

Free-surface waves on a vibrating fluid layer

Konstantin Ilin†
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09:40 on Wednesday in LTA

We study waves on the free surface of a layer of an inviscid fluid which is subject to vertical high-frequency vibrations. Three asymptotic systems of nonlinear equations that describe slowly evolving (in comparison with the vibration frequency) free-surface waves are derived. The first set of equations is obtained without assuming that the waves are long. These equations are as difficult to solve as the exact equations for irrotational water waves in a non-vibrating fluid. The other two models describe long waves. These models are obtained under two different assumptions about the amplitude of the vibration. Surprisingly, the governing equations have exactly the same form in both cases (up to interpretation of some constants). These equations reduce to the standard dispersionless shallow-water equations if the vibration is absent, and the vibration results in an additional term which is similar (although not exactly the same) to a standard surface tension term in a non-vibrating fluid. We show that the dispersive shallow water equations have both solitary and periodic travelling waves solutions and discuss an analogy between these solutions and travelling capillary-gravity waves in a non-vibrating fluid. Some weakly-nonlinear models are also considered.

Free surface flows in electrohydrodynamics with constant vorticity

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10:00 on Wednesday in LTA

The problem of free surfaces in electrohydrodynamics have been studied my many authors over the years. When studying inviscid fluids the assumption of incompressibility and irrotationality has always been assumed. In this talk we remove the assumption of irrotationality and investigate the consequences. We find that with the addition of either positive or negative vorticity, linear waves are always possible and that vorticity has a large effect on the amplitude of the wave. A weakly nonlinear model is then derived obtaining a Benjamin equation of the form:

$$(1+c^2)\left(\frac{1}{c_0}\frac{\partial u}{\partial t}+c\frac{\partial u}{\partial x}\right)+\frac{2c+c^3-\Omega}{h}u\frac{\partial u}{\partial x}+h^2c\left[\frac{c^2}{3}-B\right]\frac{\partial^3 u}{\partial x^3}-chE_b\mathscr{H}\left(\frac{\partial^2 u}{\partial x^2}\right)+\frac{1}{\rho g}\frac{\partial p}{\partial x}=0 \qquad (1)$$

Solutions to this equations are presented as well as how the vorticity affects the amplitude. Possible generalisations are then suggested for more realistic vorticity profiles.

Multiphase flows in confinement with complex geometries

Benjamin Aymard[1†]
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10:20 on Wednesday in LTA

Understanding the dynamics of immiscible fluids in confinement is crucial in numerous applications such as oil recovery, fuel cells and the rapidly growing field of microfluidics. Complexities such as microstructures, chemical-topographical heterogeneities or porous membranes, can often induce non-trivial effects such as critical phenomena and phase transitions. The dynamics of confined multiphase flows may be efficiently described using diffuse-interface theory, leading to the Cahn-Hilliard-Navier-Stokes (CHNS) equations with Cahn wetting boundary conditions. Here we outline an efficient numerical method to solve the CHNS equations using advanced geometry-capturing mesh techniques both in two and three dimensional scenarios. The methodology is applied to two different systems: a droplet on a spatially chemical-topographical heterogeneous substrate and a microfluidic separator.

Time-dependent conformal mappings with applications to nonlinear sloshing problems

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10:40 on Wednesday in LTA

In this talk we examine two key features of time-dependent conformal mappings in doubly-connected regions, the evolution of the conformal modulus, Q(t), and the boundary transformation generalizing the Hilbert transform. Results of this theory are applied to inviscid, incompressible, irrotational fluid sloshing in a rectangular vessel. It is shown that the explicit calculation of the conformal modulus is essential to correctly predict features of the flow, such as the free surface evolution.

We also present results for fully dynamic simulations which use a time-dependent conformal mapping and the Garrick generalization of the Hilbert transform to map the physical domain to a time-dependent rectangle in the computational domain. The results of this new approach are compared to the complementary numerical scheme of Frandsen (2004) and it is shown that correct calculation of the conformal modulus is essential in order to obtain agreement between the two methods.

Pipe and channel flows

Unsteady flow of thixotropic fluid in a slowly varying pipe

Andrew I. Croudace[1†]
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09:00 on Wednesday in LTD

We use lubrication theory to develop the governing equations for unsteady axisymmetric flow of thixotropic or antithixotropic fluid along a circular pipe of slowly varying radius. The strength of thixotropy is described by advective and temporal Deborah numbers, which are the ratios of the structure response timescale to, respectively, the timescales of advection and the applied pressure gradient. In the regimes of 'weak thixotropy', in which the Deborah numbers are comparable to the small aspect ratio of the pipe, we follow the expansion method proposed by Pritchard et al. (2016) to obtain the effects of thixotropy and antithixotropy as perturbations to a generalised Newtonian flow. We present illustrative results for the Moore–Mewis–Wagner rheology. Pritchard et al. (2016) showed that in a widening channel (or, by analogy, a decelerating flow), thixotropy increases the fluid velocity near the centre of the pipe and decreases it near the wall, while antithixotropy has the opposite effect. These results are generic for steady pipe flow. However, unsteady pipe flow of an antithixotropic fluid is more complicated, and the shape of the velocity profile depends on some subtle dynamical balances. We show that at least for some rheological models, the velocity perturbation for a strongly antithixotropic fluid mimics that for a thixotropic fluid. This casts doubt on the extent to which generic statements can be made about the effects of antithixotropy on lubrication flow. Initial numerical simulations using COMSOL are in agreement with the analytical solutions for steady flow, and show that our lubrication approach is qualitatively accurate for flows in a range of aspect ratios.

Shear layers in channel flow

Graham Benham[1†]
Ian Hewitt[1], Colin Please[1], Alfonso Castrejon-Pita[1], Rob Style[2] & Paul Bird[3]
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09:20 on Wednesday in LTD

We study a novel type of hydropower generation which uses the Venturi effect to amplify the pressure drop across a turbine. Efficiency depends on how the turbine wake mixes together with the main channel flow in a shear layer. We present a reduced mathematical model consisting of a set of differential algebraic equations which predicts the relationship between the channel shape and the growth of shear layers and consequent pressure recovery. A PIV experiment is used to validate the model in the two cases of a straight and a widening rectangular channel. We set up a DAE-constrained optimisation problem which can find optimal channel shapes.

The energetics of flow in flexible channel

Danyang Wang†
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09:40 on Wednesday in LTD

There are many examples of fluid-conveying vessels in the human body. When these vessels are subject to a negative transmural pressure (internal minus external) while conveying a flow, self-excited oscillations can occur. We explore the generation of self-excited oscillations in a two dimensional channel that consists of rigid and flexible segments conveying a laminar high-Reynolds number flow. In particular, we construct a general framework for analysing the energy budget of self-excited oscillations about a non-uniform basic state. We then apply this general framework to consider two particular models for the flexible wall, namely a simple membrane model with an external pressure gradient [1] and fluid-beam model [2].

[1] Stewart P S, Waters S L, Jensen O E. Local instabilities of flow in a flexible channel: asymmetric flutter driven by a weak critical layer. Physics of Fluids, 2010, 22(3): 031902.

[2] Cai Z X, Luo X Y. A fluid–beam model for flow in a collapsible channel. Journal of Fluids and Structures, 2003, 17(1): 125-146.

Oscillation mark formation in steel casting

Kevin Devine[1]†
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10:00 on Wednesday in LTD

Continuous casting has been developed industrially worldwide since the 1950s as a high-throughput method for producing, amongst other things, metal billets, blooms and slabs; more than 90% of the world's steel is produced this way, amounting globally to more than one billion tonnes of steel cast per year. During casting, liquid steel is poured into the top of a water-cooled copper mould, where intense cooling causes a solidified steel shell to form. To prevent the steel sticking to the mould wall, a flux powder is added to the surface of the steel and the mould is oscillated at high frequency; the process is further complicated by low frequency phenomena associated with the turbulent flow of molten steel and the meniscus level fluctuations. All of these combine to produce undesired imperfections on the steel surface, which are expensive to remove, and a process that is both difficult to predict and control.

In order to formulate our model we refer to the work by Hill et al. which is more amenable than some numerical models. We use a lubrication approximation in the liquid flux region. Heat flow in the steel and flux is considered and coupled with the flow equations to predict mark formation. The model is non-dimensionalised in a systematic way. By neglecting small terms we obtain a model which makes fewer a priori assumptions than Hill et al. We then proceed with our analysis by relaxing some dimensionless parameters and a qualitative result is obtained. Finally we consider a closer examination of the viscosity of molten flux.

Using drag Eulerlets

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10:20 on Wednesday in LTD

Consider a moving body in an exterior domain with large Reynolds number but steady flow. We obtain a near-field Euler flow description after matching it to a far-field Oseen flow. In particular, we use the Green's integral representation to give a near-field flow description in terms of Eulerlets. We focus our attention on drag Eulerlets for a bluff body by looking at two-dimensional steady uniform incompressible flow past a circular cylinder at Reynolds number 40 just before the onset of turbulence. By assuming a wake profile, we can replicate trends in wake pressure on the circular cylinder, wake eddies, and diffusion of the far-field wake, and compare with experiment and Direct Numerical Simulation results.

Predicting the onset of high-frequency self-excited oscillations in a channel with an elastic wall

Thomas Ward[1†] & Robert Whittaker[1]

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10:40 on Wednesday in LTD

Flow-induced oscillations of fluid-conveying elastic-walled channels arise in many industrial and biological systems. Examples include pipe flutter, Korotkoff sounds during blood pressure measurement and wheezing during forced exhalation from the pulmonary airways. To model the wall mechanics of such phenomena we consider the flow of a fluid through a rigid rectangular channel with one face replaced by an elastic surface. If the transmural pressure (internal minus external) across the elastic wall becomes sufficiently negative, the wall will buckle. In this state, small changes in the transmural pressure can induce large changes in the shape of the wall, resulting in strong fluid-structure interactions.

The mechanisms responsible for the development of the high-frequency self-excited oscillations in such a scenario are still not fully understood. Previous work on high-frequency oscillations about an undeformed base-state found that the normal mode shapes resembled the graph of $\sin((n+1/2)\pi z)$ for $z \in [0,1]$ and each $n \geq 1$, with the axial mode-1 perturbation always being the most unstable. However, if the wall oscillates about a more deformed initial configuration, the additional normal displacement from the base-state results in further axial stretching. The axial mode-1 oscillations then require significantly more axial stretching relative to the mode-2 oscillations since, in the latter case, axial displacements of the wall eliminate the need for such stretching. As stretching is much harder than bending in thin walls, the restoring forces in a mode involving significant stretching will be expected to be larger than for those that do not. For a sufficiently buckled and thin elastic walls, this may provide sufficient stabilisation to the mode-1 oscillation for the mode-2 oscillation to become the most unstable.

Building on the results of Whittaker et al. (2010), which considered perturbations about a uniform initial state, we derive a system that governs the base-state and oscillatory components of the wall displacement valid for long-wavelength high-frequency small amplitude displacements. We explain the behaviour of the system as two key dimensionless parameters, $\mathcal{F}$ and $\mathcal{K}$ are varied. These represent the strength of axial tension relative to azimuthal bending, and the strength of the base-state curvature effects relative to the restoring force from the springs. We use numerical simulations to identify distinct flow regimes in parameter space, which we explore further in detail using asymptotic analysis.

When $\mathcal{K}$ is sufficiently large, the base-state curvature effects dominate. This forces the mode solutions to become orthogonal to the curvature of the base state at leading order, i.e. there is little overall stretching and the wall becomes effectively inextensible. This forces additional spatial oscillations in the axial coordinate, increasing the number of turning points. The fundamental mode therefore resembles the axial-mode-two, the first harmonic becomes the axial-mode-three, etc. This has the effect of reducing the instability of the fundamental mode relative to the first harmonic, with the most unstable mode switching from the fundamental to the first harmonic when $\mathcal{K}$ exceeds a critical value $\mathcal{K}_c(\mathcal{F})$.

The switching in the order that the modes lose stability shows base-state curvature effects in the wall play an important role in the dynamics of the system. Further work is needed to extend this analysis to describe the the full 3D elastics tube however we anticipate that many of the physical processes described here will be applicable to flows in more realistic practical applications.

Tissue engineering I

Analysis of lattice and continuum models of bioactive porous media

Andrew Krause[1†]
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09:00 on Wednesday in LTE

In the context of tissue engineering, we recently proposed a lattice model for a bioactive porous tissue scaffold in order to understand the role of an active pore network in tissue growth. This model considered the scaffold as an evolving lattice of pores, with coupling between local cell growth in the pores, and fluid flow through the medium. Here we consider a variant of this lattice model as well as a spatially continuous analogue. We analyze these models from a dynamical systems perspective emphasizing qualitative changes in model behaviour as parameters are varied. Depending on the size of the underlying network, we observe oscillations and steady states in cell density exhibited in both models. Steady state behaviour can be described in large cell diffusion regimes via regular asymptotic expansions in the diffusion parameter. We numerically continue steady state solutions into intermediate diffusion regimes, where we observe symmetry-breaking bifurcations to both oscillatory and steady state behaviours that can be explained via local bifurcations, as well as symmetry-preserving oscillations that do not bifurcate from steady states. The spatially continuous analogue of the model only exhibits symmetric steady states and oscillatory solutions, and we conjecture that it is the finite lattice that gives rise to the more complicated symmetry-breaking dynamics. We suggest that the origin of both types of oscillations is a nonlocal reaction-diffusion mechanism mediated by quasi-static fluid flow. Finally we relate these results back to the original modelling question of how network topology influences tissue growth in a bioactive porous tissue scaffold.

Mathematical modelling of photothermal therapy

Ahmed Ismaeel[1†]
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09:20 on Wednesday in LTE

Photothermal therapy is a promising cancer treatment in which an external laser beam can be used to ablate tumor cells without need for medical surgery. This technique is suitable for use in sensitive organs like the brain and the inner wall of the stomach without affecting the healthy cells around the tumor. In addition, this kind of therapy has much reduced side effects compared to chemotherapy and radiation. The effect of heating can be magnified by introducing metal nanoparticles into the tumor through the circulation system. The efficiency of this new treatment depends on a number of factors including the concentration and distribution of delivered nanoparticles inside the targeted tumor. We use mathematical modelling to characterise nanoparticle penetration into an avascular tumour and its subsequent photothermal ablation using a concentrated heat source.

A mathematical model of a thermoresponsive drug delivery device

Niall McInerney[1†]
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09:40 on Wednesday in LTE

Thermoresponsive polymers can respond drastically to a change in temperature of the environment, which makes them the subject of extensive research with a wide variety of potential applications. In polymers with a lower critical solution temperature (LCST), the polymer undergoes a sharp transition from a swollen, wet, hydrophilic state to a collapsed, dry, hydrophobic state when the polymer is heated above the LCST. Polymers can be synthesised by copolymerising with hydrophilic or hydrophobic monomers to get a desired LCST. In a potential drug delivery device, pulsatile release (fast/slow) can be controlled by cooling/heating of the polymer.

Previous modelling work on this subject has only considered instantaneous boundary movements, where the polymer jumps from being in a condensed state to a swollen state with the temperature switch, and thus the properties of the polymer are treated as simple step functions. In this talk we will present the development of a one-dimensional non-linear double moving boundary problem based on the diffusion of a drug from such a device. This model tracks the concentrations of the penetrating solvent from the environment and the drug being released as well as the positions of the swelling/collapsing interface boundary and the boundary separating the wet swollen polymer from the still dry polymer. We will discuss analytical and numerical results and compare with experimental data.

Perspectives on constitutive models for soft tissues

Andrey Melnik[1†]
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10:00 on Wednesday in LTE

Nonlinear solid mechanics is used to model normal function and pathological conditions in soft tissues. Precise mechanical models are required to understand the physiology and pathology of organs from the theoretical point of view, as well as to evaluate a need for medical interventions and their consequences in treating diseases in practice. In many cases, the mechanical behaviour of tissues can be adequately represented by an idealised elastic material.

We review a number of (pseudo-)elastic and viscoelastic constitutive models focusing on the myocardium and other cardiovascular tissues. We examine characteristic features of structural and semi-structural approaches, highlight some basic ideas that shaped the constitutive modelling over the decades, and discuss recent efforts to overcome their limitations. We analyse how the models capture nonlinearity, anisotropy, and multi-scale aspects of the tissues. We also explore ways that residual stress can be incorporated into the models and how this fits into the framework of growth and remodelling.

This review results from our pursuit of the development of a novel multi-scale constitutive model. We believe the presentation would provide a good opportunity to learn about soft tissue modelling and to critically reflect upon the subject.

Coupled agent-based and FE modelling of left ventricle post myocardial infarction

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10:20 on Wednesday in LTE

Understanding the healing and remodelling processes induced by myocardial infarction (MI) is important in modern cardiology practice. The mechanical recovery of the myocardium post-MI can be indicative for effective treatments aimed at avoiding eventual heart failure. MI remodelling is a multiscale feedback process between the mechanical loading and cellular adaptation. In this paper, we use an agent-based model to describe the fibroblast migration regulated by chemical and mechanical cues, and couple it with a finite element (FE) mechanical model for a 3D geometry of the heart. This enables us to look at the scar healing and density growth in the local regions of the heart around the MI site.

Biorhythms

Mathematical investigation of diabetically impaired ultradian oscillations in the glucose-insulin regulation

Adam Bridgewater[1†]
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09:00 on Wednesday in LTF

The accuracy of the oscillatory nature of the glucose and insulin blood levels constitutes an important indicator of healthy regulation [5]. In this contribution, we study a two-delay mathematical model, incorporating two physiological delays as well as parameters representing diabetic deficiencies, in order to investigate the impact of a fault in either the pancreatic insulin secretion or glucose utilisation on the production of ultradian oscillations in the glucose-insulin regulatory system. The delays represent the hepatic glucose production and the time necessary for the release of insulin, and have been shown to be at least partly responsible for the oscillatory nature of the regime [3,4]. A numerical study of the non-linear mathematical model is performed to characterise the onset of the oscillations, and perturbations of the periodic solutions are used to investigate the amplitude and frequency of the oscillations. Through use of linear stability analysis and bifurcation theory, pathways to restoring appropriate cyclic regulation are mathematically described [2]. Our goal is to provide measurable indicators of deficiency in the system and a more thorough description of the contribution of insulin treatments to the reintroduction of an oscillatory behaviour which is crucial for the design of a control algorithm which could then be implemented into an insulin control system.

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The hidden rhythm in the sleep patterns of the common vole

Matthew Bailey[1†]
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09:20 on Wednesday in LTF

Sleep is essential for most mammals to function and mathematical models are playing an important role in developing our understanding of many of the physiological properties of sleep. Sleep/ wake regulation is often described as the interaction of two oscillators: a circadian oscillator with a period of 24 hours, and a homeostatic oscillator denoting the pressure to sleep. One of the most fundamental two oscillator models used to model sleep/wake regulation in mammals is the two process model. The two process model can exhibit various types of sleep-wake dynamics, it is capable of explaining monophasic sleep (one sleep per day) and polyphasic sleep (many sleeps per day) periods. The two process model can be used to explain the polyphasic patterns seen in spontaneous sleep, for the common vole, but it is not sufficient when modelling sleep deprivation. By desynchronizing the ultradian and circadian components through sleep deprivation I will show how the ultradian rhythm is hidden during spontaneous sleep. Thus with an additional, ultradian, oscillation added to the model, the voles sleep-wake behaviour under both spontaneous and deprivation conditions can be explained.

Modelling the dynamics between pathogens and the immune system, with emphasis on EPEC

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09:40 on Wednesday in LTF

In this project, the dynamics between pathogens (with emphasis on bacterial infections, in particular, EPEC infections) and the immune system are investigated mathematically. ODE models for the time courses of a general bacterial infection and for an EPEC specific infection are constructed. These models are based on the literature and on current ongoing experimental work at the CMB at the University of Nottingham. These models are analysed to unpick the important dynamics. The EPEC specific model also combines a population model with a gene network sub-model. Finally, a cellular automaton model has been constructed in MATLAB to include spatial as well as temporal dynamics. This model is then analysed parallel to the temporal ODE model to look for similarities and differences.

Nonlinear dynamical system for prostate cancer cell transdifferentiation

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10:00 on Wednesday in LTF

Prostate cancer (PCa) is the second most common cause of cancer among men worldwide. Hormone therapy is considered effective in the initial stage of PCa, as androgen is essential for PCa cell growth. As the disease progresses, the cancer cells can develop androgen independence, which leads to the failure of endocrine therapy to control PCa. Androgen independence may involve neuroendocrine (NE) transdifferentiation, but this process and its role is presently controversial. In this talk I will present a nonlinear model representing the dynamics of transdifferentiation of PCa human cells to NE-like cells. Boundedness and non-negativity of the solutions are proven. To gain a better understanding of the relationships between data, parameters and model predictions, parameter uncertainty is explored and the result of the sensitivity analysis (SA) of the model parameters is shown. The analysis of the bifurcations of the system is fed by the result of the SA. The rich dynamics of the system is explored with the aid of numerical software.

Approximating non-adiabatic transitions in two-dimensional quantum molecular dynamics

Tim Hurst†

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10:20 on Wednesday in LTF

Non-adiabatic (or avoided crossing) transitions in diatomic molecules are behind important chemical processes, such as photodissociation of sodium iodide and chemical reactions occurring in the retina. However, due to high frequency oscillations in the dynamics and exponentially small wavepackets, they are expensive to simulate. In this talk we introduce the problem in one dimension, and briefly explain the derivation of the formula in [1] used to accurately approximate the wavepacket transmitted at an avoided crossing. We then use a slicing algorithm to apply this one-dimensional formula to a two-dimensional avoided crossing, with promising results. Potential directions for future investigation are then discussed.

[1] B. Goddard and V. Betz, Phys. Rev. Lett. 103, 213001 (2009).

Feature extraction from a blood pressure signal based on attractor reconstruction

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Manasi Nandi[2], Mark Christie[2] & Ying Huang[1]

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10:40 on Wednesday in LTF

Blood pressure waveforms can be collected at a high sampling frequency for long periods of time. However, much of the data is either ignored or under analysed. Heart rate variability (HRV) methods analyse beat-to-beat intervals which involves discarding the majority of the data. Our approach to extracting key features from these large and complex datasets involves using all of the data to generate a reconstructed attractor in a three-dimensional phase space using delay coordinates for a window of data. The naturally occurring baseline variation is removed by projecting the attractor onto a plane and useful quantitative measures can be obtained from the two-dimensional attractor. The time window is then moved through the data to give a collection of continuous signals. This approach enables quantification of changes in the waveform shape and has been applied to blood pressure data for the early detection of sepsis and for detection of changes in contractility.

Geophysical fluid dynamics

Tripolar vortices in a two-layer quasi-geostrophic flow

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09:00 on Wednesday in LTH

Vortices abound in the oceans and are responsible for a large part of the transport of momentum and tracers in the medium. Determining system of vortices in mutual equilibrium which can steadily translate or rotate in the oceans can help us better understand some of the fundamental aspects of oceanic transport.

In this research we are interested in constructing equilibrium solutions for tripolar vortices in a two-layer, quasi-geostrophic flow. The tripoles consist of two like-signed vortices the upper fluid layer and an opposite-signed vortex in the lower layer.

We determine families of equilibrium solutions, address their linear stability and illustrate their nonlinear evolution for various Rossby deformation radius.

For each value of the deformation radius, there are two distinct branches of solutions. Each branch is spanned by a parameter such as the distance between the pair of like-signed vortices. The nature of the branches depends on whether the lower vortex lies on the axis of rotation of the upper pair of like-signed vortices (ordinary roundabouts) or if it is offset from it (eccentric roundabouts).

The two branches of equilibria intersect for small values of the deformation radius. In these cases, the eccentric roundabouts are neutrally stable and only ordinary roundabouts can be unstable. This is related to the fact that ordinary roundabouts can exist for vortices closer together than the eccentric roundabouts, inducing stronger shear on each other.

Eccentric roundabouts can however be unstable for large values of the deformation radius. In these cases, the two branches of solutions do not intersect. The branch of eccentric roundabouts ends at a point where one of the vortex exhibit a sharp edge where instabilities can be triggered.

The study is complemented by illustrating the nonlinear time evolution of a few generic cases.

Reconstructing ice-age palaeoclimates using model inversion and data interpolation

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09:20 on Wednesday in LTH

Site-based pollen records have been used to provide quantitative reconstructions of Last Glacial Maximum (LGM) climates, but there are too few such records to provide continuous climate fields for the evaluation of climate model simulations. Furthermore, many of the reconstructions were made using modern-analogue techniques, which do not account for the direct impact of CO_2 on water-use efficiency and therefore reconstruct considerably drier conditions under low CO_2 at the LGM than indicated by other sources of information. We show that it is possible to correct analogue-based moisture reconstructions for this effect by inverting a simple light-use efficiency model of productivity, based on the principle that a plant keeps a constant ratio between CO_2 uptake and water lost to transpiration. We give an introduction to the area of pollen based palaeoclimate reconstructions, explain the correction technique and go on to make reconstructions based on current pollen reconstructions of one or more of six climate variables (mean annual temperature, mean temperature of the warmest and coldest months, the length of the growing seasons, mean annual precipitation, and the ratio of actual to potential evapotranspiration) at individual LGM sites. We use a water-balance model to derive a moisture index (MI) at each site from mean annual precipitation and monthly values of sunshine fraction and average temperature, and correct this MI using the light-use efficiency model inversion approach. This is then used to construct three-dimensional variational (3D-Var) cost functions that can be used to derive reconstructions of all six climate variables at each site from the pollen reconstructions and a background. We consider the use of two alternative background climate states: modern climate and LGM climate derived as an ensemble average of the LGM climate model simulations which both aim to add topographic and circulation based patterns into the reconstructions. To further improve the reconstructions we discuss modified cost functions that account for the spatial proximity of sites. The use of model inversion together with a 3D-Var data assimilation scheme provides a well-formulated way of maximising the use of limited palaeoclimate data.

Probabilistic parametrization of condensation in coarse-grained moisture transport models

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09:40 on Wednesday in LTH

Large-scale circulations, such as the Hadley cell, play a crucial role in controlling the distribution of water vapour in the atmosphere. The advection-condensation (AC) model, in which moist air parcels are advected and react through condensation, provides a theoretical framework to investigate atmospheric moisture distribution. In a stochastic formulation of the AC model, small-scale turbulent motion is represented by white noise and the position and moisture content of each parcel are treated as random variables. Such stochastic model retains local fluctuation and is not overly prone to produce saturated air as in models where moisture is represented by a coarse-grained field evolving according to a partial differential equation. However, it is often not feasible to track a huge number of air parcels in weather and climate models. Here, we propose a condensation parametrization for use in coarse-grained moisture models that takes local fluctuation into account.

Internal wave attractors in stratified fluids

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10:00 on Wednesday in LTH

A disturbance in a stratified fluid can generate motion that is resisted by gravity. This competition of forces leads to oscillations that form internal waves in the fluid and these propagate along wave beams that are inclined with respect to gravity. In an asymmetric domain successive reflections from solid boundaries can force the wave beams onto a limit orbit, known as a wave attractor, focusing the wave energy of the system onto small spatial scales. In the vicinity of the wave attractor intense shear motion is generated in the fluid.

In a geophysical context, internal waves can be considered confined between lateral boundaries and the ocean surface. Examples of such lateral boundaries include the Mozambique Channel and Luzon Strait. The wave attractors formed in such domains provide a mechanism for an energy cascade, and provides insight into how energy transported to internal waves can induce mixing of the fluid.

We consider a computational model of stratified fluids using a non-dissipative Hamiltonian discontinuous finite element method. Three formulations are considered: compressible, incompressible and Euler-Boussinesq. In each case internal waves are generated by applying a body force to the fluid at rest in an asymmetric domain and the resulting system is monitored for the presence of wave attractors.

Tsunami detection and mitigation

Usama Kadri[†]
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10:20 on Wednesday in LTH

In this work, I show that it is possible to early detect tsunamis, reduce their amplitudes, and redistribute their energy over a larger space, upon interaction with two counterpart resonating acoustic–gravity waves (see [1]-[5]). Since the acoustic modes travel much faster than the tsunami, and in an opposite direction, they transfer part of the withdrawn energy outside of the original tsunami envelope, resulting in a redistribution of the latter's energy both in time and space. Thus, the two acoustic modes form a secondary tsunami envelope, then a tertiary, and so on until no further interaction is possible. I will show an example whereby the original amplitude of the tsunami drops by almost 30are positioned behind, i.e. their arrival to the coastline is delayed; if even less than that achieved in reality it will save many lives and properties.

[1] U. Kadri, 2017. Tsunami mitigation by resonant triad interaction with acoustic-gravity waves. Heliyon 3(1), pp. e00234. (10.1016/j.heliyon.2017.e00234)

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The limiting form of symmetric instability in geophysical flows

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10:40 on Wednesday in LTH

The stability of parallel flow with vertical shear, density stratification and background rotation is of fundamental importance in geophysical fluid dynamics. For a flow with vertical shear U_z and buoyancy frequency N, the dominant instability is typically a symmetric instability (sometimes known as slantwise convection) when $1/4 < R_i < 1$, where $R_i = N^2/U_z^2$. Symmetric instability, which in its simplest form has no along-stream variations, is known to be active in both the troposphere and upper ocean.

The corresponding (symmetric) inviscid linear stability problem has been well studied for the case of constant U_z and N_z, and has some interesting mathematical properties (e.g., non-separable governing PDE, an absence of normal mode solutions in rectangular domains). Here, for the first time, a general theory of symmetric instability is given when R_i varies smoothly with height, thinking of the more realistic case where an unstable layer with $R_i < 1$ lies between two stable layers with $R_i > 1$. The mathematical theory is developed for horizontally periodic disturbances to a basic state with arbitrary smooth $N_(z)$, but constant U_z. An asymptotic analysis is used to derive expressions for the most unstable mode, which occurs in the limit of large cross-isentropic wavenumber and takes the form of solutions trapped within the unstable layer; the same result is derived using an interesting generalised parcel dynamics argument, which explicitly shows how the trapping is linked to vertical variations of the potential vorticity. A separate asymptotic analysis is given for the small wavenumber limit, where only one such trapped mode may exist, as expected from the spectral theory of the Schrodinger equation. These two limiting results are shown to be consistent with an exact solution of the linear stability problem that can be obtained for a special choice of $N_(z)$.

The asymptotic analysis can be extended to allow for weak diffusion at arbitrary Prandtl number, yielding an explicit diffusive scale selection at large wavenumber. Numerical simulations show that these weakly diffusive modes dominate the early stages of the nonlinear evolution of the symmetric instability.

Inverse problems and heat transfer

Inverse coefficient problems in heat transfer

Kai Cao†

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09:00 on Wednesday in 24AA04

The thermal conductivity and /or perfusion coefficient of the one-dimensional inverse heat transfer problem are determined from interior temperature measurements using the conjugate gradient method (CGM). The inverse problem belongs to the function estimation approach since no a priori functional form of the unknown function is assumed. Regularization has been achieved by stopping the iterations at the level at which the least-squares objective functional, minimizing the gap between the computed and the measured temperature, became just below the noise threshold with which the data is contaminated, since the inverse heat transfer problem is ill-posed. Several examples are studied and the numerical results became more accurate as the amount of noise of the temperature measurement decreases. Thus, the CGM together with the discrepancy principle is an efficient and stable method for solving inverse coefficient problems.

Analysis of multiplicative regularisation for inverse problems

Yujun Qiao†

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09:20 on Wednesday in 24AA04

In this talk, a new regularisation method for nonlinear inverse problems is presented, tested and analysed. Unlike the traditional regularisation methods of the additive type which stabilises the original cost functional by adding a regularisation term, this multiplicative method regularises the original functional by multiplying it with a term slightly deviated from identity. A sequence of weight functions $\{b_n(\mathbf{x})\}$ and $\{\delta_n^2\}$ are involved in the algorithm. They are related by recurrence relations and self-updated in each iteration. The self-determination of the algorithm has profound significance: (i) Avoiding the difficulty of choosing appropriate regularisation parameter in additive methods, which takes large computational resources; (ii) Automatically identifying the important features such as the edges of the image, and applying less regularisation effect over these pixels. Numerical results have shown that the multiplicative regularisation is not only robust and reliable over symmetric and asymmetric noise distributions, but also able to tolerate much higher noise level (approximately 50%) than its additive counterpart (approximately 10%). Moreover, it does not require prior knowledge of the noise level, which is in itself an advantage in applications. We will present both numerical results for different type of noise and theoretical analysis for the algorithm.

On solvability of linear-quadratic differential game for a heat equation

Vladimir Turetsky†
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09:40 on Wednesday in 24AA04

Linear-quadratic differential game for a controlled one-dimensional heat equation in the presence of an external disturbance is considered. The equation is considered in the rectangle domain with Robin boundary conditions. The cost functional consists of an integral term, specifying the closeness of the final temperature profile to a prescribed function and two penalty terms for the control and the disturbance expenditure.The game solvability condition is derived in terms of eigenvalues of some integral operators. The optimal linear control strategy is obtained based on Riccati-type PDE. Examples of tracking problems solution by using the game optimal control are presented.

Instability in the self-similar motion of a planar solidification front

Ferran Brosa Planella[1†]
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10:00 on Wednesday in 24AA04

Silicon has many different uses, from solar cells to silicones. During the casting process, molten silicon is poured into moulds and let to solidify just by exchanging heat with the atmosphere. However, the casting conditions have a huge impact on the resulting microstructure, which is crucial in subsequent processing of the material. We present a mathematical model for solidification of silicon considering constitutional supercooling for both infinite and semi-infinite domains, and we consider the well-known self-similar solutions to these problems. We then introduce self-similar perturbations to this solutions to perform a linear stability analysis. The analysis shows that the system is absolutely unstable for self-similar perturbations in both cases, regardless of the parameter choice.

Freeze-drying, Stefan problems and level set methods

Rudolf Kohulak†
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10:20 on Wednesday in 24AA04

Freeze-drying is a process widely used in the pharmaceutical industry as a simple solution on how to reduce the water content of temperature sensitive materials and increase their stability and shelf life. However, at the moment freeze-drying remains the most expensive stage of pharmaceutical manufacturing, and hence further modelling is needed. To model the process we consider Stefan Problems. A Stefan Problem is a particular boundary value problem that arises in modelling heat transfer with phase change (water freezing, ice melting...). Hence the challenge is to capture the progression of the interface separating different phases of the material. We conclude the talk by considering different numerical methods for solving the model; in particular we focus on the level set method.

Faster PET reconstruction with a stochastic primal-dual hybrid gradient method

Matthias Ehrhardt[1†]
Pawel Markiewicz[2], Peter Richtarik[3], Antonin Chambolle[4] & Carola-Bibiane Schönlieb[1]

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10:40 on Wednesday in 24AA04

In this talk we revisit the problem of PET reconstruction with non-smooth and convex priors. As the data fidelity term in PET is the Poisson likelihood there are not many algorithms that can solve this problem. A very popular choice to solve this problem is the Primal-Dual Hybrid Gradient method proposed by Chambolle and Pock. The system matrix for clinical PET scanners is very large and cannot be stored in the memory of most computers and thus an expensive algorithm to compute matrix vector products has to be employed. In this talk we extend the Primal-Dual Hybrid Gradient method to the subset setting (like in ART, Kaczmarz or OSEM). By choosing subsets randomly we can prove that the algorithm is convergent for all sensible random subset selections. Examples based on synthetic and real data show that it is much faster (in terms of actual time) than the standard Primal-Dual Hybrid Gradient method.

Wednesday PM: Mini-Symposia

Network Science

Organised by: Marianno Beguerisse, Jonathan Ward and James Rankin

Multilayer networks: a new framework for complex systems

Ginestra Bianconi†

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13:30 on Wednesday in LTA

Multilayer networks describe complex systems formed by different interacting networks. Multilayer networks are ubiquitous and include social networks, financial markets, multimodal transportation systems, infrastructures, the brain and the cell. Multilayer networks cannot be reduced to a large single network. In this talk I will introduce relevant modelling frameworks for multilayer structures and I will discuss how it is possible to extract more relevant information from multilayer networks than from their single layers taken in isolation.

Master stability functions reveal diffusion driven instabilities in multilayer networks

Thilo Gross†

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13:50 on Wednesday in LTA

Many systems in science and technology can be described as multilayer networks, which are known to exhibit phenomena such as catastrophic failure cascades and pattern-forming instabilities. A particular class of multilayer networks describes systems where different interacting copies of a local network exist in different spatial locations, including for instance regulatory and metabolic networks of identical cells and interacting habitats of ecological populations. Here, we show that such systems can be analyzed by a master stability function approach, which reveals conditions for diffusion-driven instabilities. We demonstrate the methodology on the example of state-of-the-art meta-foodweb models, where it reveals diffusion-driven instabilities that lead to localized dynamics and spatial patterns. This type of approach can be applied to a variety of systems from nature, science and engineering to aid the understanding and design of complex self-organizing systems.

Identifying the hidden multiplex architecture of complex systems

Lucas Lacasa†

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14:10 on Wednesday in LTA

The architecture of many complex systems is well described by multiplex interaction networks, and their dynamics is often the result of several intertwined processes taking place at dierent levels. However only in a few cases can such multi-layered architecture be empirically observed, as one usually only has experimental access to such structure from an aggregated projection. A fundamental question is thus to determine whether the hidden underlying architecture of complex systems is better modelled as a single interaction layer or results from the aggregation and interplay of multiple layers. Here we show that, by only using local information provided by a random walker navigating the aggregated network, it is possible to decide in a robust way if the underlying structure is a multiplex and, in the latter case, to determine the most probable number of layers. The proposed methodology would also allow to select the optimal architecture capable of reproducing non-Markovian dynamics taking place on networks, such as human and animal mobility.

Connectivity and colouring in random geometric graphs

Ayalvadi Ganesh†

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14:30 on Wednesday in LTA

A random geometric graph is composed of vertices placed at random on the plane, or a bounded subset of it, with the edge between any pair of vertices being present with a probability that depends on the distance between them. These are widely used to model wireless communication networks, for example. In this talk, we will discuss how such models can be used as a framework to address two questions: (1) how to dynamically assign channels in a wireless network to minimise interference, and (2) how to allocate cryptographic keys in such a network in order to ensure connectivity.

Why I compute - in honour of Robert Rosner's 70th birthday

Organised by: Eun-Jin Kim and Mitchell Berger

Magnetism and dissipation in global simulations of convective stars

Matthew Browning[1†]
Laura Currie[1], Lucia Duarte[1], Lewis Ireland[1], Felix Sainsbury-Martinez[1], Maria Weber[1], Gilles Chabrier[1] & Isabelle Baraffe[1]

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13:30 on Wednesday in LTB

All stars possess magnetism somewhere in their interiors. In most cases the magnetic fields are built by the action of a magnetic dynamo, a process that converts kinetic energy to magnetic, but a comprehensive theoretical understanding of this process has remained elusive. Here, I will describe some recent results from global-scale simulations of stellar and planetary dynamos, which have helped provide some insight into how orderly fields might be generated amidst the turbulent convection in these objects. I will focus on what elements appear to play roles in establishing fields that are highly organised, either in space or in time. I will also discuss claims that particularly strong magnetic fields may affect the structure and radii of low-mass stars; in particular, I will argue that the extreme field strengths needed in some of these models are ruled out by the complementary constraints of buoyancy and Ohmic dissipation. I will briefly note some possible consequences of extraordinarily dissipative convection for the structure of low-mass stars. Finally, I will describe some of the considerable uncertainties that persist in these simulations, and speculate about how to do better.

Trefoil knot reconnection and regularity

Robert M. Kerr[†]

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14:10 on Wednesday in LTB

The reconnection timescale t_x for the evolution of a perturbed trefoil vortex knot is determined in two ways. Visually by when the last snippet of the original trefoil vortex is cut and second, by the time when v independent scaled enstrophy growth $\sqrt{v}Z(t)$ is observed. During this phase, and a bit after, there is good correspondence with the reconnection of Kleckner & Irvine (2013) and the simulated helicity is preserved, confirming the experimentally observed preservation of helicity over an equivalent timespan (Scheeler et al. 2014a). Later, a viscosity independent dissipation rate $\epsilon = vZ$ forms near $t \approx 2t_x$. It is found that maintaining the enstrophy growth as the viscosity decreases requires increasing the computational domain in a manner implied by known Sobolev space bounds for fixed domains.

A Taylor-Couette dynamo

Rainer Hollerbach[1†]
Anna Guseva[2], Ashley Willis[3] & Marc Avila[2]

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14:50 on Wednesday in LTB

We present numerical results of magnetohydrodynamic Taylor-Couette flow. The rotation rates of the inner and outer cylinders are adjusted to be in the Rayleigh-stable regime. By starting with an appropriate initial condition (MHD turbulence) we show that the flow can nevertheless act as a dynamo, with the turbulence and the magnetic field mutually supporting each others existence.

Dynamical systems and applications

Organised by: Jan Sieber

Rate-induced tipping with periodic orbits

Hassan M. Alkhayuon[1†] & Peter Ashwin[1]

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13:30 on Wednesday in LTL

Rate-induced tipping (R-tipping) is a sudden, irreversible and unexpected change that can happen when a system fails to track a continuously changing quasi-static attractor. This particular mechanism of tipping has been studied before, but only for the case where the attractors of the systems are equilibria. Even for slightly more complicated attractors such as periodic orbits, it is not well understood. We investigate this by looking at a supercritical Hopf normal form with two periodic orbits that are forced by a parameter shift. We have detected multiple-types and levels of R-tipping by considering the heteroclinic connections between different invariant sets.

Heteroclinic phenomena in the coupled replicator equations, a computer assisted approach

Cezary Olszowiec[†]

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13:50 on Wednesday in LTL

We consider a Rock-Scissors-Paper game assuming the perfect memory of the playing agents X, Y. The interaction matrices depend on two parameters $\epsilon_X, \epsilon_Y \in (-1, 1)$ and the dynamics are described by the coupled replicator equations.

We provide the description of naturally appearing heteroclinic network and investigate asymptotic and chaotic behavior in its neighbourhood. It turns out that certain types of behaviours are never possible or appear in the system only for some parameter values, e.g. finite switching. In particular the infinite switching happening near the network cannot be described by the full-shift on two symbols and its form strongly depends on the parameter values. In the system we observe different bifurcation scenarios: e.g.transition from order to chaos (through Hamiltonian case where invariant tori and Hamiltonian chaos might be observed), loss of one dimension of the local stable manifold of the subcycle or disappearance and appearance of the local stable and unstable manifolds of the different subcycles at the same time. As well we investigate numerically the existence of the heteroclinic connection between different heteroclinic subcycles (i.e. a superheteroclinic orbit) and its bifurcation to the different heteroclinic connections (forward and backward) from the hyperbolic fixed point (i.e. Nash equilibrium) to the subcycles.

Remote tactile surface texture sensing: from rats to robots

Thibaut Putelat[1†]
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14:10 on Wednesday in LTL

Rats collect tactile information about their surrounding environment while actively rubbing and taping their whiskers against surfaces and objects. The physico-cognitive processes involved in rats perception and in particular the nature of the physical signals that rats use for the recognition and discrimination of the texture of a surface, namely its lay, roughness and waviness, are still poorly known. We investigate the mechanical modalities of the signal transference from the whisker's tip/object contact zone to the base/mechanoreceptors interface, both theoretically from a nonlinear mathematical model of whiskers described as tapered elastic rods, and experimentally from the spatiotemporal record by high-speed videography of the motion of natural rat whiskers sliding on surfaces of different texture. Our results suggest that stick-slip transitions caused by dynamic friction play a key role for the stimulation of vibrations and bending waves propagating along the whisker, conveying in turn textural information to the neuronal mechanoreceptors at the whisker base. We shall present results showing how the rod geometry determines the modulation and filtering of the received signal at the base.

Energy growth and scattering maps for a billiard inside a time-dependent symmetric domain close to an ellipse

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14:30 on Wednesday in LTL

We study billiard dynamics inside an ellipse with a time-dependent periodic perturbation of the axes and and small $O(\delta)$ quartic polynomial deformation of the boundary. In this situation the energy of the billiard is no longer conserved. We prove a type of Fermi acceleration, namely that given a sufficiently large initial energy, there exists a billiard trajectory that reaches an arbitrarily larger energy value in some finite time. The proof depends on reduction of the billiard map to two Hamiltonian flows defined on the normally hyperbolic invariant manifold Λ parametrised by energy and time in the phase space of the billiard. The two flows approximate inner and scattering maps, which are common tools that arise in the studies of Arnol'd diffusion. Melnikov type calculations imply that the *scattering map* is only defined on a subset of Λ that increases with δ and becomes empty for $\delta = 0$.

Blow-up and asymptotic growth in Volterra equations

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14:50 on Wednesday in LTL

Blow-up solutions of Volterra equations have attracted considerable attention due to their deep connections with parabolic PDEs. In particular, nonlinear Volterra equations arise naturally in models of combustion in a reactive-diffusive medium. Research on such blow-up problems has sought to answer three central questions: when do solutions explode, at what time do solutions explode, and what is the asymptotic behaviour of solutions at blow-up? I will focus on the last question by discussing estimates on the rates of blow-up of explosive solutions and unbounded growth of global solutions. In contrast to the majority of the literature in this area, our methods are constructive, and hence particularly well suited to analysing the qualitative behaviour of solutions. Our approach also provides new proofs of well-known necessary and sufficient conditions for the finite-time blow-up of solutions.

Applications of string theory

Organised by: Jan Gutowski

How category theory simplified my life

Christian Saemann†

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13:30 on Wednesday in 22AA04

I will show how some areas in Mathematical Physics, mostly connected to String Theory, can benefit from the application of Category and Higher Category Theory. In particular, I will discuss how categorified gauge theory can help in the search of a mysterious superconformal field theory in six dimensions, how categorified symmetries clarify issues in an extension of supergravity called Double Field Theory and how higher category theory might even help with calculating scattering amplitudes. After this extensive motivation, I will present a convenient route to higher categories using Kan complexes and show how to derive an explicit solution in categorified gauge theory.

Supersymmetry and representation theory

Mathew Bullimore†

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14:30 on Wednesday in 22AA04

Representation theory is ubiquitous in physics, from atomic structure to condensed matter. In recent years, representation theory has arisen in unexpected and remarkable ways in the study of supersymmetric quantum field theory. I will outline some striking recent results in this area and explain why this topic holds great potential for both mathematicians and physicists.

Wednesday PM: Contributed Talks

Flow stability, boundary layers and bubbles

Roughing up wings: a promising technique in laminar flow control

Pietro Servini[1 †]
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13:30 on Wednesday in LTD

Ensuring that laminar boundary layers remain attached to the surface of an airfoil, be that the wing of a bird or drone or the blade of a turbine or helicopter, is important both for drag reduction and the prevention of stall. Numerical and experimental work carried out by Rothmayer and Huebsch, among others, suggests that the use of dynamic roughness elements (small oscillating bumps) may delay the separation of a laminar boundary layer.

In this talk, we will show that both dynamic and static elements, appropriately placed, are able to delay the positions of local and breakaway separation for flow over a hump embedded within the boundary layer. Conversely, negative roughness elements can result in a sudden and dramatic advancement of breakaway separation once their depth is above a certain threshold.

Destabilisation and modification of Tollmien-Schlichting disturbances by a three dimensional surface indentation

Hui Xu[1 †]
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13:50 on Wednesday in LTD

We consider the influence of a smooth three-dimensional (3D) indentation on the instability of an incompressible boundary layer by linear and nonlinear analyses. The numerical work was complemented by an experimental study, to investigate indentations of approximately 11 and 22, δ_{99} width at depths of 45%, 52% and 60% of δ_{99}. For these indentations a separation bubble confined within the indentation arises. Upstream of the indentation, spanwise-uniform Tollmien-Schlichting (TS) waves are assumed to exist, with the objective to investigate how the 3D surface indentation modifies the two-dimensional (2D) TS disturbance. Numerical corroboration against experimental data reveals good quantitative agreement. Comparing the structure of the 3D separation bubble to that created by a purely 2D indentation, there are a number of topological changes particularly in the case of the widest indentation; more rapid amplification and modification of the upstream TS waves along the symmetry plane of the indentation is observed. For the shortest indentations, beyond a certain depth there are then no distinct topological changes of the separation bubbles and hence on flow instability. The destabilising mechanism is found to be due to the confined separation bubble and is attributed to the inflectional instability of the separated shear layer. Finally for the widest width indentation investigated ($22\delta_{99}$), results of the linear analysis are compared with direct numerical simulations. A comparison with the traditional criteria of using N -factors to assess instability of properly 3D disturbances reveals that a general indication of flow destabilisation and development of strongly nonlinear behaviour is indicated as N=6 values are attained. However N $-$factors, based on linear models, can only be used to provide indications and severity of the destabilisation, since the process of disturbance breakdown to turbulence is inherently nonlinear and dependent on the magnitude and scope of the initial forcing.

Investigating the possibility of using Oseen flow to model the boundary layer

Rabeea Darghoth[1†] & Edmund Chadwick[1]

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14:10 on Wednesday in LTD

Oseen flow has recently given good results for modelling manoeuvring roblems and so we investigate the possibility of continuing this into the boundary layer noting that the oseenlet tends to the stokeslet near the Green's function origin. In particular, we look at two-dimensional incompressible flow past a semi-infinite flat plate and compare the Oseen flow and Blasius solution with the Navier-Stokes solution across a range of parameter values. The Oseen solution is given by a boundary integral distribution of oseenlets whose strength is determined by complex analysis from the Wiener-Hopf technique. The Blasius solution is obtained from neglecting certain axial changes and using dimensional analysis to give an ordinary differential equation solved numerically. The Navier-Stokes solution is determined using the Finite Difference Method and appropriate far-field boundary conditions.

Stability of a twisted plateau border with line tension and bending stiffness

Robert J. Whittaker[1†] & Simon Cox[2]

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14:30 on Wednesday in LTD

We investigate the effects of line tension and bending stiffness on the stability of a Plateau border at the junction of three soap films using analytical and numerical techniques. A simple geometry is considered, in which the border initially lies in equilibrium along the axis of a circular cylinder, with three equally-spaced films radiating outwards to meet the inside wall of the cylinder. The films are pinned at the two ends of the cylinder with a fixed relative twist, so the initial film surfaces are helicoids. The stability boundary for this system is found as a function of the relative twist and cylinder aspect ratio, for different tensions and stiffnesses of the border. The system generally becomes unstable as the length and/or twist is increased. Analytically, finding the stability boundary can be reduced to looking for zeros of the first eigenvalue of an infinite matrix. Solutions from this approach are in good agreement with full numerical simulations. The border mechanics can be stabilising or destabilising at different border lengths. Even a very small positive tension or bending stiffness has a significant stabilising effect on highly twisted borders, since the unstable mode driven by the films in the absence of any border mechanics involves a highly contorted border.

Selection of axisymmetric bubbles in the limit of small surface tension

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14:50 on Wednesday in LTD

It is well known that some two-dimensional free surface flows possess a continuum of solutions when surface tension is neglected but only a discrete set of solutions when surface tension is taken into account. Moreover, this discrete set of solutions reduces to a unique solution as the surface tends to zero. These problems include fingering in a Hele-Shaw cell and two-dimensional rising bubbles. In this talk we will show (using a finite difference scheme) that similar properties hold for axisymmetric rising bubbles in a tube.

The onset of double-diffusive convection with evolving background temperature gradients

Oliver Kerr†

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15:10 on Wednesday in LTD

When a deep body of fluid with a stable salinity gradient is heated from below at a horizontal boundary a destabilizing temperature gradient develops. As the heat diffuses into the fluid the effective thermal Rayleigh number based on the diffusion length-scale grows and, for the case of a sudden increase in the temperature by a fixed amount, is proportional to $t^{3/2}$. Hence, one may expect the fluid to be stable initially, and only becoming unstable after a finite time. However, the effective salt Rayleigh number grows as t^2, and so becomes increasingly important. In these circumstances the temporal evolution of the background gradients can be important. Here we look at the onset of linear instabilities taking into account these changing background gradients, and find the optimal instabilities. This optimization requires an appropriate choice of a measure for the magnitude of the instabilities. We will also consider, briefly, the heating of a salinity gradient from a vertical boundary.

Tissue engineering II

Modelling drug elution from polymer-free drug-eluting stents with microporous surfaces and drug-filled stents

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13:30 on Wednesday in LTE

Polymer-free drug-eluting stents (DESs) are an innovative new treatment for cardiovascular disease which is the leading cause of death globally. They have the potential to overcome problems associated with the traditional polymer-coated DESs (e.g. late stent thrombosis). However, the absence of a rate-controlling polymer layer makes optimisation of the drug release profile a particular challenge. The use of microporous stent surfaces to modulate the drug release rate is an approach that has recently shown particularly promising clinical results. A mathematical model has been developed to describe drug release from stents with microporous surfaces. The model predicts a two-stage release profile, with a relatively rapid initial release of most of the drug, followed by a slower release of the remaining drug. In the model, the slow release phase is accounted for by an adsorption/desorption mechanism close to the stent surface. The theoretical predictions are compared with experimental data and good agreement is found. The model of drug elution from a drug-filled stent is also developed. The drug is stored in the inner layer of a tri-layer wire which acts as a reservoir releasing the drug through laser-drilled holes on the outer surface of the stent struts. The general model is simplified using the assumption of low drug solubility and two special cases where the dissolution occurs in a uniform downward direction and in a spherical outward direction are considered. The main advantage of the simplified models is the ability to achieve analytical solutions. These solutions allow for calculating the drug release profile rapidly and for identifying the dependence of the various parameters of the system. The valuable insights provided by these models will serve as a useful guide for designing the enhanced polymer-free stents of the future.

Bloch waves in blood vessels with stents

Sara Frecentese[1]†
Giorgio Carta[2], Luca Argani[1], Alexander Movchan[1], Natasha Movchan[1] & Michael Wall[3]
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13:50 on Wednesday in LTE

In recent years, a considerable amount of research has been carried out to study stent-artery interaction in order to improve design of arterial stents. The subject of this talk is a novel approach to the mathematical modelling of stented blood vessels.

We model the stent as a periodic system using Bloch-Floquet analysis. The elementary cell includes the artery, modelled as full 3D linear elasticity, two stent coils and the blood, which is modelled using linearized fluid-solid interaction theory. This allow us to obtain dispersion properties for waves and analyse trapped modes. Computer simulations were performed for three different designs, and comparison of the resultant dispersion diagrams are presented.

The financial support of the GTA PhD studentship from the University of Liverpool is gratefully acknowledged.

Mathematical modelling of ureteroscope irrigation

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14:10 on Wednesday in LTE

Background: Ureteroscopy is a procedure to examine the urinary tract, and is often used for kidney stone removal surgery. Successful ureteroscopy relies on a good intrarenal view, which is in turn dependent on having constant irrigation of the system, both to clear the field of view of debris, and to open up the ureter to provide access for the scope. Successful irrigation is determined by properties of the ureteroscope and the operating room set-up. Mathematical modelling can be used to predict the impact of different ureteroscope designs, as well as the effect of working tools (laser fibres and baskets), on flow.

Methods: We constructed a mathematical framework, based on systematic reductions of the Navier Stokes equations, to relate pressures and volumetric flow rates throughout the urinary system during ureteroscopic procedures with straight and curved ureteroscopes, with and without working tools. We focused on analysing how working tools and the extent of scope deflection affect the flow rate of irrigation fluid through the ureteroscope. We validated these theoretical models via comparison with wet-lab experiments. Additionally, we considered how having elliptical, instead of circular, cross-sections for the working tools and channel would affect the flow of irrigation, and also modelled the effect of varying the position of the working tool within the channel. We also analysed the stability of the position of the working tool within both circular and elliptical channels.

Results: The model for flow rate in a straight and deflected flexible ureteroscope demonstrated an excellent correlation with the wet-lab findings. The impact of working tools on flow can also be accurately modelled, when considering the working tool to lie non-concentrically within the channel. Elliptically shaped channels and tools lead to higher flow rates under certain conditions.

Discussion: Mathematical modelling is applied in many industry settings to improve efficiency. Mathematical models can be used to understand the impact of using different ureteroscopic equipment, to optimise the use of existing equipment and to guide the design of the next generation of instruments.

Using dynamic flux balance analysis to predict non-steady state behaviour in continuous metabolic models

Matthew Coleman[†]
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14:30 on Wednesday in LTE

Flux balance models have long been used to analyse steady state behaviour in metabolic networks. By enforcing mass balance constraints and using linear programming we are able to find predicted optimal states for a metabolic network. However, these models cannot be used to observe dynamic behaviour, such as periodic solutions, or diauxic growth (growth in two phases) often observed in bacteria. Dynamic flux balance analysis (dFBA) is an extension to flux balance analysis that allows us to model the dynamic behaviour of fluxes in metabolic networks, and also provides estimates on metabolite concentrations.

We apply dFBA to a metabolic network for Clostridium autoethanogenum. This network has previously, depending on the relative levels of uptake rates, been seen to exhibit a range of states with linear and sublinear growth in one or more metabolites. By applying dFBA to this network under the same range of uptake rates, we observe similar growth patterns in metabolite concentrations. This allows us to consider which dynamic behaviours are intrinsic to the network, and which may be more related to specific choices for how we model reaction rates in more complex models.

Modelling cell heterogeneity using multi-dimensional population balance equations

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14:50 on Wednesday in LTE

When considering cell culture, many models assume a homogeneous mixture of identical cells, which can be modelled by a system of ODEs. However, if this assumption is relaxed, a different approach must be used. The most rigorous way to model heterogeneous cells is by using population balance equations. This framework considers cells to be distributed by one or more internal variable, such as cell volume or age. Thus, the progression of the cell through the cell cycle by growth and division is considered as movement through an internal state space. The population balance equation describes this process, which results in an integro-partial differential equation for the number density of cells with a given internal state.

While one dimensional population balance models for cells are relatively well understood, multi-dimensional models for cells have not been well explored, in part because they are mathematically complex and very computationally expensive to solve. However, it is often useful to consider cells that are heterogeneous in several respects – for example, cell volume, DNA content and protein content. In this work, we have studied two-dimensional population balance equations with differing single cell models, using both numerical and analytical methods. Slight changes in the structure of the single cell model are found to have significant effects of the shape of the number density function, and thus on the extent of heterogeneity for each internal variable. These results should generalise to higher dimensions, thus giving information about the heterogeneity of a cell population without having to compute the full solution.

Mathematical study of active contraction in myocyte and left ventricle from health to non-ischemic dysfunction

Hao Gao[1†]
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15:10 on Wednesday in LTE

Cardiovascular disease is one of the leading causes of death worldwide, in particular, myocardial dysfunction, which may lead to heart failure eventually. Understanding the electro-mechanics of the heart function from healthy state to failing state will help in developing effective clinical treatments of myocardial dysfunction, and preventing heart failure. In this work, we present a multi-scale electro-mechanics model of myocardial active contraction.

We first studied the electro-mechanics coupling in a single myocyte in the healthy and failing states, and then the single cell model was embedded in a dynamic left ventricular (LV) model to investigate the compensation mechanism of LV pump function due to the non-ischemic dysfunction for the first time. The Holzapfel-Ogden constitutive law was used to describe the passive myocardial response at the tissue level, a modified Grandi-Pasqualini-Bers model was adopted to model calcium dynamics of the individual myocytes, and the active tension was described using the Niederer-Hunter-Smith's myofilament model. The multi-scale LV model was implemented using a hybrid immersed boundary method with finite element elasticity. The predicted myocardial dynamics of both the healthy and diseased LV models is consistent with the clinical measurements and observations. In particular, we found that a low level of intracellular Ca2+ transient in individual myocytes can result in a LV pump function failure despite the adaptations that the heart takes to maintain the stroke volume. These adaptations include an increased myocardial contractility, decreased systolic blood pressure, and increased diastolic filling pressure.

In conclusion, our work suggests that treatments targeted at increased contractility or lowering the systolic blood pressure alone are not sufficient in preventing LV pump failing. The more effective strategy may be to restore a balanced physiological Ca2+ handling mechanism in the myocytes.

Gao H., et al, Dynamic finite-strain modelling of the human left ventricle in health and disease using an immersed boundary-finite element method. IMA Journal of Applied Mathmatics, doi:10.1093/imamat/hxu029
Funded by British Heart Foundation (PG/14/64/31043) and EPSRC (EP/N014642/1), NNSF China (11471261, 11101333) Type of presentation: Talk

Asymptotics

Asymptotic analysis of solutions to transmission problems in solids with many inclusions

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13:30 on Wednesday in LTF

We construct an asymptotic approximation to the solution of a transmission problem for a body containing a region occupied by many small inclusions. The cluster of inclusions is characterised by two small parameters that determine the nominal diameter of individual inclusions and their separation within the cluster. These small parameters can be comparable to each other. Remainder estimates of the asymptotic approximation are rigorously justified. Numerical illustrations are presented that demonstrate the efficiency of the asymptotic approach when compared with benchmark finite element algorithms.

A functional analytic approach for singular perturbation problems in perforated domains

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13:50 on Wednesday in LTF

In this talk, we present the so-called "functional analytic approach" for the analysis of the asymptotic behavior of the solutions of singular perturbation problems in perforated domains. Such an approach, based on integral equations and functional analysis, allows to convert for example a (linear or nonlinear) boundary value problem in a domain with a small hole of size ϵ into a system of ϵ-dependent integral equations, that can be then analyzed by means of the implicit function theorem. The final goal is to represent the solutions in terms of power series expansions of the singular perturbation parameter or of known functions of it. Moreover, the coefficients of these series can be explicitly computed by means of the solutions of recursive systems of integral equations. Some applications of such method to the effective properties of dilute composite materials will be described.

Asymptotic analysis of a silicon furnace model

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14:10 on Wednesday in LTF

Silicon is produced from quartz rock in electrode-heated furnaces by using carbon as a reduction agent. We have developed a full continuum model for the time evolution of chemical concentrations, gas partial pressures, velocity, and temperature within a one-dimensional vertical section of a furnace. In this talk we present a simplified model, in which we find the chemistry and temperature in the furnace are strongly coupled. This model depends on diffusion, an endothermic reaction, and the external heating input to the furnace. A free boundary problem arises and we examine a suitable asymptotic limit of the system. We also comment on how the findings may be relevant for the operation of silicon furnaces.

Complexity asymptotics

Bwebum C. Dang[1†]
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14:30 on Wednesday in LTF

We are developing a new method called complexity asymptotics to efficiently solve complex problems such as the non-linear Navier-Stokes flow with a free surface and exterior domain. The method linearises the far-field which is represented by a boundary integral distribution of Green's functions giving a far-field asymptotic. The near-field non-linear complex behaviour is modelled by a radial basis function domain distribution of these Green's functions. The idea is that this distribution is negligible in regions where the non-linearities in the flow are negligible and so amenable to mesh adaptation and efficient solution. We start with investigating a test problem of two-dimensional flow past a circular cylinder.

Demystifying the G-spot in contact mechanics

Alan Champneys[1†]
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14:50 on Wednesday in LTF

The so-called G-spot is a singularity that occurs in contact mechanics that underlies the so-called Painleve paradox in contact mechanics of rigid bodies with friction, where there is sufficient coupling between normal and tangential degrees of freedom. The canoncial example is that of the instability that happens when chalk is pushed rather than dragged across a blackboard. We use multiple scale asymptotic expansion in the presence of a small parameter that regularises the rigid contact into a stiff elastic one. The dynamics in the inner region is found to feature a canard-like trajectory in some cases, perturbations from which are found to be represented by a certain generalised hypergeometic function. The asymptotic behaviour of this function for large argument is found to determine whether continuation from the G-spot is via direct lift-off or via a so-called impact without collision in which there is an O(1) change in contact velocity over an infinitesimal time interval. The results are backed up by numerical computations on both a simple toy example and on a frictional impacting mechanism.

Novel applications

Experimental characterisation and modelling of an aeroelastic energy harvester

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13:30 on Wednesday in 24AA04

Aeroelastic energy harvesters are a promising technology for the operation of wireless sensors and microelectromechanical systems [1], as well as providing the possibility of harvesting wind energy in applications were conventional wind turbines are ine ective, such as highly turbulent ows [2]. The development of aeroelastic energy harvesters to date has focused mainly on the utter of airfoils, the galloping of prismatic structures and vortex induced vibrations as a means to generate energy. The work presented relates to the development of a novel form of aeroelastic energy harvester sharing characteristics with both utter based and galloping energy harvesters. The dynamics of a prototype device have been characterised with the use of a 6 axis accelerometer gyroscope. A lumped parameter mathematical model has been developed and is compared to experimental results.

[1] A. Abdelke , International Journal of Engineering Science 100, 112 (2016).

[2] J. McCarthy, S. Watkins, A. Deivasigamani, S. John, and F. Coman, Journal of Wind Engi- neering and Industrial Aerodynamics 136, 101 (2015).

The security market line and short sale constraints

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13:50 on Wednesday in 24AA04

In finance literature it is known that on a financial market in which short-selling of risky assets is restricted, the market portfolio is not typically efficient (see e.g. Fama and French (2004)). This work analyses two different kinds of regulatory policies of short-sales on financial markets in which asset returns are given by an exogenously generated stochastic process. The first constraint allows investors short-selling of a reasonable size whereas the second prohibits short-selling of any size. For both policies a modified market portfolio is introduced. These portfolios are mean-variance efficient with the highest possible Sharpe ratio and satisfy the corresponding short-sale constraint. For both cases a time-dependent security market line is implemented which allows for a decomposition into systematic and non-systematic risk. It is shown that, from a viewpoint of a regulator, the restricted short-sale constraint is a more suitable regulatory policy. In contrast to the prohibited short-sale constraint, the restricted short-sale constraint, does not reduce the possibility of diversification. Hence, the portfolio return in this case has no diversifiable risk, a higher Sharpe ratio and a smaller coefficient of variation, which yields a smaller default risk, provided the return process is multivariate elliptically distributed.

Local and global dynamics in a discrete time growth model with VES production function

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14:10 on Wednesday in 24AA04

In this work we analyze the local and global dynamics shown by the discrete time neoclassical one-sector growth model with differential savings while assuming a Variable Elasticity of Substitution (VES) production function. We prove the existence of fluctuations and complex dynamics when the elasticity of substitution between production factors is lower than one and shareholders save more than workers. We show that the model can exhibit unbounded endogenous growth and multistability phenomena. Moreover we demonstrate that, if workers save more than shareholders, then the growth path is bounded from above and the boundary is independent from the saving rate of shareholders.

Reduced-order approximation of consensus dynamics in networks with hybrid adaptive communication protocols

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14:30 on Wednesday in 24AA04

This talk will focus on a hybrid decentralised adaptive control (or communication protocol) that is used to drive a network of agents toward consensus. A network is a weighted graph, that is, a set of elements called nodes or vertices, which may be connected to one another via relational links (edges). To each node we assign a state and to each edge a weight (or gain). We want our states and gains to evolve over time until consensus is achieved. Consensus occurs when our node states evolve to a common value.

The state and gain evolutions are governed by a system of coupled differential equations.

We develop an approximation to the system to capture the qualitative behaviour of the most distant node state and gain dynamics, collapsing a system of order N^2 to one of 3 differential equations, thus serving as a powerful reduction of order of the larger system. Numerical evidence is presented to argue the robustness of the approximation over a range of differing initial network sizes and parameter configurations.

Computing contact angles from molecular dynamics simulations of nanodroplets

Srikanth Ravipati[1†]
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14:50 on Wednesday in 24AA04

With the computational tools available today it is straightforward to simulate using Molecular Dynamics (MD) a sessile nanodrop on a substrate. However, estimation of the contact angle by e.g. analysing the shape of the nanodrop, remains ambiguous. The most commonly employed methods either use diverse choices of parameters to analyse the interface profile, different interface profile fitting functions or require a large number of particles. To address these problems, we propose a novel geometry-based methodology together with an appropriate coarse-grained description of the fluid density in an appropriate MD framework. To validate our method, we estimate the contact angle of Lennard-Jones (LJ) and SPC/E water nanodroplets on planar LJ walls and compare them with Young contact angle values computed employing test-area perturbation method. Our method allows us to estimate accurately the Young contact angle with only a few thousands of particles.

Hydrodynamic instabilities and pattern formation in active cholesterics

Carl A. Whitfield[1†]
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15:10 on Wednesday in 24AA04

Active liquid crystals have come to represent an archetype for active matter, offering a framework for organising ideas about biological processes and biologically inspired materials. In this talk I will describe the basic properties and consequences of introducing active stresses, with principal direction along the local director, in cholesteric liquid crystals.

Cholesteric textures are found in a wide range of biological systems such as dinoflagellate chromosomes, the carapaces of insects and crustaceans, fish eggshells and compact bones. Given these examples, it is natural to expect that active stresses in cholesteric phases should have broad relevance to a diverse range of biological and biologically inspired materials. Moreover we are motivated by the possibility constructing active cholesterics in vitro.

We find that the helical ground state is linearly unstable to extensile stresses, without threshold in the limit of infinite system size, whereas contractile stresses are hydrodynamically screened by the cholesteric elasticity to give a finite threshold. This is confirmed numerically and the non-linear consequences of instability, in both extensile and contractile cases, are discussed. We also consider the stresses associated to defects in the cholesteric pitch (lambda lines) and show how the geometry near to the defect generates threshold-less flows reminiscent of those for defects in active nematics. At large extensile activity lambda lines are spontaneously created and can form steady state patterns sustained by constant active flows.

Posters

Osmotic effects on membrane bound organelles

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17:30 on Monday in University Hall

Membrane bound organelles are present in all eukaryotic cells. Understanding their morphology and dynamics is important due to their relevance in many medical pathologies, e.g. Parkinson's and Alzheimer's. Using ideas from fluid dynamics and statistical physics we examine the effects of osmotic stress due to ion pumps on the dynamics of the contractile vacuole complex, an organelle regulating volume in freshwater algae; and to understand the dynamics of mitochondria fission due to osmotic shock.

Theoretical analysis of a fractal ultrasonic transducer

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17:30 on Monday in University Hall

The dominant technology that is used to ensure that safety critical structures such as nuclear plants, aircraft, and oil pipelines are safe and free from defects is ultrasound imaging. Of course ultrasound imaging also plays a vital role in medical imaging and therapies. A periodic composite structure is used in piezoelectric ultrasonic transducers to enhance their transmission and reception sensitivities. This structure has one principal length scale which, due to the resonant nature of the device, determines its operating frequency. Inspired by nature's ultrasound users (insects, bats, dolphins, for example) there is a drive to create multi-frequnecy (that is, wide bandwidth) devices. As these are resonant devices then it follows that resonators that span a range of length scales should be used. This paper describes a mathematical model which can be used to predict the dynamics of a fractal ultrasound transducer whose piezoelectric components span a range of length scales. The Sierpinski gasket is the fractal design used in this study but importantly we use the complement of the standard Sierpinski gasket. The electrical and mechanical fields that are within this structure are expressed in terms of a finite element basis. A renormalisation approach is then used to calculate a solution to this model from the discrete matrices that arise. The principal performance measures of the fractal ultrasonic transducer are then compared to an equivalent standard device. It transpires that the fractal device has a significantly higher reception sensitivity (18 dB) and a significantly wider bandwidth (3 MHz). So much so that this work has led to the manufacture of the world's first fractal ultrasonic transducer.

Hybrid asymptotics-numerical method for efficient simulation of excitable system

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17:30 on Monday in University Hall

Waves propagating in excitable systems such as cardiac tissues are typically modeled by reaction-diffusion equations including one or several small parameters. One the major problems in the numerical simulation of excitable waves is the difference in time and space scales of the waves in the medium. The time scale of excitation is orders of magnitude faster that the time scale of recovery. For this reason, full simulations of excitable waves such those arising in the connection detailed ionic models of cardiac electrophysiology are prohibitively expensive and time-consuming. Following Barkley [Physica D, 49, 61-70, 1991], we propose a semi-implicit numerical scheme for efficient simulation of excitable systems. In particular, the method has the potential to significantly speed-up the simulations of the model at small computational cost without neglecting their excitability characteristics. The numerical method is compared with standard numerical schemes such as the explicit Euler method. We next extend the method for one and two spatial variations and observe the formation of spiral waves in finite domains.

Lift-off of a solid object from a horizontal rigid surface

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17:30 on Monday in University Hall

We study conditions under which a solid body lying at rest upon a horizontal rigid surface can be removed from the surface by fluid forces and we also study the ensuing motion. The criteria for lift-off of a thin body from the surface are a main issue to be addressed as part of the general picking-up of the body and transporting it. A nonlinear theoretical study is presented on fluid-solid interaction in which the motion of the body and the unsteady two-dimensional motion of the inviscid fluid interact with each other. Once the fluid flow starts up, the solid object can lift off almost immediately from the surface due to fluid forces. Another contribution concerns the influences of thickness and various shapes of the body, which lead to a different and more general form of lifting off. The investigation involves numerical and analytical studies and comparisons over small, order unity and large time scales. The body can return to the surface or fly far away.

Stability of a non-Newtonian fluid in the boundary layer

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17:30 on Monday in University Hall

The mechanical behaviour of many real fluids is well described by the Navier-Stokes theory. This theory is based on the assumption of a Newtonian constitutive equation. More specifically, the extra-stress tensor can be expressed as a linear, isotropic function of the components of the velocity gradient. Many common fluids, such as water and air can be assumed to be Newtonian. However, many rheological complex fluids such as polymer solutions, soaps, blood, paints, shampoo, ketchup are not well described by a Newtonian constitutive equation, which does not take into account any relaxation and retardation phenomena. Viscoelastic fluids are examples of non-Newtonian fluids, they exhibit both viscous and elastic properties when undergoing deformation. Elasticity is the tendency of the material to return to its original shape once the external force is removed. One of the first class of material models proposed consists of fluids of differential type. The fact that this class of fluids cannot take account of stress relaxation may suggest that they are suitable to model materials where relaxation effects are insignificant, such as slurry fluids and fluids involved in the food industry. We consider a subclass of differential type fluids known as Rivlin-Ericksen fluid of second grade. The aim of the research is to understand the stability behaviour of such fluids in boundary layers. Specifically, a simple two-dimensional configuration of a flow over a semi-infinite wedge is considered. A linear stability analysis leads to a modified Orr-Sommerfeld equation that is solved by a Chebyshev collocation method. The numerical experiments conducted show that elasticity in this model has a stabilising effect.

Locality of matrix functions

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17:30 on Monday in University Hall

It is well known that if a biinfinite matrix $A(i,j)$ with $i,j \in \mathbb{Z}$ is *exponentially localised*, i.e. $|A(i,j)| \leq C \exp(-\gamma\,|i-j|)$ for some $C,\gamma > 0$, and f is a function analytic in a neighbourhood of the spectrum of A, then $f(A)$ is again exponentially localised but with a different rate $\tilde{\gamma} > 0$. This fact is of fundamental importance in quantum mechanics, where it implies that the interactions between electrons are exponentially localised and enables computations scaling linearly in the number of atoms (compared to cubic scaling for the traditional approach based on eigendecompositions). In numerical mathematics, such localisation results have been used to prove local convergence of spline interpolation and motivate preconditioners based on sparse approximate inverses, among many other applications. This poster reviews several proofs of the aforementioned result and assesses them in terms of generality and sharpness of the resulting decay rate $\tilde{\gamma}$.

Mathematical modelling of a magnetic immunoassay

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17:30 on Monday in University Hall

An important stage in immunoassay design is optimizing the sensitivity of the assay's dose response to the desired analyte concentration range. The dose response is generally dependent upon surface concentrations of capture antibody used in the assay. Correctly specifying these quantities is therefore vital but may require time- and resource-consuming experiments.

We propose a mathematical model of a novel, magnetic-particle based immunoassay with the aim of using the model to predict optimal assay parameters. The model is deterministic, non-spatial and considers three distinct time regimes that correspond to different stages of the immunoassay process. The model variables are governed by coupled ODEs that use mass action kinetics to describe the antigen-antibody reactions govern. We non-dimensionalize the model and derive an analytical approximation for the immunoassay dose response by assuming a set of antibodies in the assay saturate during the incubation period.

We demonstrate that for certain parameter values the approximation and full solution are in good agreement. Dose responses produced by the model compare well with those obtained experimentally in that they replicate the 'hook' effect, a form of interference observed in immunoassays that is due to competition for capture antibodies on the sensor surface. We also identify 5 key dimensionless parameters and show that by modifying the amount of capture antibody present relative to the desired target range we can control the immunoassay sensitivity and limit of detection. This approach of mathematically modelling the immunoassay process therefore shows promise as an effective method for efficiently optimizing immunoassays without the need for large numbers of experiments.

Thermo-viscous effects in shaping glass vials

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17:30 on Monday in University Hall

Glass vials exhibit cracking when the pressing of the molten glass is performed too fast. We propose a model for this process, comprising two fluid phases – a fluid lubricant that separates the molten glass – and the roller forming the vial. This model yields an effective Navier-like slip boundary condition and admits an interesting family of physical solutions. A weakly coupled thermal flow produces an effective convective-heat-transfer boundary condition. The fluid and thermal flows are then used in a quasi-steady scheme modelling the vial pressing to analyse the shear stress exerted on the glass as a function of the operating speed.

Linking physiologically based pharmacokinetics, mathematical modelling and systems biology

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17:30 on Monday in University Hall

Physiologically Based Pharmacokinetics (PBPK) is the study of what the body does to a drug after it has been administered, by considering physical, chemical and physiological factors. We aim to see how mathematics can be used to model the concentration of a general drug solute as it moves through organs within the body. Once we know how to model the organs individually, we can then integrate these models into a larger systems model that can be implemented using PBPK software. To begin with we are looking to model the kidney as a whole by understanding the processes on a small scale and seeing how these can be integrated into a whole-kidney model.

Diffusion mediated bubble growth

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17:30 on Monday in University Hall

Extrusion in cereal manufacturing is the process of mixing a starch based powder with water at high temperatures/pressure, which are induced by shearing the mixture. This mixture is then pushed out of the extruder through a "die" where it experiences a sharp pressure drop. Simulating the "flash", which occurs in specific extrusion processes as dissolved gas vaporises within the "extrudate mixture", poses significant modelling and numerical difficulties. The flash is characterized by the rapid change in volume of the mixture as it exits an extruder. A major contribution to the total volume change comes from the nucleation, and subsequent growth, of bubbles in the extrudate, as the mixture becomes supersaturated near the end of the extruder.

Understanding the growth of bubbles, and the dependence of this growth on the mixture components (e.g. the amount of dissolved gas), is imperative in constructing a full model for the flash of extrudate leaving an extruder. This work looks at a model for bubble growth which incorporates the diffusion of heat and dissolved gas around the liquid surrounding the bubbles, as well as the rheological properties of the extrudate. Suitable reductions to this model are made based on the typical size of the dimensionless parameters in a typical extrusion setup. Finally, the small time asymptotic solutions for all the relative variables are found, as well as numerical solutions. The consequence being that, if a given external pressure function is applied, the time dependent evolution of the bubble can be determined.

Investigating cell mechanics with atomic force microscopy

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17:30 on Monday in University Hall

Mechanical properties of cells and their extracellular matrix (ECM) play important roles in many macro-scale biological and pathological processes including cell migration and myocardium infarction. Fibroblasts are known to excrete components that make up the ECM, such as collagen, giving rise to an architectural framework or 'scaffold' that supports the cells and are found to be a key component in cancer tumor growth and invasion. The exact function of fibroblasts during metastasis is unknown. However, it is thought that an insight into the mechanical properties of this cell type can offer opportunities in therapeutics to the control the growth and migration of cancer. In recent years, atomic force microscopy (AFM) has been established as a popular tool to measure the mechanical stiffness of fibroblast cells. However, most of the measured data are interpreted using the classical Hertzian law, which applies for linear elasticity only. In this study, fibroblast cells are indented with a spherical AFM tip, and the applied forces are measured from the bending of the AFM cantilever. The cells are adhered to multi-layers of collagen gel and/or glass substrates with different depths. Finite element analysis will be used to quantify the cell anchorage dependence on the stiffness and structure of substrates. Both linear and nonlinear composite continuum models are used to describe the cellular behaviours, and we also discuss a homogenization method to upscale a single cell by overall quantities.

"Mapping the battleground": A systems biology approach to modelling plant-microbe interactions

Lydia Rickett[1†]
Silke Robatzek[1], Richard Morris[2], Jürg Felix[3], Justin Lee[4], Fabian Trempel[4], Jospeh McKenna[5], Julia Vorholt[6] & Frank Menke[1]

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17:30 on Monday in University Hall

Disease-causing bacteria present a major threat to the agricultural industry, and by extension to global food security. But what ammunition does a plant have to detect and defend against these bacteria? Pathogens may be identified through tell-tale bacterial molecular patterns, and plants have a whole host of tools to perceive and respond to these signals. We will introduce a probabilistic model which uses systems biology approaches to map out the major components of this complex process.

Comparison with biological experiments shows that this model successfully predicts some conditions under which the system of perception breaks down, a phenomenon that has been noted, but previously unexplained in the literature. Ongoing work is focussing on building in additional key components, including multi-species perception and the potentially negative side-effects of an efficient bacterial perception system on plant growth.

Particle and fluid dynamics within a blender

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17:30 on Monday in University Hall

Have you ever forgotten to replace the lid of the blender before beginning to purify your mango and passion-fruit smoothie? If you have, you'll have witnessed the catastrophic explosion of fruit and yoghurt flung haphazardly around the kitchen in an unpredictable manner. This is consequential of the complicated and turbulent fluid dynamics present within the machine, the exact behaviour of which is unknown. Sharp, angular blades rotate at extremely high speeds to mix and chop the fruit into a purée of particles that are as small and uniform in size as possible. But what characteristics of the blender are responsible for the outcome? While experimental evidence gives intuition into blade and vessel design, along with operational parameters such as speed and blend time, there is a knowledge gap surrounding the precise impact on the particle and fluid dynamics. How interdependent are the chopping and mixing mechanisms? What determines the minimum particle size and how can it be lowered?

This seemingly simple, user-friendly appliance is governed by complex and intricate mathematics. In this poster I present a predictive model for the resulting particle size distribution of a smoothie, based on a system of integral ordinary differential equations inspired by Becker-Döring theory. The result: knowledge of the precise operating regime that will create the finest, most homogeneous purées in the most efficient manner.

Silicon anodes in lithium batteries

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17:30 on Monday in University Hall

Lithium-ion batteries are becoming increasingly common in everyday life with mobile phones and laptops being essential to everyday life and electric cars coming into the mainstream market. The demand for high-power and long-lasting batteries has caused vast research to be put into finding new materials for anodes and cathodes. One promising anode material is silicon, as this has the largest volumetric and gravimetric capacities for lithium ions currently known. However, upon lithiation, silicon increases volume by up to 300%, causing high stresses on the battery as well as cracking in the anode itself. Many anode material companies, including Nexeon (with whom this project is in collaboration), have developed methods of mitigating this expansion and reducing the stresses. These include using micro-scale particles (known as secondary particles) or nanofibres to decrease the stresses and using graphite shells around a silicon core to constrain the expansion. We look to mathematically model this expansion using solid mechanics to predict the behaviour of the anode using different designs of secondary particles. This will potentially shed light on an optimal anode design for future lithium-ion batteries.

We use a homogenisation technique to relate the expansion in a secondary particle to the expansion of the whole anode. We extend the current model by Chakraborty, Chapman, Goriely and Please (who worked with unimaterial designs) to analyse the use of silicon core/graphite shell structures and investigate the effect of different core sizes, central voids and porosity. We find that the graphite is too weak to constrain solid silicon, which appears to expand freely. Further, when a central void is used in the middle of the silicon core, the anode material continues to expand outwards as opposed to inwards, thus ignoring the void. Finally, we investigate designs using porous silicon layers and find that the expansion can be slightly mitigated without sacrificing the high capacity of silicon.

Multi-parametric analysis of a contact problem for a coated elastic half-space in case of high contrast material parameters

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17:30 on Monday in University Hall

A contact problem dealing with the indentation of an elastic half-space coated with a thin layer is investigated. The consideration is focused on the effect of contrast in material properties of the layer and the substrate. The two natural parameters arising in this problem are ratio of the thickness of the coating to typical radius of contact, which is assumed small, and the ratio of stiffnesses of the layer and the substrate, being large or small for relatively hard or soft coating layer, respectively.

A multi-parametric analysis is established using straightforward asymptotic integration, revealing a full classification relying on these two small parameters. 2D exact solution is derived for the indenter of a sinusoidal shape to verify the asymptotic behavior of stresses and strains with asymptotic results. Numerical example is presented for paraboloid indenter shape for a relatively soft and hard layer.

Modelling low-current discharge of lead-acid batteries

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17:30 on Monday in University Hall

One of the greatest challenges in developing renewable energy sources is finding an efficient energy storage solution to smooth out the inherently fluctuating supply. One cheap solution is lead-acid batteries, which are used to store off-grid solar energy in developing countries. However, modelling of this technology has fallen behind other types of battery; the state-of-the-art models are either overly simplistic, fitting black-box functions to current and voltage data, or overly complicated, requiring complex and time-consuming numerical simulations. Neither of these methods offers great insight into the chemical behaviour at the micro-scale.

In our research, we use asymptotic methods to explore the Newman porous-electrode model for a constant-current discharge at low current densities, a good estimate for real-life applications. In this limit, we obtain a simple yet accurate formula for the cell voltage as a function of current density and time. We also gain quantitative insight into the effect of various parameters, such as electrode porosity or initial electrolyte concentration, on this voltage. Further, our model allows us to quantitatively investigate the effect of ohmic resistance and mass transport limitations, as a correction to the leading order cell voltage. Finally, we explore the effect on cell voltage of other secondary phenomena, such as growth of a discharge-product layer in the pores and reaction-induced volume changes in the electrolyte.

Modelling drug release from polymer-free drug-eluting stents

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17:30 on Monday in University Hall

Polymer-free drug-eluting stents are an innovative new treatment for coronary heart disease which is the leading cause of death globally. In these polymer-free stents, the drug is either sprayed directly onto a bare metal surface or infused in a metallic porous medium. They have the potential to overcome problems associated with the current best treatment: polymer-coated drug-eluting stents. However with no polymer to control drug release, it is unclear how desired release rates can be achieved. In this poster, the first model of drug elution from polymer-free stents is presented. The generalised model is capable of predicting the drug release from a number of polymer-free systems including those that exhibit nanoporous, nanotubular and smooth surfaces. The model is based principally on dissolution theory and the theory of diffusion in porous media. Analytical solutions are derived to determine the important parameters that control the release rate. Drug release profiles are also provided, and design recommendations are offered so that the release profile may be tailored to achieve the desired outcome.

Critical phenomena and integrability in network models

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17:30 on Monday in University Hall

Mathematically, networks models are defined as set of nodes and relative edges (links between nodes) with assigned statistical rules for their state. The definition of such rules induces the definition of a set of quantities (order parameters) that characterise the global state of the network. We propose an approach that treats networks as a statistical mechanical system and we analyse critical phenomena and phase transitions via a mapping of the problem into a suitable set of integrable nonlinear PDEs. In particular we show how the Curie-Weiss model for spin systems, is exactly solvable, can be formulated in terms of a network. Finally, the statistical mechanics of the networks analysed has been tested using the Metropolis-Hastings algorithm.

Modelling urban population dynamics using coupled integro-differential equations

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17:30 on Monday in University Hall

Current urban population models do not explain how population distributes within a city or the expected locations of growth and decay over longer time periods. This research seeks to identify key factors which determine such urban population distributions by using an integro-differential equation model to simulate population change in symbiosis with service provision. The assumption is that people and services benefit from close proximity but only where there is space available. This system may tend towards a spatially homogeneous distribution or a spatial pattern. Using Gaussian spatial weight kernels, linear stability analysis performed at the spatially homogeneous steady state shows that stability depends on a key function; the carrying capacity for services given a local population density. In particular, instability can only occur where the carrying capacity function is convex with respect to population density. Furthermore, this spatial instability can occur only for perturbations with a sufficiently long lengthscale. The analysis may explain how the spatial and local interactions between populations and services can drive the emergence of patterns in cities, and could predict the characteristic scale of such patterns.

Films, layers and droplets: The effect of near-wall fluid structure on spreading dynamics

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17:30 on Monday in University Hall

In this poster we present a study of the spreading of liquid droplets on a solid substrate at very small scales. One technique available to study such small droplets is to introduce a so-called effective wetting energy (or binding potential) into the governing equations, which captures the effect of density oscillations that would be present in a coarse-grained manner. Additionally surface tension effects significantly influence both steady and spreading droplets at the scales we are interested in. In particular, we focus on strong packing and layering effects in the liquid near the substrate due to underlying density oscillations in the fluid caused by attractive substrate-liquid interactions using a thin-film model and explore the effects it has on steady droplet shapes and the spreading dynamics of droplets. We modify the usual thin-film model to enable a diffusive dynamics at scales where the adsorption layer is sub-monolayer in thickness, i.e., the dynamics along the layer consists of single-particle hopping rather than the collective hydrodynamic motion implicit in standard thin-film models.

Surfactant effects on film relaxation during topological T1 process

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17:30 on Monday in University Hall

Foams have several application fields from the food sector to the medical and (bio)pharmaceutical industry. Particularly in microfluidic applications, friction with the walls of the channel through which a foam flows will have a large effect on the foam structure [1]. At the same time, the flow induces a flow of surfactant within the films [2], leading to variations in surface tension, and hence deviations from the usual geometrical equilibrium conditions known as Plateau's laws. Macroscopically, an aqueous foam acts as a viscoelastic medium, whose flow depends on its surfactants, liquid fraction, bubble size and shear rate. Durand and Stone considered the relatively simple 2D system of five films connecting four pins between two parallel sheets of glass to investigate the influence of the surface viscoelastic properties on the topological T1 process [3]. Particularly, their experiment quantified surface viscosity and Gibbs elasticity connected with the surfactant transport. Grassia et al extended this model to consider the evolution of the film lengths and the related surfactant transport in the time [4]. They suggested to compare the experimental time scale for the film growth and the growth times from model predictions to quantify the surfactant transport between adjacent films. In neither of these works was the friction with the walls taken into account; this friction is significant because it causes four of the films to bend, potentially invalidating the models. Here, we combine the viscous froth model of Kern et al [5], which takes into account the viscous friction with the bounding plates of an experiment, with a population balance model for surfactant. We start with the simpler situations of film stretching to explain our numerical method, before returning to the five-film system and exploring the effect of surfactant motion on the dynamics, for example the time taken to trigger a topological change, considering two cases: surfactant confined to a film, or surfactant that can move around three-fold vertices.

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Wave reflection and propagation at staircase boundaries in finite-difference models

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17:30 on Monday in University Hall

Finite-difference models are widely used for numerical solutions of wave propagation and reflection in various physical settings, including acoustics, oceanography, seismology, and electromagnetism. Here we investigate a standard set of equations for (acoustic) waves in two spatial dimensions, using a popular second-order finite-difference staggered grid formulation, which is known as the Yee grid in electromagnetism and the Arakawa C-grid in fluid dynamics. However, smooth boundaries in physical space are then replaced by staircase boundaries in the computational domain, and such staircases degrade the accuracy of the numerical solutions. Here this degradation is quantified analytically for two important model problems with time-harmonic waves: reflection at a wall, and propagation along a channel. For wave reflection, the complex amplitude of the reflected wave is recovered to second order in grid spacing h when the boundary is aligned with the grid, but there is a first-order phase error when the boundary is aligned at 45 degrees to the grid. However, in both cases, surprisingly the wavevector of the reflected wave is recovered exactly, for an arbitrary incident wavevector. For wave propagation, the along-channel wave speed is recovered to second order in grid spacing h when the boundary is aligned with the grid, but only first order in h when the boundary is aligned at 45 degrees to the grid. Such first-order errors inevitably degrade the accuracy of the numerical solutions in the interior of the domain, despite the second-order finite-differences used there.

Index by presenting author